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TRANSMISSION LINES
DESIGN, CONSTRUCTION, AND PERFORMANCE

TRANSMISSION LINES

DESIGN, CONSTRUCTION AND PERFORMANCE

BY

FRED C. DEWEESE

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TRANSMISSION LINES
DESIGN, CONSTRUCTION AND PERFORMANCE

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PREFACE

There has long been need for a comprehensive work dealing exclusively with the design, construction, and performance of lines for the transmission of electric power, in which the material is so presented that it will be useful to those who are interested in the subject, from beginners to more advanced engineers. The subject matter pertains to the design, construction and performance of transmission lines, and for all practical purposes, is compiled to cover the subject in that sequence. It comprises a step-by-step procedure from short-line problems requiring only the simplest of arithmetic, through long high-voltage line problems requiring involved mathematics for their solution.

The method of presentation is largely new. The equations in Chap. IV for finding the economical size of a.c.s.r. and copperweld-copper, as well as the new method for calculating sag in long spans (mean length method) in Chap. IX, were developed by the author and have not previously been published. The critical load (Chap. XV) is treated from a practical operating point of view and should be of considerable interest and value to operating engineers. In the application of the hyperbolic theory of solution, an attempt has been made to simplify the work and offer such explanations as will serve to promote more extensive use of this method of analysis. This material should be of special interest to more advanced practicing engineers and to teachers in schools of engineering. It should also provide incentive for those who are not familiar with the method to learn more about it.

The book contains several numerical examples, accompanied by their solution, which should serve as an aid in the application of the theories and methods employed.

Approximately one-third of the book, comprising Chaps. X to XIII, is devoted to field work or work closely related thereto, covering line location and survey; modern devices, tools, and equipment; and successful methods of transmission-line construction. This material is suitable for use by construction men, contractors, and others engaged in this line of work.

Where the word "utility" is used throughout this book, it should be taken to mean any organization, Govermental or otherwise, engaged in supplying electric service to the public Any expressions of opinion are those of the author.

Data and material pertinent to the design of transmission lines were furnished by manufacturers, for which credit is thankfully acknowledged. The author is indebted to R. C. Bradbury and H. H. Rodee, of the Aluminum Company of America; and F. E. Leib, Copperweld Steel Company, for checking table values and other data. He expresses gratitude to C. W. Moseley, Manager, Construction Division, R. H. Bouligny, Inc., Charlotte, N.C., for assistance in preparing the material on the subject of transmission-line construction; and to Carl H. Dunlap, Chief Engineer, Electrical Department, American School, Chicago, for reading the manuscript and for many valuable suggestions and comments.

FRED C. DEWEESE.

RALEIGH, N.C.,
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CONTENTS

PAGE
PREFACE. v
CHAPTER
I. Transmission Systems 1
II. Conductors and Circuit Comparisons 7
III. Electrical Properties of Conductors 16
IV. Short Transmission Lines. 35
V. Medium-length and Long Transmission Lines. . . . 58
VI. Mechanical Design of Transmission Lines. 76
VII. Mechanical Calculations 92
VIII. Sag and Tension Calculations. 110
IX. New Method of Calculating Sag in Long Spans (Mean Length Method) 120
X. Line Location and Survey 146
XI. Transmission-line Devices, Structures, and Structure Spotting 159
XII. Construction of Wood-pole Transmission Lines . . . 189
XIII. Construction of Steel-tower Lines 219
XIV. Performance of High-voltage Transmission Lines with Phase Control. 228
XV. Transmission of Power at Critical Load 240
XVI. Mathematical Operations and Corona-loss Calculations. 259
INDEX. 293

TRANSMISSION LINES
DESIGN, CONSTRUCTION, AND PERFORMANCE

CHAPTER I

TRANSMISSION SYSTEMS

The function of a transmission system is to provide facilities for conveying electric energy from its source of production, the generating stations, to areas in which the energy is consumed. The power-consuming areas may be considered as centralized at points where the potential is reduced to some lower voltage for further transmission and distribution or for actual consumption.

Probably no major engineering development has more served the advancement of civilization or provided to a greater degree for the comforts of life than have electric transmission systems. It would be difficult, indeed, to enumerate any everyday comforts made possible in the home, office, or factory by the manipulation of a machine or appliance in which electricity does not play a vital part. Few people realize, however, that many of these comforts would never have been possible without the great transmission systems of today.

Transmission systems in general are designed and constructed to provide capacity for transferring comparatively heavy loads throughout the system and from the system to other power-consuming areas. In order to transmit heavy loads or large blocks of power economically for any considerable distance, comparatively high voltage is required.

Transmission systems as we know them today are of comparatively recent origin. Twenty-five years ago, there were possibly not more than two or three transmission lines operating at a nominal voltage higher than 132 kv.

The older transmission systems, as a rule, did not provide a nucleus for any particular basis of design. Many of the more modern systems are the result of a race to keep step with the ever-increasing demand for system capacity. This has necessitated the rebuilding of the smaller systems to increase the voltage or the size of the conductor or both and the building of additional lines for interconnection with adjacent systems to meet the constantly growing demand for power.

Several systems are often tied together electrically, covering large areas and several hundreds of miles in distance. The frequency of the combined systems is controlled from one of the larger generating stations. Power may be transferred through the interconnecting lines from one system into the next adjoining system, where it may be consumed, thus releasing power from that second system to still a third, etc. This method of operation provides for a minimum of transmission losses and a high degree of service, and economical transmission of power is thereby attained.

When two or more systems are tied together, the generating stations of one or more of the systems may consist largely of fuel-fired plants, while those of other systems in the group may by hydro. Interconnection and parallel operation are desirable in such situations, since the top-heavy hydro systems may draw power from fuel-burning plants during times of low water. During times of high or normal water supply, fuel may be conserved by drawing power from systems with ample water-power facilities. With this method of power interchange, only a minimum of reserve generating capacity and standby stations need be maintained.

Figure 1 shows schematically a typical transmission network and indicates quite clearly the possible direction of power flow throughout the system. Generating stations are indicated by G_1, G_2, and G_3. The main switching and substations are indicated by S_1, S_2, and S_3. Interconnections are indicated by I_1, I_2, and I_3. The power flow may be in either direction, depending on the load setup, over each section of line in the loop with the exception of those between generating station G_2, substation S_2, and interconnection I_1, respectively. Transmission lines radiating from the network proper are indicated by the transformer stations and the short lines terminating in arrow points. Extensive transmission networks even in their simplest form are seemingly complicated. However, when one becomes familiar with the system as a whole, many of the apparent complexities become clear.

In the early days of power transmission, standardization of voltage was unheard of, and many systems grew to substantial size before the advantages to be gained by voltage standards were recognized. The necessity for standardization became clear, however, when transmission engineers and operators began to realize the need for interconnecting their systems to provide for more ample load-carrying capacity, to afford more economical and reliable service, and to relieve manufacturers of building and carrying in stock transformers and other apparatus with a wide variety of voltages. Present nominal standard receiver voltages for high-voltage transmission, for low-voltage transmission, and for distribution and rural-line service are given in Table 1.

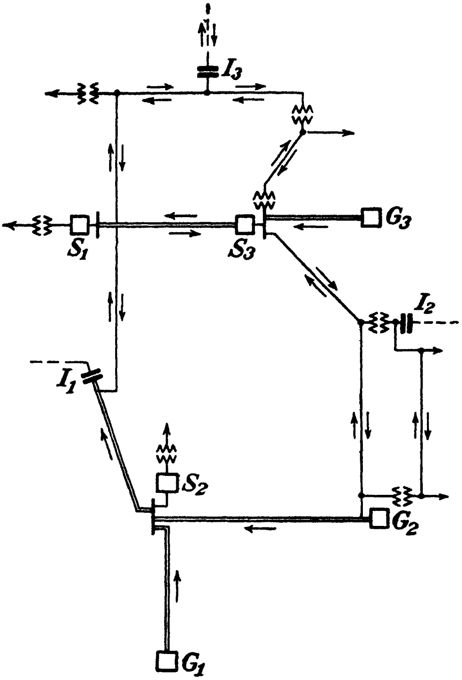


FIG. 1.—Transmission system showing generating stations, substations, and interconnections.

The lowest nominal voltage class that should in general be considered as "high-voltage transmission" is 66,000 volts. The highest voltage that should be considered "low-voltage transmission" is 44,000 volts. Three-phase 13,200 volt three-wire delta lines are used extensively for both low-voltage transmission and distribution feeders and are therefore included in both classifications. Three-phase three-wire 11,000-volt circuits are generally considered low-voltage transmission. Three-phase four-wire lines with a voltage of 6,900/11,950Y are used for both urban and rural service and are considered to be a distribution voltage. Single-phase 6,900 volts phase wire to neutral is more or less standard for rural-line service and is placed in the category with distribution systems.

TABLE 1.—NOMINAL RECEIVER VOLTAGES

High-voltage transmission	Low-voltage transmission	Distribution and rural
66,000	11,000 (3 wire)	2,400 delta
110,000	13,200	2,400/4,160Y
132,000	22,000	6,900
154,000	33,000	6,900/11,950Y
220,000	44,000	13,200

In general, under present-day conditions, any new transmission line will be either a tie line within the major system network itself or an extension from the main system. Obviously the voltage of a tie line will be fixed by the system voltage. The voltage of an extension from the system will be dictated by the transmission distance and load requirements. Since the higher the voltage the more costly the line and substations, obviously the lowest voltage that will give adequate, reliable, and economical service should be used. This is not a matter, however, that is susceptible to any general conclusion, since the solution in each particular case constitutes a problem in itself.

Usually the first information a designer receives about a pending proposition is that negotiations are under way for a proposed load at a given location, which will impose a certain additional demand on the system. At the same time, he is usually requested to make an investigation and recommend the most logical method of serving the load, including the estimated cost of making the proposed extension. If the designer can consider the extension from two or more points on the system, comparative estimates are required.

The first step in a problem of this nature is to determine from maps the approximate distance and available route or routes for a line to the site of the proposed load from possible points on the system. From the transmission distance, size, and nature of the load, a conductor size is determined that will be of ample capacity to provide adequate service at the available or desired transmission voltage.

Time is usually an element of importance, and cost estimates must often be prepared without a field inspection. If the designer is not familiar with the character of the country to be traversed, valuable information about unusual conditions and major obstructions, excessive land values, etc., can usually be secured from maps and other sources that will assist in the preparation of the preliminary cost estimates.

Estimates for the line are usually made on a cost per mile basis. It is well to keep the estimate in three groups: poles, crossarms, pole hardware, and guying material in one group; conductor, insulators, and insulator hardware in another group; pole labor, conductor labor, right-of-way clearing, surveying, and field engineering in another group. These items, plus the estimated cost of right-of-way purchase, constitute the total estimated direct cost. Tool depreciation and insurance are taken as a percentage of the pay roll.

In the design of transmission lines, it is highly important to keep in mind that the purpose of a given extension is to provide a profitable means for disposing of electric energy, the sale of which constitutes the only source of revenue for the utility. On the other hand, it should be kept in mind that the utility is engaged in a highly competitive business. Regardless of whether it is a city, community, or industrial concern, power must be supplied at a cost commensurate with that for which it may be purchased elsewhere or otherwise produced.

If the business is not profitable to the utility, the system cannot be properly maintained for long periods of time. Plant facilities may then so depreciate that it will be difficult to provide adequate and continuous service.

It is not the purpose of the author to include in this book matter dealing with the design, construction, and performance of transmission system networks as such. Calculations for extensive networks require, if the results are to approach a fair degree of accuracy, the use of a calculating board. Such methods of analysis are beyond the scope of this work. The material, however, is presented in practical and usable form, and it is hoped that the average practicing engineer may find it applicable for use in his everyday problems. The material should also be of assistance to engineering students in their studies of the power-transmission phase of their work. That part of the work dealing with the performance of high-voltage lines and the critical load should be of interest to the more advanced engineer who is concerned with electric-power transmission.

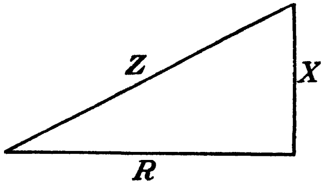
Since problems of a special nature and materials that may be considered as non-standard require special handling, an effort has been made to confine the discussions to problems and materials with which the engineer is likely to come in daily contact. An effort has also been made to include data and information that are not generally otherwise available, especially in the form presented here.

CHAPTER II

CONDUCTOR AND CIRCUIT COMPARISONS

The selection of a transmission-line conductor should be governed to a certain extent by the nature of the load to be served. If the load is temporary, for example, like the power supplied to a construction project, it will generally be found feasible to install a smaller conductor than for a more permanent load.

The ohmic resistance R and inductive reactance X combine vectorially to form the impedance Z of the circuit. When the load is inductive, the impedance will oppose the flow of current and produce what engineers know as "voltage drop."



Since the voltage at the receiving end of the line must be maintained at a reasonably constant and predetermined value, it follows that, in order to compensate for the voltage drop, or the voltage absorbed by the impedance of the circuit, the voltage at the sending end of the line, under load conditions, must be maintained at a higher value than at other points along the line. When the load is temporary, voltage requirements are not generally so exacting, and it is therefore usually permissible to use a smaller conductor. For copper conductor No. 2 and smaller and also for aluminum cable steel reinforced (a.c.s.r.) 1/0 and smaller, at normal spacing, the ohmic resistance will generally exceed the inductive reactance (see impedance triangle above).

Overhead transmission lines are sometimes subject to severe ice and wind loads, and the conductor must be of ample mechanical strength, with a predetermined factor of safety, to withstand the resultant stresses imposed by such loads. The severity of ice and wind loads will depend to a large extent on the section of the country in which the line is located. Loading districts are discussed in Chap. VI.

When the load is permanent and may be expected to have a normal growth, the electrical properties of the conductor should be such that at maximum load the impedance will not produce a voltage drop under initial conditions greatly in excess of about 6 percent of the delivered voltage. This will provide a line of adequate capacity to permit an increase in load of approximately 100 percent without excessive voltage drop. Voltage drop between 10 and 15 percent can usually be compensated for by means of voltage taps in the transformers. Drop in excess of about 15 percent requires additional line capacity. This may be obtained by increasing the size of the conductor, raising the voltage, correcting the power factor, or building another line. When it is necessary to build an additional line, it should generally follow a different route from that of the first line; because when lines are closely paralleled, trouble on one line often affects both lines.

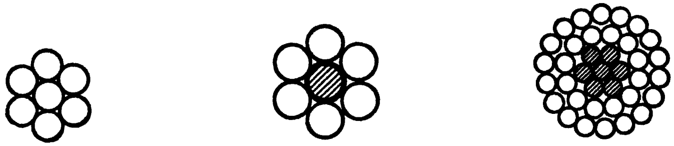
It is generally not advisable to increase the size of conductor in order to reduce voltage drop, since the results to be obtained are usually negligible compared with the expenditure required. The most logical method of obtaining increased load-carrying capacity is almost always to increase the voltage.

There are three types of conductors that may at present be considered more or less standard for transmission lines. These are hard-drawn (h.d.) copper, copperweld-copper, and aluminum cable steel reinforced (a.c.s.r.). There are of course other types, such as copperweld, Amerductor, and various types of hollow conductor. Hollow conductors are usually built up of copper to obtain maximum over-all diameter with a minimum amount of conductor material. The special built-up types are used mainly for very high voltages and at high altitudes, where the effect of corona[1] may be the determining factor. The discussion here will be confined principally to the first three mentioned types, cross sections of which are shown in Fig. 2.

Copper Conductors.—Hard-drawn copper of standard stranding, 97.3 percent conductivity, is used extensively for transmission-line conductor for voltages up to and including 110 kv.

[1] See Chap. XVI.

Copper has the advantage of maximum transmission capacity for a given power loss and voltage drop. The relatively small diameter of copper with standard stranding as compared with its load-carrying capacity presents a minimum of projected area to wind loads. This gives a higher factor of safety for the poles or structures and necessitates only a minimum of side guying against transverse loads. Because copper is a homogeneous material and follows definite laws as regards sagging characteristics and temperature coefficients, sag calculations for any assumed or known conditions are relatively simple.



4/0 Copper 7 strand 0.522 in. diameter

4/0 Copper equivalent copperweld-copper 6 strand copper, 1 strand copperweld steel 0.55 in. diameter

4/0 Copper equivalent a.c.s.r. 30 strand aluminum, 7 strand steel

FIG. 2.—Cross section of equivalent sizes of copper, copperweld-copper and a.c.s.r.—full size.

On the other hand, when the line voltage is relatively high or when the line is at extremely high elevations, the effect of corona may be the determining factor, and copper of standard stranding will usually be uneconomical. Excessive weight is required to provide a sufficient diameter to prevent corona formation. Because of its comparatively low ratio of strength to weight, copper necessarily requires greater sag for a given factor of safety. In addition, comparatively high structures or short spans must be used to provide adequate ground clearance at high temperature or at maximum load.

Copperweld-copper.—Type F seven-strand copperweld-copper, consisting of six strands of hard-drawn copper and one strand of copperweld steel, has a ratio of strength to weight of 30 to 35 percent in excess of that for the equivalent size of copper. This permits of higher tensions than for copper, and consequently of longer spans for a given ground clearance and height of structure. Further, for an equivalent size, the diameter is slightly greater, thus making it available for use at higher voltages and higher elevations. The copperweld-steel core is depended on for a small amount of conductivity (about 5 percent), which permits of a minimum weight for a given equivalent conductivity.

For equivalent conductivity, the weight of copperweld-copper is between 8 and 9 percent more than for copper. The cost, at the same unit price, for the extra weight offsets to a certain extent the savings made possible by the use of longer spans. The projected area exposed to transverse loads is also somewhat greater than for the equivalent size of copper.

A.C.S.R.—Aluminum cable steel reinforced is universally adaptable for use as a transmission-line conductor. It has a ratio of strength to weight about double that of copper and about 1.6 times that of copperweld-copper. As a result of this high strength, minimum sag is required, and shorter poles may therefore be used for a given span length. The diameter is comparatively large for a given capacity, thus making it available for use at high voltages and at high elevations.

To absorb stresses at or near the insulator support, armor rods are installed. The armor rod not only absorbs the shock due to conductor vibration but also serves to prevent burning of the strands from insulator flashover. Vibration dampers, for economic reasons, are generally used on the larger sizes of conductor. Armor rods and vibration dampers are standard equipment for a.c.s.r. They are used for overhead ground wires and to a limited extent for other types of conductor.

A.c.s.r. has the disadvantage, however, of having greater projected area exposed to wind loads, which may in some cases require the use of a pole of higher class to give the required factor of safety.

In all probability, the sizes most commonly used for transmission-line conductor for voltages up to and including 110 kv are 2/0 copper, 2/0F copperweld-copper, and 4/0 a.c.r.s. Since these sizes are equivalent in conductivity, their relative mechanical properties are compiled in Table 2.

Frequency.—Frequency throughout the United States is now practically standardized at 60 cycles. Some of the older systems are still operated at 25 cycles, and on the west coast there is one large system operating on a frequency of 50 cycles. If constants for 25 and 50 cycles or for any other frequency are not available, any given line may readily be converted to its equivalent length at 60 cycles. Assume, for example, a 25-cycle line, 4/0 copper, having a length of 150 miles. The equivalent 60-cycle length = $^{25}/_{60} \times 150 = 62.5$ miles. The 25-cycle resistance is given as 0.2767 ohm per mile at 25°C, from which we have $0.2767 \times 60/25 = 0.664$ ohm per mile equivalent 60-cycle resistance. After the actual length of the 25-cycle line is converted to its equivalent 60-cycle length, it can be treated in the calculations as if it were a 60-cycle line.

TABLE 2.—COMPARISON OF MECHANICAL PROPERTIES OF COPPER, COPPERWELD-COPPER AND A.C.S.R.

Size	Wt. per 1,000 ft, lb	Area, sq in.	Stranding	Ult. ten., lb	Over-all diam., in.	Ratio strength to wt.
2/0cu	410.9	0.1045	7	5,927	0.414	14.40
2/0F	446.8	0.1162	6cu/1cu-w	8,094	0.436	18.11
4/0a.c.s.r.	292.1	0.1939	6al/1st	8,420	0.563	28.82

Power Factor.—Electrical engineers dealing with problems in alternating current (a.c.) circuits are frequently confronted with the term "power factor." An understanding of the effects of power factor (pf) in a.c. circuits is essential to the analysis and design of transmission lines. It is well known that the power factor of a given circuit is the ratio of the actual power in kilowatts (kw) to the apparent power in kilovolt-amperes (kva). The apparent power in a three-phase circuit supplying a balanced load is equal to the product of volts × amperes × $\sqrt{3}$. The power in kw is equal to the product of

$$\text{kv} \times \text{amp} \times \text{pf} \times \sqrt{3}.$$

Energy Losses.—Information that is often not available is required for the determination of the energy loss in a transmission circuit with any degree of accuracy. In general, it may be said that a conductor that will give a voltage drop consistent with good design will give a fairly economical line. A given transmission line can operate most economically at only one value of load. Consequently, it may sometimes be desirable to design and construct a given line on the basis of the trend of load growth. When information is available to show the trend of load growth over an extended period of time for loads of a similar character, it may be advisable to use a conductor that will be most economical for a load that may be expected after a certain number of years. A line so constructed could be expected to give best average economy over its expected life.

There are three methods in use by which the most economical size of conductor for a given line and load may be approximated. These are as follows:

1. Mean annual current method.
2. Equivalent hour method.
3. Loss factor method.

Method 1 requires the determination of the mean annual current, which is that value of current required to flow continuously to give the same annual loss (I^2R) as the loss that actually occurs over the same period of time.

Method 2 requires determination of the equivalent hours, *i.e.*, the number of hours that maximum current must flow to produce the same power loss that would occur over the time the line is actually in service.

Method 3 is based on a loss factor that may be derived as follows: Take a load curve for a typical day in March, and compute the I^2R loss for each hour. Follow the same procedure for a typical day in June, September, and December. Multiplying the sum of the losses calculated from the four curves by 91, the number of days in each quarter of the year, will give the approximate total annual loss in kilowatt-hours (kwh). The ratio of the total loss, thus computed, to the loss if the circuit had been loaded to maximum current for every hour in the year is called the "loss factor."

Power-loss calculations are not always required in connection with transmission-line problems, but it is well for the designer to familiarize himself with methods for making the calculations in order to be able to furnish such information when desired.

Comparison of Single-phase and Three-phase Circuits.—Since practically all lines classified as transmission consist of three-phase circuits, this work, except for purposes of comparison, will be confined to the discussion of three-phase circuits of transmission voltage.

In the analysis of three-phase circuits, supplying balanced loads, it is necessary to take into consideration only one of the three conductors of the circuit, the voltage-to-neutral, and the equivalent spacing of the conductors. Except for electrified railroads, a special application, single-phase circuits are used for power transmission only in a very minor capacity. However, the comparative weights of conductors required for transmitting a given load are of considerable interest.

Probably the most satisfactory method of comparing single-phase and three-phase circuits is by the line-to-neutral method. This makes it possible to compare lines of equal length, using as a basis one wire of each circuit. The line-to-neutral voltage of a single-phase circuit is equal to $E/2$, where E is the phase-to-phase voltage. The voltage to neutral for a three-phase circuit equals $E/\sqrt{3}$.

Let R_1 and R_3 be the resistance per wire, single-phase and three-phase, respectively. For equal power and equal percentage power loss, the weight of conductor varies inversely with the square of the voltage-to-neutral and inversely as the resistance R_1 and R_3. The ratio of weight of each conductor in a three-phase circuit to the weight of each conductor in a single-phase circuit is as follows:

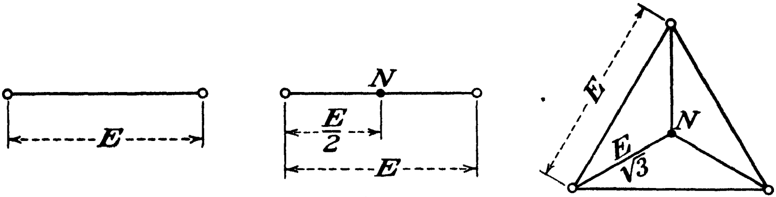
$$\frac{2}{3} \times \frac{(E/2)^2}{(E/\sqrt{3})^2} = \frac{R_1}{R_3} = 0.5 \tag{1}$$

The total weight of the three conductors in a three-phase circuit is therefore $(3 \times 0.5)/2 = 0.75$, or 75 percent of the weight of the two conductors in a single-phase circuit. The following analysis will make quite clear how these values are derived.

> *Example.* Assume a two-wire single-phase circuit, and a three-wire three-phase circuit, both of the same length, 22,000 volts between phase wires, each circuit transmitting 5,000 kw at unity power factor.

Solution. The voltage-to-neutral single-phase = 22,000/2 = 11,000 volts. The voltage-to-neutral three-phase = 22,000/1.732 = 12,700 volts. The load-to-neutral single-phase = 5,000/2 = 2,500 kw. The load-to-neutral three-phase = 5,000/3 = 1,667 kw. I_1 = 2,500/11 = 227.27 amp. I_3 = 1,667/12.7 = 131.21 amp. Assume a power loss of 5 percent of the delivered power = 0.05 × 2,500 = 125 kw single-phase; = 0.05 × 1,667 = 83,33 kw three-phase. The resistance per wire $R_1 = 125{,}000/(227.27)^2 = 2.42$ ohms. $R_3 = 83{,}333/(131.21)^2 = 4.84$ ohms. Since the weight of conductor for a given length varies inversely as the resistance, the weight per wire for the three-phase circuit = 2.42/4.84 = 0.5 times the weight per wire single-phase. This derives the value given by (1). Use of the left-hand member of (1) will eliminate considerable calculations.

Using a base weight of unity per wire single-phase, the total weight = 2 × 1 = 2. By the same method, the total weight of conductor three-phase = 3 × 0.5 = 1.5.



(a) (b) (c)

FIG. 3.—Schematic connections of single-phase and three-phase systems.

For equal power and equal percentage of power loss, the total weight of conductor in the three-phase circuit is equal to 1.5/2 = 0.75 times the weight required single-phase. With this method, it is not necessary to know the length of either circuit, the only stipulation being that the length of both lines (not conductors) be the same. Schematic diagrams of single-phase and three-phase connections are shown in Fig. 3. Relative weights for various combinations of those connections are given in Table 3.

In three-wire single-phase and four-wire three-phase circuits, the neutral conductor bears no fixed relation to the size of the phase wires. The size of the neutral conductor is usually based on the neutral current that may be expected to flow from any unbalanced load condition. For this reason, the relative weights cannot be determined from any theoretical consideration. Table 3 has been compiled to give, respectively, the relative weights for combinations when the neutral is one-half and when it is equal to the phase wires. However, the relative weights for combinations other than those in the table may be readily determined as follows:

Assume a three-phase four-wire circuit, the neutral having 63 percent of the weight of the phase wires. Assume further that this is to be compared with a single-phase three-wire circuit

Table 3.—Relative Conductor Weights for Various Combinations of Single-phase and Three-phase Circuits in Fig. 3

Figure	Number of wires	Relative size N	Ratio 3-phase to 1-phase	Relative cond. wt.
a	2	0		100 (base wt.)
c	3	0	1.5/2	75
c	4	½	1 75/2	87.5
c	4	1	2/2	100
b	3	½		100 (base wt.)
c	3	0	1.5/2.5	60
c	4	½	1.75/2.5	70
c	4	1	2/2.5	80
b	3	1		100 (base wt.)
c	3	0	1.5/3	50
c	4	½	1.75/3	58.3
c	4	1	2/3	66.7

having a neutral half the size of the phase wires (see Table 3). The ratio is seen to be 1.5/2.5 for a three-phase three-wire circuit. The neutral weighs 63 percent of the phase wires, and the total base weight (see page 14) is therefore

$$1.5 + 0.5 \times 0.63 = 1.815.$$

The weight of the three-phase conductors in terms of the weight of the single-phase conductors is then 1.815/2.5 = 72.6 percent. The relative values for other combinations are determined in the same manner.

CHAPTER III

ELECTRICAL PROPERTIES OF CONDUCTORS

Copper.—Table 4 gives the effective 60-cycle resistance at 25°C conductor temperature, and inductive reactance at various conductor spacings for hard-drawn copper wire and cable, up to and including 500,000 cir mils. This covers the range of conductor sizes required to meet the demand for most conditions, except possibly for extremely high voltages at which diameter is the limiting factor. Since it is fairly common practice to use solid or stranded wire up to and including No. 1, the values for these sizes, except No. 6, are given for both solid and stranded conductors.

The conductivity of other types of conductors is usually based on their copper equivalent, *i.e.*, designated as being equivalent to a certain size of copper. It is therefore essential that the basic values should be highly accurate. Accordingly, all resistance and reactance values in Table 4 are extended to four places. Because of spiraling of the strands around a central core, the outer strands are slightly longer than the central core. As a result of this extra length and contact resistance between the strands, the A.S.T.M. specifications permit an allowance of 2 percent increase in the weight and resistance for 7, 12, 19, and 37 strands, and of 1 percent increase for 3 strands.

Further, owing to what is known as "skin effect," the 60-cycle resistances are appreciably higher than d.c. resistance for the larger size conductors. The magnetic center of a conductor is linked by all the flux lines, but the magnetic density diminishes as the surface of the conductor is approached. Accordingly, the inductive reactance is higher near the center, and the inductance of the wires in the cable will therefore be different. The higher reactance of the wires near the center produces an unequal distribution of the current and crowds the current to the surface. This produces skin effect and results in a higher *IR* drop near the surface of the conductor.

TABLE 4 —60-CYCLE RESISTANCE AND REACTANCE, OHMS PER MILE, HARD-DRAWN COPPER

Size, AWG or MCM	No. of wires	Diam, in	Res 25°C	Equivalent spacing, ft										
				2	3	4	5	7	9	11	13	15	17	19
6	1	0 1620	2 1849	0 7210	0 7702	0 8051	0 8321	0 8730						
4	1	2043	1 3738	6928	7419	7769	8039	8448	8753					
4	7	2320	1 4020	6861	7353	7702	7973	8381	8687					
2	1	2576	0 8643	6647	7139	7488	7758	8167	8472	0 8715				
2	7	2920	0 8819	6579	7071	7420	7691	8099	8404	8648				
1	1	0 2893	0 6853	0 6506	0 6998	0 7347	0 7618	0 8026	0 8331	0 8575	0 8777	0 8951		
1	3	3600	6926	6409	6901	7250	7521	7929	8234	8478	8680	8854		
1	7	3280	6995	6439	6931	7280	7550	7959	8264	8507	8710	8884		
1/0	7	3680	5549	6298	6790	7139	7410	7818	8123	8366	8569	8743	0 8895	0 9029
2/0	7	4140	4403	6157	6649	6998	7269	7677	7982	8226	8429	8602	8754	8889
3/0	7	0 464	0 3495	0 6017	0 6509	0 6858	0 7129	0 7537	0 7842	0 8085	0 8288	0 8462	0 8614	0 8759
3/0	12	492	3495	5891	6383	6732	7003	7411	7716	7960	8162	8336	8488	8623
4/0	7	522	2776	5876	6368	6717	6988	7396	7701	7944	8147	8321	8473	8608
4/0	12	552	2776	5751	6243	6592	6862	7271	7576	7819	8022	8196	8347	8482
4/0	19	528	2776	5809	6301	6650	6921	7329	7634	7877	8080	8254	8405	8540
250	12	0 600	0 2353	0 5649	0 6141	0 6490	0 6761	0 7170	0 7474	0 7718	0 7921	0 8094	0 8246	0 8381
250	19	574	2353	5708	6200	6549	6820	7229	7533	7807	7980	8153	8305	8440
300	12	657	1966	5539	6031	6380	6651	7059	7364	7608	7810	7984	8136	8271
300	19	628	1966	5598	6090	6439	6710	7118	7423	7667	7869	8043	8195	8330
350	12	710	1691	5445	5938	6287	6557	6966	7271	7514	7717	7890	8042	8177
350	19	0 679	0 1691	0 5504	0 5996	0 6345	0 6616	0 7024	0 7329	0 7573	0 7775	0 7949	0 8101	0 8236
400	19	725	1484	5423	5915	6264	6535	6943	7248	7492	7695	7868	8020	8155
450	19	770	1324	5351	5343	6193	6463	6872	7177	7420	7623	7797	7948	8083
450	37	772	1324	5332	5824	6173	6443	6852	7157	7401	7603	7777	7928	8064
500	19	811	1197	5288	5780	6129	6400	6808	7113	7356	7559	7733	7886	8020
500	37	814	1197	.5268	5760	6109	6378	6788	7093	7337	7539	7713	7865	8000

With a sufficient degree of accuracy for all practical purposes, the skin-effect factors at a frequency of 60 cycles, or the ratio of a.c. to d.c. resistance for the range of conductors in Table 4, may be derived as follows:

For solid copper wires $F = 1 + 0.0315(r \times 2.54)^4$ (2)
For stranded copper $F = 1 + 0.0202(r \times 2.54)^4$ (3)

where F = skin-effect factor.
r = radius of conductor, in.

Equations (2) and (3) are an adaptation of the Rayleigh formula for effective resistance of copper conductor to alternating current in order to convert the radius in terms of centimeters to that in terms of inches.[1] The reader is cautioned against putting too much dependence on the use of these equations for conductor sizes other than those in Table 4.

TABLE 5.—EQUIVALENT RADIUS r IN.

Size	Number of wires					
	1	3	7	12	19	37
6	0.0810					
4	.1022	0 1106	0.1080			
2	.1288	.1395	.1362			
1	.1446	.1566	.1529			
1/0	1625		.1717			
2/0	0.1824		0.1928			
3/0	.2048		.2165	0.2400	0 2286	
4/0	.2300		.2431	2695	.2568	
250				.2930	.2791	0 2837
300				.3209	.3057	.3108
350				0.3466	0 3302	0.3357
400				.3706	.3530	.3589
450				.3930	.3745	.3806
500				.4143	.3947	.4012

The resistance values in Table 4 apply to hard-drawn copper (97.3 percent conductivity) and were derived from the fundamental resistivity of 0.15328 ohm (meter, gram), which is the international annealed copper standard at 20°C. The temperature coefficient of resistance of annealed copper (100 percent conductivity) at 20°C is 0.00393 per deg C. The resistance of annealed copper (100 percent conductivity) at 20°C is 10.371 ohms (mil, foot). The temperature coefficient of resistance of hard-drawn copper (97.3 percent conductivity) is $0.00393 \times 0.973 = 0.00382$ per deg C. The resistivity of hard-drawn copper at $20°\text{C} = 100/97.3 \times 0.15328 = 0.15753$ ohm (meter, gram). The resistance of hard-drawn copper at $20°\text{C} = 100/97.3 \times 10.371 = 10.659$ ohms (mil, foot). The resistance of hard-drawn copper at

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[1] See TAYLOR and NEALE, "Electrical Design of Power Transmission Lines," p. 38, D. Van Nostrand Company, Inc., New York.

$$25°\text{C} = 10.659(1 + 0.00382 \times 5) = 10.863 \text{ ohms (mil, foot).}$$

In terms of practical units, the resistivity in ohms (meter, gram) is the resistance of a wire of uniform cross section one meter in length (39.37 in.) weighing one gram (0.002205 lb). The resistance in ohms (mil, foot) is the resistance of a wire having a diameter of one mil (0.001 in.) and a length of one foot.

The inductive reactance values in Table 4 were derived from the following equation:

$$X = 2\pi f\left(0.08047 + 0.74113 \times \log_{10} \frac{D}{r}\right) \qquad (4)$$

where D = equivalent spacing of conductors.

r = equivalent radius of conductor, in same units as D.

For solid wires, r is equal to the actual radius. For stranded conductors, r is slightly less than the actual radius but slightly more than the radius of a solid wire of equal cross section.

Substitute r from Table 5 in (4) for any particular size of copper and stranding for computation of the reactance.

Table 6 gives the capacitance to neutral for copper wire and cable at a frequency of 60 cycles.

Table 7 gives the capacity susceptance in micromhos per mile of single bare conductor for copper wire and cable at 60 cycles. The values in this table are derived from the product of the capacitance in Table 6 and $2\pi f$. For example, 2/0 copper, seven strand, 7-ft spacing: $377 \times 0.01488 = 5.6098$. The table value is 5.61.

TABLE 6.—CAPACITANCE TO NEUTRAL IN MICROFARADS PER MILE OF SINGLE BARE CONDUCTOR COPPER WIRE AND CABLE

Size, AWG or MCM	No. of wires	Equivalent spacing, ft										
		2	3	4	5	7	9	11	13	15	17	19
6	1	0.01571	0.01466	0.01400	0.01353	0.01288						
4	1	.01638	.01524	.01453	.01449	.01332	0.01284					
4	7	.01676	.01558	.01484	.01431	.01357	.01308					
2	1	.01710	.01587	.01510	.01455	.01380	.01328	0.01290				
2	7	.01753	.01623	.01543	.01486	.01407	.01354	.01314				
1	1	0.01749	0.01621	0.01540	0.01483	0.01405	0.01351	0.01312	0.01280	0.01255		
1	3	.01827	.01687	.01600	.01539	.01455	.01398	.01355	.01322	.01294		
1	7	.01793	.01658	.01574	.01515	.01433	.01378	.01336	.01304	.01277		
1/0	7	.01836	.01693	.01607	.01545	.01460	.01403	.01360	.01326	.01299	0.01275	0.01256
2/0	7	.01881	.01733	.01641	.01577	.01488	.01429	.01384	.01349	.01321	.01297	.01276
3/0	7	0.01928	0.01773	0.01677	0.01610	0.01518	0.01456	0.01410	0.01373	0.01344	0.01319	0.01298
3/0	12	.01952	.01793	.01695	.01627	.01533	.01469	.01422	.01386	.01356	.01330	.01309
4/0	7	.01977	.01814	.01714	.01644	.01548	.01484	.01436	.01398	.01368	.01342	.01320
4/0	12	.02002	.01833	.01733	.01661	.01564	.01498	.01449	.01411	.01380	.01354	.01331
4/0	19	.01982	.01819	.01718	.01648	.01552	.01487	.01430	.01401	.01370	.01344	.01322
250	12	0.02040	0.01868	0.01762	0.01687	0.01587	0.01519	0.01469	0.01430	0.01398	0.01371	0.01348
250	19	.02020	.01850	.01746	.01673	.01574	.01508	.01458	.01420	.01388	.01362	.01339
300	12	.02083	.01904	.01794	.01717	.01613	.01543	.01491	.01451	.01418	.01390	.01357
300	19	.02062	.01885	.01778	.01702	.01600	.01531	.01480	.01440	.01408	.01380	.01360
350	12	.02122	.01935	.01822	.01743	.01635	.01564	.01511	.01469	.01435	.01407	.01383
350	19	0.02099	0.01917	0.01806	0.01728	0.01622	0.01551	0.01499	0.01458	0.01425	0.01397	0.01373
400	19	.02133	.01945	.01830	.01750	.01642	.01570	.01516	.01474	.01440	.01412	.01388
450	37	.02165	.01971	.01854	.01772	.01661	.01587	.01532	.01490	.01455	.01426	.01401
500	37	.02193	.01995	.01874	.01790	.01677	.01602	.01546	.01503	.01468	.01438	.01413

The capacitance between phase wires is one-half the table values.

Aluminum Cable Steel Reinforced (A.C.S.R.).—Table 8 gives the resistance and reactance of single-layer and multiple-layer a.c.s.r. corresponding in equivalent sizes to the copper wire and cable in Table 4.

Table 7.—60-cycle Capacity Susceptance to Neutral in Micromhos per Mile of Single Bare Conductor—Copper Wire and Cable

Size, AWG or MCM	No. of wires	Equivalent spacing, ft										
		2	3	4	5	7	9	11	13	15	17	19
6	1	5.923	5.527	5.278	5.101	4.856						
4	1	6.175	5.745	5.478	5.463	5 022	4.841					
4	7	6.318	5.874	5.595	5.395	5.116	4.931					
2	1	6.447	5.983	5.693	5 485	5 202	5.006	4.863				
2	7	6.609	6.119	5.817	5.602	5.304	5.104	4.954				
1	1	6 594	6.111	5.806	5.591	5 297	5 093	4 946	4.825	4 731		
1	3	6.888	6.360	6.032	5.802	5.485	5 270	5.108	4.984	4.878		
1	7	6.759	6.250	5.934	5.711	5.402	5 195	5 037	4 916	4 814		
1/0	7	6.922	6.383	6.058	5.824	5.504	5.289	5.127	4.999	4.897	4.807	4 735
2/0	7	7.091	6.533	6.186	5.945	5.610	5.387	5.218	5.086	4.980	4 890	4.810
3/0	7	7.268	6.684	6.322	6.070	5.723	5.489	5.316	5.176	5.067	4.972	4.893
3/0	12	7.359	6.759	6.390	6.134	5.779	5.538	5.361	5.225	5.112	5.014	4 935
4/0	7	7.453	6.839	6.462	6.198	5.836	5.595	5.413	5.270	5.157	5.059	4 976
4/0	12	7.547	6.910	6.533	6.262	5.896	5.647	5.463	5.319	5.202	5.104	5.018
4/0	19	7.472	6.857	6.477	6.213	5 851	5.606	5.425	5.282	5.165	5.067	4.984
250	12	7.691	7 042	6.643	6.360	5.983	5.726	5.538	5.381	5.270	5.169	5.082
250	19	7.615	6.974	6.582	6.307	5.934	5.685	5.497	5 353	5.233	5.135	5.048
300	12	7.853	7.178	6.763	6.473	6.081	5.817	5.621	5.470	5.346	5 240	5.116
300	19	7.773	7.106	6.703	6.416	6.032	5.772	5.579	5.429	5.308	5.202	5.127
350	12	8.000	7 295	6 869	6 571	6.164	5.896	5.696	5.538	5.410	5.304	5 214
350	19	7.913	7 227	6.808	6.514	6.115	5.847	5.651	5.497	5.372	5.267	5.176
400	19	8 041	7.332	6.899	6.597	6.190	5.919	5.715	5.557	5.429	5.323	5.233
450	37	8.162	7.430	6.989	6.680	6.262	5.983	5.775	5.617	5.485	5.376	5.282
500	37	8.267	7.521	7.065	6.748	6.322	6.039	5.828	5.666	5.534	5.421	5.327

The capacity susceptance *b* between phase wires is one-half the table values.

One important characteristic of the single-layer conductor is that the resistance and reactance are appreciably affected by the current density in the larger sizes. On the other hand, for multiple-layer conductors, the resistance and reactance are practically independent of the current density. This may be explained as follows:

The comparatively low conductivity of the steel core permits only a small portion of the total current to flow in the central part of the conductor, from which the current in the aluminum strands surrounding the steel core is practically uniform. The skin effect is therefore not appreciable at standard frequencies and conductor sizes. There is, however, because of the hysteresis or iron losses, an increase of resistance with alternating current produced by alternating the flux in the steel core. For single-layer conductors, *i.e.*, those with only a single layer of aluminum over the steel core, the resistance and reactance vary with the current density, increasing as the current density increases. This increase in resistance is not appreciable for single-layer conductors in sizes below 1/0.

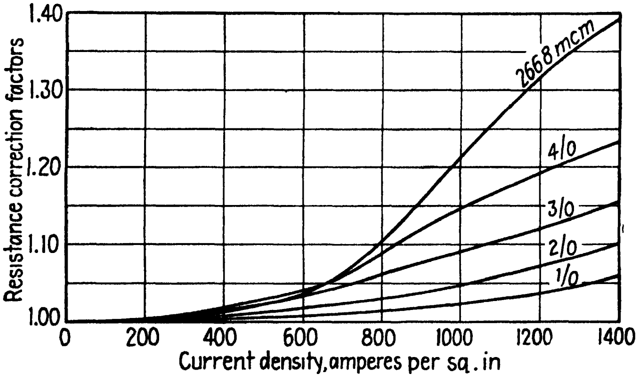

FIG. 4.—Resistance correction factors for single layer a.c.s.r. at various current densities.

The curves in Fig. 4 have been plotted for use in correcting for resistance of single-layer conductors at various current densities. The values at the left, plotted against current density at the bottom, are the percentage of resistance in terms of the table values.

Assume, for example, a 4/0 a.c.s.r. single-layer conductor in which the current density is 800 amp per sq in. From a density of 800 amp at the bottom, follow up to the 4/0 curve and then to the left. The correcting factor is 1.09. From Table 8, 0 amp per sq in., the resistance is given as 0.445 ohm per mile. At 800 amp per sq in., the resistance = 1.09 × 0.445 = 0.485 ohm per mile. Other values are found in the same manner.

It is interesting to note that, for 1/0 and 2/0 a.c.s.r., the resistance curves are still bending upward at 1,400 amp per sq in.; the 3/0 curve is practically a straight line from 800 to 1,400 amp;

TABLE 8.—60-CYCLE RESISTANCE AND REACTANCE OF A.C.S.R., OHMS PER MILE

Size, AWG or MCM	No. of wires	Copper equiv.	Res. 25°C, ohms	Equivalent spacing, ft											
				2	3	4	5	7	9	11	13	15	17	19	21
Single-layer conductors, 0 amp per sq in															
4	6/1	6	2.240	0.701	0.751	0.785	0.812	0.853							
2	6/1	4	1.410	.676	.726	.760	.788	.828	0.859						
1	6/1	3	1.120	.664	.714	.748	.776	.816	.847	0.871					
1/0	6/1	2	0.888	.652	.701	.736	.763	.804	.834	.859	0.879	0.897			
2/0	6/1	1	0.706	.638	.688	.722	.750	.790	.821	.845	.866	.883	0.898	0.912	0.924
3/0	6/1	1/0	0.560	0.624	0.673	0.708	0.735	0.776	0.806	0.831	0.851	0.869	0.884	0.897	0.909
4/0	6/1	2/0	0.445	.608	.657	.692	.719	.760	.790	.815	.853	.852	.868	.881	.893
266.8	6/7	3/0	0.351	.550	.600	.635	.662	.702	.733	.757	.778	.795	.810	.824	.836
Multiple-layer conductors, all current densities															
266.8	26/7	3/0	0.350	0.549	0.598	0.633	0.660	0.701	0.731	0.756	0.776	0.793	0.802	0.822	0.834
336.4	26/7	4/0	.278	.534	.584	.619	.646	.687	.717	.742	.762	.779	.794	.808	.820
336.4	30/7	4/0	.278	.529	.578	.613	.640	.681	.712	.736	.756	.774	.789	.802	.814
397.5	26/7	250MCM	.235	.525	.574	.609	.636	.677	.707	.732	.752	.769	.780	.798	.810
397.5	30/7	250	.235	.519	.568	.603	.630	.671	.701	.726	.746	.763	.779	.792	.804
477.0	26/7	300	0.196	0.514	0.563	0.598	0.625	0.665	0.696	0.720	0.741	0.758	0.773	0.787	0.799
477.0	30/7	300	.196	.508	.557	.592	.619	.660	.690	.715	.735	.752	.768	.781	.794
556.5	26/7	350	.168	.504	.553	.588	.615	.656	.687	.711	.731	.748	.764	.777	.789
556.5	30/7	350	.168	.498	.548	.582	.610	.650	.681	.705	.726	.743	.758	.772	.784
636.0	26/7	400	.147	.496	.546	.580	.607	.648	.679	.703	.723	.741	.756	.769	.782
636.0	30/19	400	0.147	0.491	0.540	0.575	0.602	0.643	0.673	0.697	0.718	0.735	0.751	0.746	0.774
715.5	26/7	450	.131	.489	.538	.573	.600	.641	.671	.696	.716	.733	.748	.762	.774
715.5	30/19	450	.131	.483	.533	.568	.595	.636	.666	.691	.711	.728	.743	.757	.769
795.0	26/7	500	.117	.483	.532	.567	.594	.635	.665	.689	.710	.727	.742	.756	.768
795.0	30/19	500	.117	.477	.526	.561	.588	.629	.660	.684	.704	.722	.737	.750	.762

Table values are given by courtesy of the Aluminum Company of America.

TABLE 9.—60-CYCLE REACTANCE OF A.C.S.R., OHMS PER MILE

Single-layer conductors, 600 amp per sq in.

Size, AWG or MCM	Wires	Copper equiv.	Equivalent spacing, ft											
			2	3	4	5	7	9	11	13	15	17	19	21
4	6/1	6	0.702	0.752	0.786	0.814	0.854							
2	6/1	4	.681	.730	.765	.792	.833	0.863	0.888					
1	6/1	3	.670	.720	.754	.782	.822	.853	.877					
1/0	6/1	2	.659	.708	.743	.770	.811	.842	.866	0.886	0.904			
2/0	6/1	1	.648	.698	.733	.760	.800	.831	.855	.876	.893	0.908	0.922	0.934
3/0	6/1	1/0	0.638	0.687	0.722	0.749	0.790	0.820	0.845	0.865	0.883	0.898	0.911	0.923
4/0	6/1	2/0	.628	.677	.712	.739	.780	.810	.835	.855	.872	.888	.901	.913
266.8	6/7	3/0	.568	.618	.652	.679	.720	.751	.775	.796	.813	.828	.842	.854
Single-layer conductors, 1,200 amp per sq in.														
4	6/1	6	0.706	0.755	0.790	0.817	0.858							
2	6/1	4	.695	.744	.779	.806	.847	0.877	0.902					
1	6/1	3	.690	.740	.774	.802	.842	.873	.897					
1/0	6/1	2	.684	.734	.768	.796	.836	.867	.891	0.912	0.929			
2/0	6/1	1	.677	.727	.762	.789	.829	.860	.884	.905	.922	0.937	0.951	0.963
3/0	6/1	1/0	0.668	0.717	0.752	0.779	0.820	0.851	0.875	0.895	0.913	0.928	0.941	0.953
4/0	6/1	2/0	.655	.705	.739	.766	.807	.838	.862	.883	.900	.915	.929	.941
266.8	6/7	3/0	.648	.697	.732	.759	.800	.830	.855	.875	.892	.907	.921	.933

Table values are given by courtesy of the Aluminum Company of America.

TABLE 10.—CAPACITANCE TO NEUTRAL, MICROFARADS PER MILE OF SINGLE BARE CONDUCTOR A.C.S.R.

Size, AWG or MCM	No. of wires	Copper equiv.	Equivalent spacing, ft											
			2	3	4	5	7	9	11	13	15	17	19	21
4	6/1	6	0.0170	0.0158	0.0150	0 0145	0 0137							
2	6/1	4	.0178	.0165	.0156	.0150	.0142	0.0137	0.0133					
1	6/1	3	.0182	.0168	.0160	0153	.0145	.0139	.0135	0.0132	0.0129	0.0127		
1/0	6/1	2	.0187	.0172	.0163	.0157	.0148	.0142	.0138	.0134	.0131	.0129	0.0127	0.0125
2/0	6/1	1	.0191	.0176	.0166	.0160	.0151	.0145	.0140	.0137	.0134	0131	.0129	.0127
3/0	6/1	1/0	0.0196	0.0180	0.0170	0.0163	0 0154	0.0147	0.0143	0.0139	0 0136	0 0133	0.0131	0.0129
4/0	6/1	2/0	0201	.0184	.0174	.0167	.0157	.0150	.0145	.0142	0138	.0136	.0134	.0132
266 8	6/7	3/0	.0207	.0189	.0178	.0170	.0160	.0153	.0148	.0144	.0141	.0138	.0136	.0134
266.8	26/7	3/0	.0207	.0189	.0178	.0170	.0160	.0153	.0148	.0144	.0141	.0138	.0136	.0134
336.4	26/7	4/0	.0213	.0194	.0183	.0175	.0164	.0157	.0152	0147	0144	.0142	.0139	.0137
336.4	30/7	4/0	0 0214	0 0195	0.0184	0.0176	0.0165	0.0158	0 0152	0.0148	0 0145	0 0142	0.0139	0.0137
397.5	26/7	250	.0218	0198	.0186	.0178	.0166	.0159	.0154	0149	0146	.0143	.0140	0138
397 5	30/7	250	.0219	0199	.0187	.0179	.0167	.0160	.0154	0150	0147	.0144	.0141	.0139
477 0	26/7	300	.0222	0202	.0190	.0181	.0169	.0162	.0156	.0152	.0148	.0145	.0142	.0140
477.0	30/7	300	.0224	.0203	.0191	.0182	.0170	.0163	0157	.0152	.0149	0146	.0143	.0141
556.5	26/7	350	0 0226	0 0205	0.0193	0.0184	0 0172	0.0164	0.0158	0 0154	0 0150	0 0147	0.0144	0.0142
556 5	30/7	350	0228	0207	.0194	.0185	.0173	.0165	.0159	.0154	.0151	0148	.0145	.0143
636.0	26/7	400	.0231	.0208	.0195	.0186	.0174	.0166	0160	0155	0151	.0148	.0145	.0143
636.0	30/7	400	0232	0210	.0197	.0187	0175	.0167	.0161	0156	0152	.0149	.0146	.0144
715.5	26/7	450	.0234	.0211	.0198	.0189	.0176	.0168	.0162	0157	.0153	.0150	.0147	.0145
715.5	30/19	450	0 0236	0 0213	0.0199	0.0190	0 0177	0 0169	0.0163	0.0158	0 0154	0 0151	0 0148	0 0146
795.0	26/7	500	.0237	.0214	.0200	.0191	.0178	0169	.0163	0158	.0154	0151	.0148	.0146
795.0	30/19	500	0239	0216	.0202	0192	.0179	.0170	.0164	0159	.0155	.0152	.0149	.0147

Table values are given by courtesy of the Aluminum Company of America.
The capacitance c between phase wires is one-half the table values.

TABLE 11.—60-CYCLE CAPACITY SUSCEPTANCE TO NEUTRAL, MICROMHOS PER MILE OF SINGLE BARE CONDUCTOR, A.C.S.R.

Size, AWG or MCM	No. of wires	Copper equiv.	Equivalent spacing, ft											
			2	3	4	5	7	9	11	13	15	17	19	21
4	6/1	6	6 41	5 95	5 67	5 46	5 18							
2	6/1	4	6 71	6 21	5 90	5 67	5 37	5 16	5 01					
1	6/1	3	6 87	6 34	6 02	5 79	5 47	5 26	5 10	4 97	4 87			
1/0	6/1	2	7 03	6 48	6 15	5 90	5 58	5 35	5 19	5 06	4 95			
2/0	6/1	1	7 21	6 63	6 28	6 03	5 69	5 46	5 28	5 15	5 04	4 95	4 87	4 80
3/0	6/1	1/0	7 39	6 79	6 42	6 15	5 80	5 56	5 38	5 24	5 13	5 03	4 95	4 88
4/0	6/1	2/0	7 58	6 95	6 56	6 29	5 92	5 67	5 48	5 34	5 22	5 12	5 03	4 96
266 8	6/7	3/0	7 79	7 12	6 71	6 43	6 04	5 78	5 59	5 44	5 31	5 21	5 12	5 04
266 8	26/7	3/0	7 80	7 12	6 71	6 43	6 03	5 77	5 58	5 43	5 32	5 20	5 13	5 04
336 4	26/7	4/0	8 03	7 31	6 90	6 60	6 18	5 92	5 73	5 54	5 43	5 34	5 25	5 17
336 4	30/7	4/0	8 08	7 37	6 93	6 63	6 22	5 94	5 74	5 58	5 45	5 34	5 25	5 17
397 5	26/7	250MCM	8 22	7 46	7 02	6 71	6 26	5 99	5 77	5 62	5 50	5 39	5 28	5 20
397 5	30/7	250	8 25	7 50	7 05	6 74	6 31	6 03	5 82	5 66	5 52	5 41	5 32	5 24
477 0	26/7	300	8 37	7 62	7 16	6 82	6 37	6 11	5 88	5 73	5 58	5 47	5 35	5 28
477 0	30/7	300	8 43	7 66	7 19	6 86	6 42	6 13	5 91	5 75	5 61	5 49	5 40	5 31
556 5	26/7	350	8 52	7 73	7 28	6 94	6 48	6 18	5 96	5 81	5 65	5 54	5 43	5 35
556 5	30/7	350	8 60	7 80	7 31	6 97	6 52	6 22	5 99	5 82	5 68	5 56	5 46	5 38
636 0	26/7	400	8 71	7 84	7 35	7 01	6 55	6 24	6 02	5 84	5 70	5 59	5 49	5 39
636 0	30/19	400	8 75	7 92	7 43	7 05	6 60	6 30	6 07	5 88	5 73	5 62	5 50	5 43
715 5	26/7	450	8 84	7 98	7 46	7 13	6 64	6 33	6 11	5 92	5 77	5 65	5 54	5 47
715 5	30/19	450	8 90	8 03	7 50	7 16	6 67	6 37	6 15	5 96	5 81	5 69	5 58	5 50
795 0	26/7	500	8 96	8 09	7 54	7 19	6 71	6 38	6 14	5 96	5 81	5 69	5 59	5 49
795 0	30/19	500	9 03	8 16	7 62	7 24	6 75	6 41	6 18	5 99	5 84	5 73	5 62	5 54

Table values are given by courtesy of the Aluminum Company of America.
The capacity susceptance *b* between phase wires is one-half the table values.

while the curve for the 266.8 MCM cable shows a decided tendency to droop at the higher current densities.

The resistance and reactance values compiled in Table 8 and the reactances in Table 9 were derived from actual test. The curves of Fig. 4 indicate quite clearly how the resistance is affected by the current density for single-layer a.c.s.r. These curves provide a simple means for determining the resistance for intermediate current density values between 250 and 1,400 amp per sq in. Below 250 amp per sq in., the increase in resistance is negligible. The peculiar shape of the curves indicates that direct interpolation between the table values for various current densities would in some cases produce appreciable errors. For this reason, the curves greatly enhance the value of the tabulations.

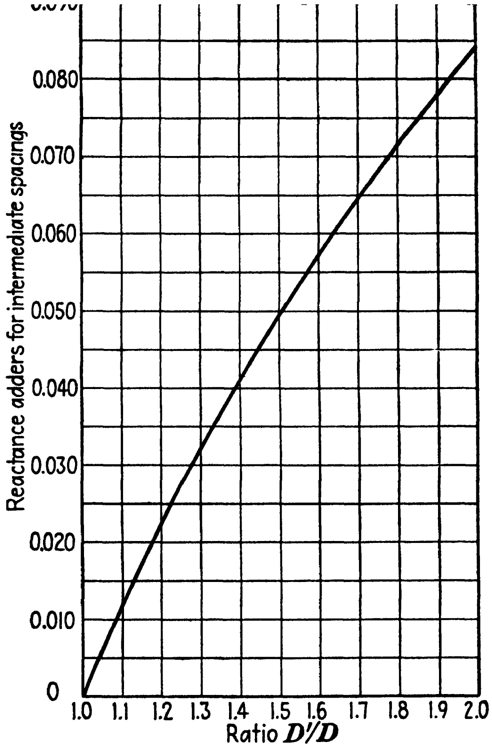

FIG. 5.—Curve of reactance corrections for any ratio of D' to D, where D' equals actual or assumed spacing and D equals next smaller table value for any type of conductor.

For multiple-layer conductors (see Table 8), the resistance is practically independent of the current density, as is also the inductive reactance. This is made understandable by the fact that, since the aluminum strands in the two layers are wound around the steel core in opposite directions, the magnetizing effect of the current is neutralized and the flux lines in the steel core are practically constant, regardless of the current density.

The equivalent spacing of the conductors will usually not correspond exactly with the table values. It is unnecessary, however, to interpolate between table values, since the reactance x may readily be determined from Fig. 5 for any desired spacing. This curve is applicable for use with any type of conductor.

Assume, for example, a 2/0 copper cable having a spacing of 3.9 ft. The ratio $D'/D = 3.9/3 = 1.3$. From this value at the bottom, follow up to the curve and then to the left to 0.032. The table value for 3-ft equivalent spacing is 0.665 ohm per mile. $0.665 + 0.032 = 0.692$ ohm reactance per mile at 3.9 ft. This curve is especially convenient for finding reactance values for spacings outside the range of the table. Assume a 4/0 copper 19-strand cable, with an equivalent spacing of 25 ft. The reactance x as computed by Eq. (4) is 0.8873 ohm per mile. The last table value is for 19 ft. $D'/D = {}^{25}\!/_{19} = 1.316$ nearly. Starting with this value, we find from the curve that the adder is 0.0333. The table value for 19 ft is 0.8540. Adding, we have $0.8540 + 0.0333 = 0.8873$ ohm per mile, which checks the calculated value.

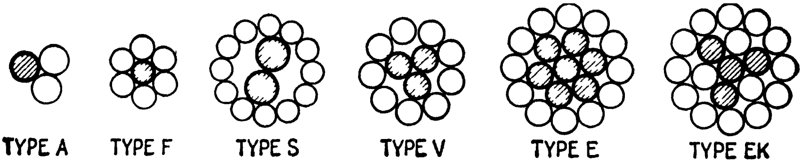

FIG. 6.—Copperweld-copper

Copperweld-copper.—Copperweld-copper conductor consists of a combination of copper and copperweld-steel strands, as indicated by the various types in Fig. 6. The strength requirements will govern the relative number of strands of each.

The copperweld-steel strand will have 30 percent of the conductivity (in some cases 40 percent) of each strand of copper of equal total cross section. For example, in type F, the con-

TABLE 12.—60-CYCLE RESISTANCE AND REACTANCE, OHMS PER MILE COPPERWELD-COPPER

Equivalent conductance, AWG or MCM	Type	No. of wires	Res., 25°C	Equivalent spacing, ft											
				2	3	4	5	7	9	11	13	15	17	19	21
4	A	2/1	1.400	0 704	0.753	0.788	0 815	0 856							
2	A	2/1	900	675	.724	759	786	827	0.858						
2	F	6/1	0.900	0.659	0.708	0.743	0 770	0 811	0 842	0.866					
1	F	6/1	720	.645	.694	729	756	797	.828	.852	0 872				
1/0	F	6/1	575	631	.680	.715	742	783	.814	.838	858	0 876			
2/0	F	6/1	460	617	.666	.701	.728	.769	.800	.824	844	.862	0.877	0.890	0.902
3/0	F	6/1	.365	603	.652	.687	714	.755	.786	.810	.830	.848	.863	.876	.888
4/0	F	6/1	.295	.589	638	.673	.700	.741	.772	.796	816	.834	.849	.862	.874
2/0	S	12/2	0 460	0.590	0.639	0 674	0.701	0.742	0.773	0.797	0 817	0 835	0.850	0.863	0.875
3/0	S	12/2	.365	.575	.624	659	.686	.727	758	.782	.802	.820	.835	.848	.860
4/0	S	12/2	295	.561	.610	.645	.672	.713	744	768	.788	.806	.821	.834	.846
2/0	V	9/3	0 460	0 602	0 651	0 686	0 713	0 754	0.785	0 809	0 829	0.847	0.862	0.875	0.887
3/0	V	9/3	365	588	.637	.672	.699	.740	.771	.795	.815	.833	.848	.861	.873
4/0	V	9/3	.295	.574	623	.658	.685	726	.757	.781	801	.819	.834	.847	.859
250	V	9/3	.250	.564	.613	.648	.675	716	.747	.771	.791	.809	.824	.837	.849
300	V	9/3	.210	.553	602	.637	664	.705	736	.760	.780	.798	813	.826	.838
350	V	9/3	.180	.544	593	.628	.655	696	.727	.751	.771	.789	.804	.817	.829
3/0	E	12/7	0.365	0 585	0.634	0.669	0.696	0 737	0.768	0.792	0.812	0.830	0.845	0.858	0.870
4/0	E	12/7	295	.570	.619	.654	.681	722	.753	.777	.797	.815	.830	.843	.855
3/0	EK	15/4	0.365	0.580	0 629	0.664	0.691	0 732	0.763	0.787	0 807	0.825	0.840	0.853	0.865
4/0	EK	15/4	.295	567	.616	.651	678	719	.750	.774	.794	.812	.827	.840	.852
250	EK	15/4	.250	.556	.605	.640	667	708	.739	763	.783	.801	.816	.829	.841
300	EK	15/4	.210	.545	.594	.629	.656	.697	.728	.752	.772	.790	.805	.818	.830
350	EK	15/4	.180	.536	.585	620	647	.688	.719	743	.763	.781	.796	.809	.821

Table values are given by courtesy of the Copperweld Steel Company.

TABLE 13.—CAPACITANCE TO NEUTRAL, MICROFARADS PER MILE OF SINGLE BARE CONDUCTOR COPPERWELD-COPPER

Equivalent conductance, AWG or MCM	Type	Equivalent spacing, ft											
		2	3	4	5	7	9	11	13	15	17	19	21
4	A	0 0175	0 0160	0 0154	0 0148	0.0141							
2	A	0183	0169	.0161	.0154	.0146	0.0140						
2	F	0.0177	0 0164	0 0156	0 0150	0 0142	0.0136	0 0132					
1	F	.0181	0167	.0159	.0153	0145	0139	.0135	0 0131				
1/0	F	.0186	0171	.0162	.0156	0147	0141	0137	0134	0 0131			
2/0	F	0190	.0175	.0166	.0159	.0150	0144	.0140	.0136	0133	0 0131	0 0129	0 0127
3/0	F	0195	0179	.0169	0163	0153	0147	.0142	.0139	.0136	0123	.0131	.0129
4/0	F	.0200	.0183	0173	.0166	0156	0150	.0145	.0141	0138	0135	.0133	.0130
2/0	S	0.0196	0.0180	0 0170	0 0163	0 0154	0 0147	0.0143	0.0139	0 0136	0.0133	0 0131	0.0129
3/0	S	.0201	0184	0174	.0167	0157	0150	.0145	.0142	0138	0136	0134	0132
4/0	S	0207	0189	0178	0171	0160	0153	.0148	0144	.0141	0138	.0136	.0134
2/0	V	0.0193	0 0177	0 0168	0 0161	0 0152	0 0146	0 0141	0 0137	0 0134	0 0132	0 0130	0.0128
3/0	V	0198	0182	0172	0164	0155	0148	0144	.0140	0137	0134	.0132	.0130
4/0	V	0203	0186	.0175	0168	.0158	0151	.0146	.0142	.0139	0137	.0134	.0132
250	V	0207	0189	.0178	0171	0160	0154	0148	0144	0141	0138	.0136	.0134
300	V	.0211	0193	.0182	0174	0163	0156	0151	.0147	0143	0140	.0138	0136
350	V	.0215	.0196	.0185	0176	0165	0158	0153	0148	0145	0142	0140	.0137
3/0	E	0.0200	0 0183	0 0173	0 0166	0 0156	0 0150	0 0145	0 0141	0 0138	0 0135	0 0133	0.0131
4/0	E	.0205	.0188	0177	0169	0159	.0153	0147	.0144	0140	0138	0135	.0133
3/0	EK	0.0197	0.0181	0.0171	0 0164	0 0154	0 0148	0 0143	0 0139	0 0136	0 0134	0.0132	0.0130
4/0	EK	0202	0185	.0175	0167	0157	.0151	.0146	.0142	0139	0136	.0134	.0132
250	EK	.0206	0188	0177	0170	.0160	0153	0148	0144	0141	.0138	.0136	.0134
300	EK	0210	.0192	.0181	0173	0162	0155	0150	.0146	0143	.0140	.0137	.0135
350	EK	0204	.0195	0184	.0176	0165	0157	.0152	.0148	0144	0142	.0139	.0137

The capacitance values in the table were calculated from the equation $C = \frac{0.038829}{\log_{10} (D/r)}$.

D = distance between conductor centers. r = radius of conductor.

TABLE 14 —60-CYCLE CAPACITY SUSCEPTANCE TO NEUTRAL, MICROMHOS PER MILE OF SINGLE BARE CONDUCTOR, COPPERWELD-COPPER

Equivalent conductance AWG or MCM	Type	Equivalent spacing, ft											
		2	3	4	5	7	9	11	13	15	17	19	21
4	A	6 398	6 036	5 810	5 595	5 297							
2	A	6 910	6 383	6 051	5 817	5 500	5 282						
2	F	6 677	6 179	5 870	5 651	5 350	5 142	4 992					
1	F	6 835	6 315	5 991	5 764	5 448	5 237	5 078	4 954				
1/0	F	6 997	6 454	6 115	5 877	5 553	5 331	5 169	5 037	4 931			
2/0	F	7 171	6 598	6 247	6 002	5 663	5 433	5 259	5 127	5 018	4 927	4 848	4 780
3/0	F	7 352	6 756	6 386	6 126	5 776	5 534	5 361	5 222	5 108	5 010	4 931	4 859
4/0	F	7 540	6 914	6 530	6 258	5 893	5 644	5 463	5 316	5 203	5 101	5 014	4 905
2/0	S	7 393	6 786	6 417	6 156	5 798	5 557	5 380	5 240	5 127	5 029	4 950	4 878
3/0	S	7 582	6 948	6 560	6 285	5 915	5 666	5 482	5 335	5 218	5 120	5 033	4 958
4/0	S	7 785	7 122	6 711	6 428	6 040	5 779	5 587	5 436	5 312	5 210	5 123	5 044
2/0	V	7 269	6 684	6 322	6 070	5 723	5 489	5 316	5 180	5 067	4 973	4 894	4 822
3/0	V	7 453	6 843	6 466	6 198	5 836	5 595	5 414	5 271	5 157	5 059	4 976	4 905
4/0	V	7 649	7 005	6 613	6 337	5 957	5 704	5 516	5 369	5 252	5 150	5 063	4 988
250	V	7 800	7 125	6 722	6 435	6 047	5 787	5 591	5 440	5 320	5 214	5 127	5 052
300	V	7 966	7 269	6 850	6 549	6 145	5 877	5 678	5 523	5 399	5 289	5 199	5 120
350	V	8 117	7 393	6 956	6 650	6 236	5 957	5 753	5 595	5 463	5 353	5 263	5 180
3/0	E	7 525	6 903	6 518	6 247	5 885	5 636	5 451	5 308	5 191	5 093	5 010	4 935
4/0	E	7 729	7 073	6 669	6 386	6 002	5 749	5 557	5 410	5 286	5 184	5 097	5 022
3/0	EK	7 412	6 805	6 447	6 172	5 813	5 572	5 391	5 252	5 139	5 041	4 958	4 886
4/0	EK	7 608	6 967	6 579	6 303	5 930	5 678	5 493	5 346	5 729	5 127	5 044	4 969
250	EK	7 716	7 091	6 688	6 402	6 017	5 761	5 568	5 421	5 297	5 195	5 108	5 033
300	EK	7 917	7 231	6 809	6 515	6 119	5 851	5 655	5 500	5 376	5 271	5 180	5 101
350	EK	8 064	7 352	6 918	6 616	6 205	5 934	5 730	5 572	5 440	5 335	5 240	5 161

The values for susceptance were derived from the equation $b = 2\pi f C$ The charging current in amperes per mile of single conductor = (susceptance from table) × (volts to neutral) × 10^{-6}.

ductivity of the core will have 0.3/6 = 0.05, or 5 percent of the conductivity of the six copper strands. Since the copper is at the periphery of the steel core, 30 percent of the total cross section requires only a thin layer of copper; and permits a steel core of adequate strength for economical transmission-line construction.

The 60-cycle resistance of copperweld-copper, though not entirely independent of the current density, may for all practical purposes be considered so.

For convenience in calculating transmission-line performance, resistance and reactance values are given in Table 12; capacitance in Table 13; and capacity susceptance in Table 14.

Steel as Transmission-line Conductor.—Steel may be used to reasonably good advantage as a transmission-line conductor, but care should be exercised in this respect, since, as pointed out below, it is limited in its application. From what is now known about the properties of steel when used as a current-carrying medium, it should generally be considered as limited to lines of such voltage that considerable power may be transmitted at comparatively low current value. This is understandable, since, owing to the magnetic properties of steel conductor, the ohmic resistance and inductive reactance are functions of the current density, both increasing with increased current density.

Manufacturers of steel wire and cable have for some time been actively engaged in research work in order to learn more about the electric and magnetic properties of steel strand under electrical load. Because few tables giving such properties are available, it is usually necessary for the designer to calculate the values and compile tables for the kind, size, and stranding of steel with which he is most concerned. It is necessary to know only the ohmic resistance at any given temperature and current value and the internal inductance in order to calculate the values from which tables similar to Table 15 may be compiled.

The unit ohmic resistance and internal reactance values in Table 15 are in terms of 60-cycle frequency and were furnished by the American Steel & Wire Company. The resistance values, however, have been changed to read in terms of 25°C instead of 20°C, as furnished by the manufacturer. The geometrical mean radius (GMR) and all values of total reactance at the given spacings were calculated by the author. Either the GMR or internal inductance must be known in order to calculate x_a, the

TABLE 15.—60-CYCLE RESISTANCE AND REACTANCE OF SIEMENS MARTIN STEEL CONDUCTOR, OHMS PER MILE

Amp	GMR	Res., 25°C	Internal react.	Equivalent spacing, ft									
				1	2	3	4	5	7	9	11	13	15
							¼ in.						
5	0.0001076	13.38	0.550	1.109	1.193	1.242	1.277	1.304	1.345	1.375	1.400		
10	0.0000868	13.39	0.576	1.135	1.219	1.268	1.303	1.330	1.371	1.401	1.426		
15	0.0000599	13.43	0.621	1.180	1.264	1.313	1.348	1.375	1.416	1.446	1.471		
20	0.0000476	13.45	0.649	1.208	1.292	1.341	1.376	1.403	1.444	1.474	1.499		
25	0.0000318	13.47	0.698	1.257	1.341	1.390	1.425	1.452	1.493	1.523	1.548		
							5/16 in.						
5	0.0001621	7.938	0.532	1.059	1.143	1.192	1.227	1.254	1.295	1.326	1.350		
10	0.0001225	7.944	0.566	1.093	1.177	1.226	1.261	1.288	1.329	1.360	1.384		
15	0.0000874	7.966	0.607	1.134	1.218	1.267	1.302	1.329	1.370	1.401	1.425		
20	0.0000682	7.997	0.637	1.164	1.248	1.297	1.332	1.359	1.400	1.430	1.455		
25	0.0000511	8.004	0.672	1.199	1.283	1.332	1.367	1.394	1.435	1.466	1.490		
							⅜ in.						
5	0.0001934	5.978	0.528	1.037	1.121	1.170	1.205	1.232	1.273	1.304	1.328	1.348	1.366
10	0.0001473	5.984	0.561	1.071	1.155	1.204	1.239	1.266	1.307	1.337	1.362	1.382	1.399
15	0.0001123	5.997	0.594	1.104	1.188	1.237	1.272	1.299	1.340	1.370	1.395	1.415	1.432
20	0.0000884	6.026	0.623	1.133	1.217	1.266	1.301	1.328	1.369	1.399	1.424	1.414	1.461
25	0.0000702	6.031	0.651	1.161	1.245	1.294	1.329	1.357	1.397	1.427	1.452	1.472	1.489
							½ in.						
5	0.0002510	3.176	0.535	1.006	1.090	1.139	1.174	1.201	1.242	1.273	1.297	1.317	1.335
10	0.0002077	3.182	0.558	1.029	1.131	1.162	1.197	1.224	1.265	1.296	1.320	1.340	1.357
15	0.0001835	3.185	0.573	1.044	1.128	1.177	1.212	1.239	1.280	1.311	1.335	1.355	1.373
20	0.0001609	3.197	0.589	1.060	1.144	1.193	1.228	1.255	1.296	1.327	1.351	1.371	1.387
25	0.0001309	3.203	0.614	1.085	1.169	1.218	1.253	1.280	1.321	1.352	1.376	1.396	1.414

unit reactance at 1-ft spacing. The reactance x_d at spacings in excess of 1 ft is a function of the frequency and spacing between centers of conductors. These values are constant for all types of conductor at a given spacing.[1] The calculations may be performed as follows:

$$x = x_a + x_d = 0.004657f \log_{10} \frac{1}{\text{GMR}} + 0.004657f \log_{10} D \quad (5)$$

$$x_a \text{ at 60 cycles} = 0.2794 \log \frac{1}{\text{GMR}}$$

from which $$\text{GMR} = \frac{r}{\text{antilog } (x_i/0.2794)}$$

where r = radius of conductor, ft.
x_i = unit internal reactance.

Take, for example, ¼-in. steel strand with a current of 5 amp at a spacing of 5 ft. The actual outside diameter is 0.24 in. $r = 0.01$ ft $x_i = 0.55$ ohm. $0.55/0.2794 = 1.9684$. The antilog, or number belonging to this logarithm, is 92.97.

$$\text{GMR} = 0.01/92.97 = 0.0001076,$$

$1/0.0001076 = 9{,}297$, the log of which is 3.9684.

$x_a = 0.2794 \times 3.9684 = 1.1088$ ohms per mile at 1-ft spacing.
x_d at 5-ft spacing $= 0.2794 \times \log 5$.
$x_d = 0.2794 \times 0.69897 = 0.1953$.
$x_a + x_d = 1.1088 + 0.1953 = 1.3041$ ohms total reactance per mile.

The table value to three places = 1.304. Other values are derived in the same manner.

Siemens Martin steel strand is probably more widely used for transmission-line conductor than are any of the other strength classes. It is readily adaptable for general-purpose use, such as conductor and overhead ground wire, and is easily installed as anchor guys. Since it has ample strength for most span lengths and loading, it is often necessary to purchase only one size and type for a given transmission line.

The values in Table 15 may be used with safety for high-strength steel. The latest available information indicates that internal inductance is slightly lower for high-strength steel than for Siemens Martin. The Siemens Martin table values will therefore give values that will be on the safe side.

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[1] See "Standard Handbook for Electrical Engineers," 7th ed., Table 53, p. 1201.

CHAPTER IV

SHORT TRANSMISSION LINES

For transmission lines not over 50 miles in length, the effect of capacitance is so small as to have no appreciable effect on the performance of the circuit. Calculations based on the assumption that the effect of capacitance is negligible for a line 50 miles long result in an error in the order of only one-half of 1 percent. Accordingly, lines up to 50 miles in length will be considered as short lines.

The fundamental basis for transmission-line calculations is the receiver voltage. The nominal receiving-end voltage will generally depend on the load to be supplied and the transmission distance. It is possible to use the supply-end voltage as a base; but, since the receiver voltage must remain practically constant at a given value, it will generally be more satisfactory as a base for calculations.

The question, "How much load will a line carry?" is frequently asked. The answer is that the capacity of a given line is not, without qualifications, a fixed quantity, measured in terms of its load-carrying capacity. For example, the power that can be transmitted over a given line may be, and quite often is, limited by the transformer capacity at one or both ends of the line. On the other hand, with practically unlimited generation and transformer capacity available, the line capacity, for all practical purposes, is limited by the permissible voltage drop.

The capacity of a transformer is limited by the heating of the windings and is therefore measured in kva. In a transmission line, however, the heating of the conductors, at transmission voltage, does not enter into the problem; and the transmission of power is generally measured in kw at a given power factor. Curves are included, however, that may be used for converting the values at the basic power factor (0.8 lagging) to their equivalent value at the known or desired power factor.

The most valuable information a designing engineer can have is a knowledge of authentic short-cut methods, provided that such methods can be relied upon to give results as accurate as those derived by more rigorous calculations. In fact, the designer who does not possess such methods for computing voltage drop and similar values may find himself at a decided disadvantage, since busy executives expect the answers to such problems in a matter of minutes, not hours.

TABLE 16.—VOLTS DROP PER KW PER MILE AT 100 VOLTS 60 CYCLES THREE-PHASE 80 PERCENT POWER FACTOR

Size, AWG or MCM	No. of wires	Equivalent spacing, ft										
		2	3	4	5	7	9	11	13	15	17	19
6	1	27 58	27.92	28.15	28 34	28 62						
4	1	18 98	19.34	19 59	19 79	20 09	20.31					
4	7	19 22	19 58	19 83	20 03	20.32	20.55					
2	1	13.63	14 00	14.25	14 47	14.78	15 01	15 20				
2	7	13.73	14.13	14.37	14.59	14 90	15 13	15 32				
1	3	11.74	12.11	12.38	12.59	12.90	13 14	13.30	13.49	13 63		
1	7	11.83	12 21	12 47	12.68	13 00	13 23	13 42	13 58	13 72		
1/0	7	10.29	10.68	10.94	11 16	11.47	11 71	11 90	12 07	12 20	12.32	12.43
2/0	7	9.057	9.441	9.715	9.925	10.25	10 49	10 67	10 84	10 98	11.10	11.21
3/0	7	8.061	8.446	8.720	8 935	9 257	9 500	9.696	9.859	9.997	10.12	10.23
3/0	12	7.963	8 347	8.622	8 835	9 141	9 401	9 595	9.756	9 895	10 02	10.13
4/0	7	7.250	7 637	7 913	8 128	8 451	8 697	8 893	9 055	9.195	9 318	9 425
4/0	12	7 152	7.539	7 814	8 028	8 353	8 597	8 791	8 954	9.094	9 216	9 325
4/0	19	7.197	7.577	7 860	8.075	8 400	8 643	8 838	9.001	9 141	9 263	9 372
250	12	6.660	7.048	7.324	7.540	7.866	8 109	8.305	8 468	8.608	8 731	8 840
250	19	6.706	7.094	7.368	7 586	7.914	8 156	8 353	8.516	8 655	8.778	8.889
300	12	6.198	6 586	6 863	7.079	7 405	7.650	7.846	8 009	8.150	8 272	8.382
300	19	6.242	6.632	6 909	7.125	7 453	7.697	7.894	8 055	8.195	8.320	8.431
350	12	5.857	6.246	6.524	6.740	7.066	7.311	7.508	7.669	7.812	7.935	8 045
350	19	5.903	6.293	6.570	6.787	7.113	7.358	7.550	7.720	7.859	7.983	8.094
400	19	5.640	6.029	6.308	6.524	6 851	7.097	7.292	7.456	7.597	7 720	7.830
450	19	5.427	5.817	6 095	6.312	6.639	6.885	7.082	7.245	7.385	7.510	7.620
450	37	5.411	5.802	6.078	6.296	6.625	6.869	7.066	7 231	7.371	7.494	7 604
500	19	5.246	5.636	5.913	6.131	6.459	6.704	6.900	7.065	7.206	7.330	7.039
500	37	5.230	5.624	5.900	6.115	6.445	6.690	6.885	7.050	7.191	7.313	7.423

Drops for No. 1 solid and three strand are the same.
Table values are for hard-drawn copper.

Table 16, which gives volts drop per kw per mile at 100 volts, three-phase, 60 cycles, 0.8 power factor has been compiled to facilitate voltage-drop calculations. One hundred volts is used as a base because it lends itself readily to conversion to actual line voltage. Voltage-drop values derived from Table 16 are as accurate as those computed by any known method, when the effect of capacitance of the circuit is considered negligible. The table values were computed from the following equation:

$$E_{sn} = \sqrt{(E_{rn} \cos \theta + IR)^2 + (E_{rn} \sin \theta + IX)^2} \qquad (6)$$

where E_{sn} = sending-end voltage to neutral.

E_{rn} = receiving-end voltage to neutral = $100/\sqrt{3}$ = 57.735 volts.

$\cos \theta$ = pf = 0.8.

$\sin \theta$ = reactive factor = 0.6.

$$I = \frac{1{,}000}{100 \times 1.732 \times 0.8} = 7.217 \text{ amp at 100 volts.}$$

The reader will recognize Eq. (6) immediately as being fundamental and should have no hesitancy in using the table values.

As an example, refer to Table 4, for 2/0 seven-strand copper, 4-ft spacing, for which the resistance r and reactance x are 0.4403 and 0.7 ohm per mile, respectively. From this,

$$Ir = 7.217 \times 0.4403 = 3.1776 \text{ resistance volts drop}$$

$Ix = 7.217 \times 0.7 = 5.0519$ reactance volts drop. Substituting these values in (6) and solving, we have

$$\begin{aligned} E_{sn} &= \sqrt{(57.735 \times 0.8 + 3.1776)^2 + (57.735 \times 0.6 + 5.0519)^2} \\ &= \sqrt{4{,}012.4888} = 63.344 \text{ volts to neutral} \end{aligned}$$

1.732(63.344 − 57.735) = 9.715 volts drop per kw per mile between phase wires, which checks the table value.

Calculating voltage drop from the values in Table 16 is extremely simple, regardless of the line voltage or load, and is carried out as follows:

$$\frac{\text{kw} \times \text{miles} \times \text{table value}}{0.01 \times E_r} = \text{volts drop} \qquad (7)$$

Voltage-drop Calculations.—The most satisfactory method of demonstrating how the work is actually carried out is to set up the problem and perform the operations step by step, which requires only a few minutes.

Example 1. For a transmission circuit 20 miles long consisting of 2/0 seven-strand copper, equivalent spacing 5 ft, 22,000 volts, supplying a load of 1,500 kw at 0.8 power factor, three phase, 60 cycles, it is required to find the following:

a. Voltage drop in volts and percentage at 0.8 power factor.

b. Voltage drop at 0.7 and 0.95 power factor at 1,500 kw.

c. Kw load at 0.7 power factor and 0.95 power factor for the same drop as in *a*.

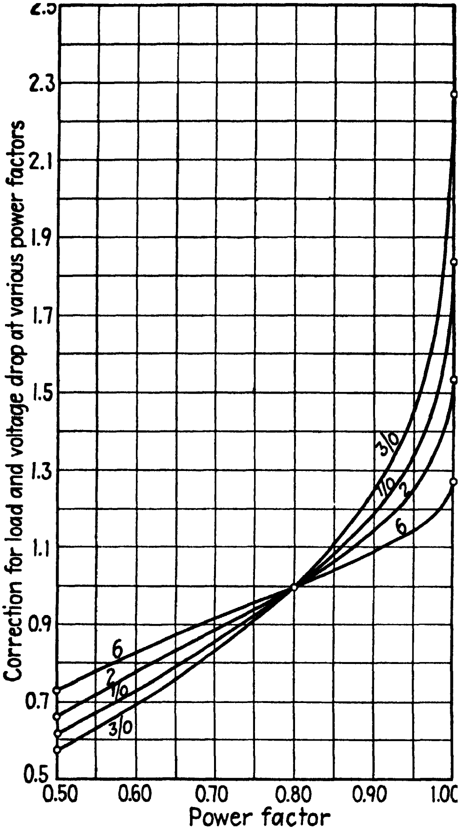

FIG. 7.—Equivalent spacing 3 ft.

Solution for a. From Table 16 for 2/0 copper, 5-ft spacing, the drop per kw per mile is seen to be 9.925 volts. Substituting in (7) and solving, we have

$$\frac{1{,}500 \times 20 \times 9.925}{220.00^*} = 1{,}352 \text{ volts drop} = 6.14 \text{ percent of } 22{,}000$$

---

* Point off two places from the right.

Solution for b. Refer to Fig. 12 and follow from 0.7 power factor up to the curve for 2/0 copper and then to the left. The correction factor is 0.86. Dividing the drop at 0.8 power factor

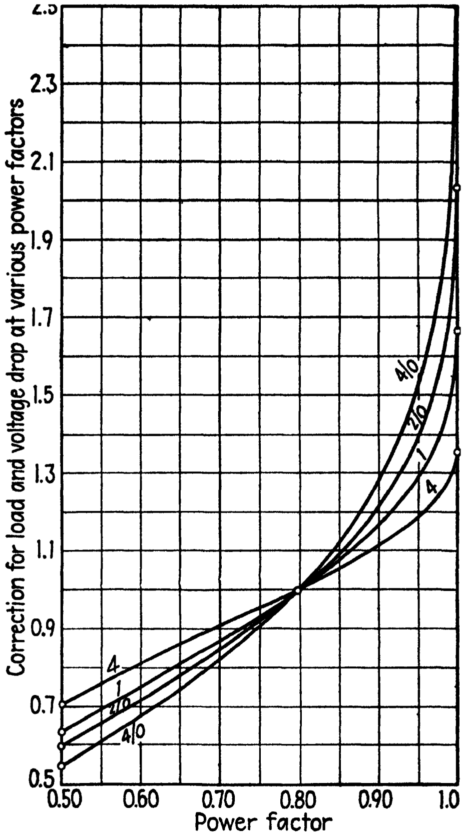

Fig. 8.—Equivalent spacing 3 ft.

by this value = 1,352/0.86 = 1,572 volts drop. The correction for 0.95 power factor is 1.426, from which 1,352/1.426 = 948 volts drop.

Solution for c

1,500 × 0.86 = 1,290 kw at 0.7 pf
1,500 × 1.426 = 2,139 kw at 0.95 pf

One of the problems most commonly met with in practice is that of a transmission line supplying loads at intermediate points along the line. Values such as those compiled in Table 16 are practically indispensable in problems of this nature, for they reduce the solution to the simplicity of that for a line supplying only a single load.

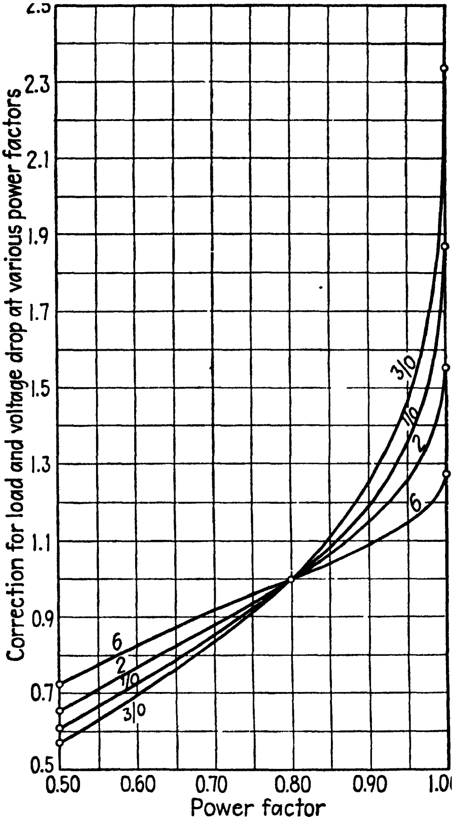

FIG. 9.—Equivalent spacing 4 ft.

Example 2. The physical characteristics of the line in this example are illustrated by Fig. 15 (page 46). Voltage at D is 33,000, equivalent spacing 5 ft, 0.8 lagging power factor. The problem is to find the voltage drop from C to D, B to C, and A to B.

From Table 16, the drop per kw per mile at 100 volts for No. 2 seven-strand copper at 5-ft spacing is 14.59 volts; for 1/0 seven-strand copper, it is 11.16 volts. Substituting in (7) for each section of the line, we have

$$\text{Drop from } C \text{ to } D = \frac{1{,}000 \times 10 \times 14.59}{330} = 442 \text{ volts}$$

Voltage at C = 33,000 + 442 = 33,442

$$\text{Drop from } B \text{ to } C = \frac{2{,}500 \times 15 \times 11.16}{334.42} = 1{,}250 \text{ volts}$$

Voltage at B = 33,442 + 1,250 = 34,692

$$\text{Drop from } A \text{ to } B = \frac{3{,}700 \times 8 \times 11.16}{346.92} = 952 \text{ volts}$$

Voltage at A = 34,692 + 952 = 35,644

Total drop = 442 + 1,250 + 952 = 2,644 volts

= 8.01 percent of 33,000 volts

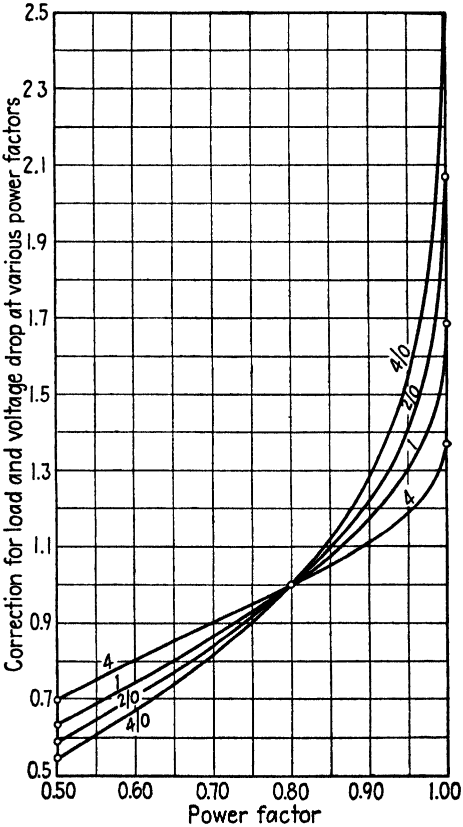

Fig. 10.—Equivalent spacing 4 ft.

Problems of this nature can readily be solved by using Figs. 7 to 14, regardless of the power factor at the various load points.

Assume, for example, the power factor at load *D* to be 0.9, that in the line at the tap point for load *C* to be 0.8, and that at the tap point for load *B* to be 0.75. The correction for 0.9 power factor No. 2 copper from Fig. 11 is 1.153. The calculated drop from *C* to *D* (previously given) is 442 volts, from which the drop at 0.9 power factor = 442/1.153 = 383 volts. Voltage at *C* is now 33,000 + 383 = 33,383 volts.

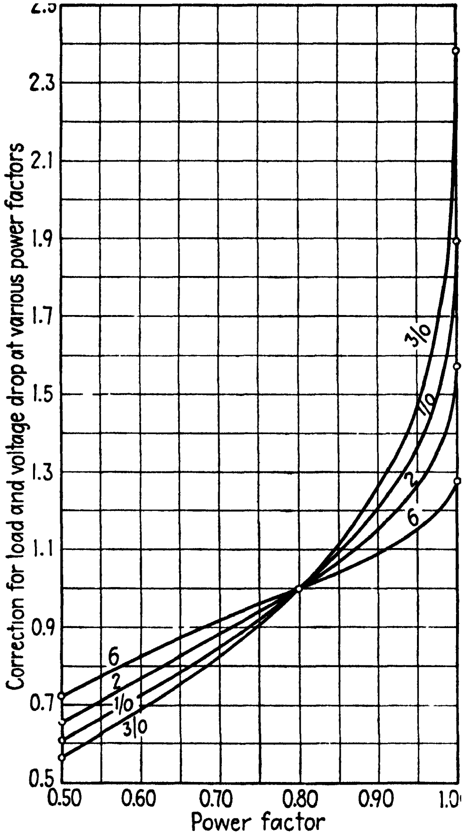

FIG. 11.—Equivalent spacing 5 ft.

$$\text{Drop from } B \text{ to } C = \frac{2{,}500 \times 15 \times 11.16}{333.83} = 1{,}253 \text{ volts}$$

$$\text{Voltage at } B = 33{,}383 + 1{,}253 = 34{,}636 \text{ volts}$$

From Fig. 11, for 1/0 copper, the correction for 0.75 power factor is 0.921, from which the drop at

$$0.75 \text{ power factor} = 11.16/0.921 = 12.11 \text{ volts}$$

$$\text{Drop from } A \text{ to } B = \frac{3{,}700 \times 8 \times 12.11}{346.36} = 1{,}035 \text{ volts}$$

$$\text{Voltage at } A = 34{,}636 + 1{,}035 = 35{,}671 \text{ volts}$$

$$\text{Total drop} = 383 + 1{,}253 + 1{,}035 = 2{,}671 \text{ volts}$$

$$= 8.09 \text{ percent of } 33{,}000 \text{ volts}$$

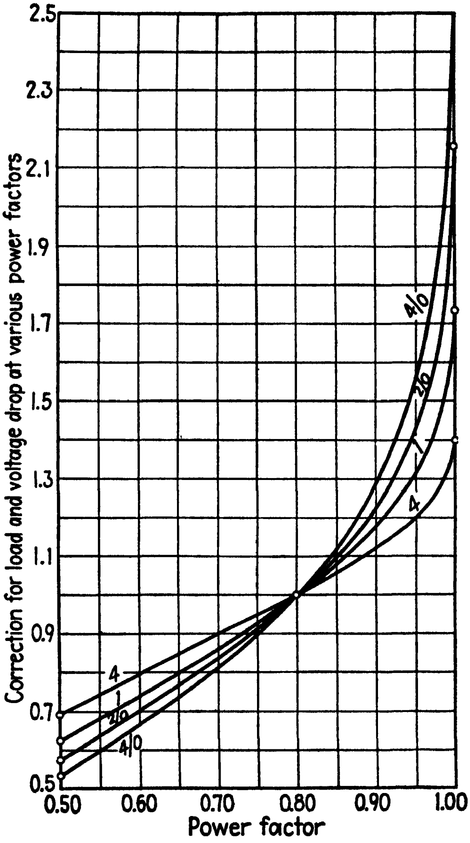

FIG. 12.—Equivalent spacing 5 ft.

The flexibility of the above method is seen to be almost unlimited. It should also be noted that it is not necessary to calculate the current value, nor is it necessary to refer to tables for resistance and reactance values. The enormous amount of time and effort eliminated by the use of the table values can be better appreciated by considering that, if it were necessary to resort to the use of Eq. (6), a complete solution would be required for each section of the line, with a greater liability of error in the final results.

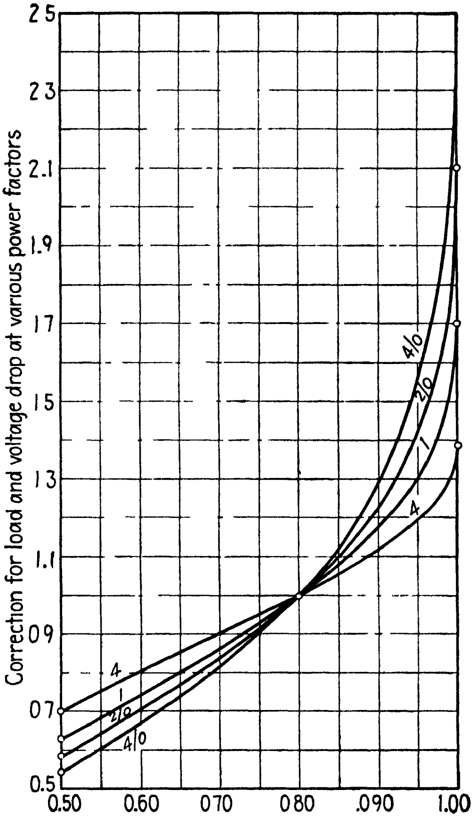

FIG. 13.—Equivalent spacing 7 ft.

The calculations required for solutions to Examples 1 and 2 are based on the use of hard-drawn copper. The results will usually be sufficiently accurate, however, for equivalent sizes of a.c.s.r. and copperweld-copper. If, instead of using the 2/0 copper, as in Example 1, we had used 4/0 a.c.s.r. (equivalent size), the current density would be in the order of 250 amp per sq in. By referring to Fig. 4, it will be seen that, for a current density of 250 amp per sq in., the increase in resistance is negligible. The increase in reactance at this current density is also negligible.

When the current density exceeds about 500 amp per sq in. for single-layer a.c.s.r. conductors, the voltage-drop calculations, if a high degree of accuracy is desired, should be made from the resistance derived from Table 8, corrected for the given current density in Fig. 4; and the reactance should be taken from Table 9.

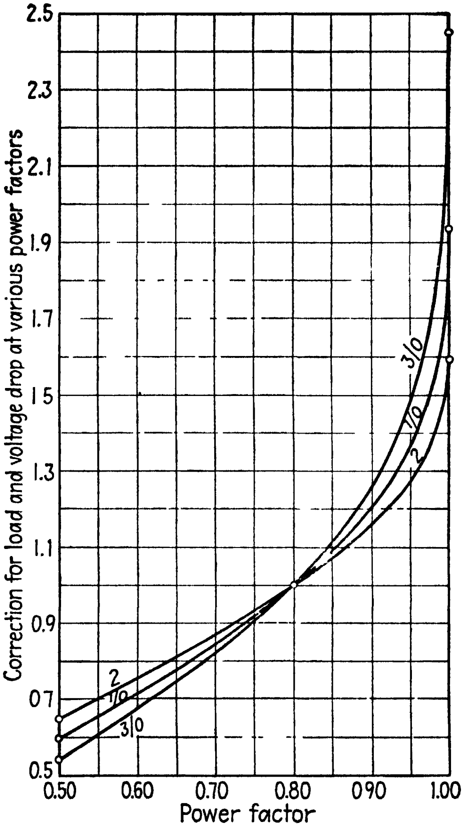

FIG. 14.—Equivalent spacing 7 ft.

Power-loss Calculations.—It is a well-known fact that, if all items of cost that make up a transmission line, exclusive of the conductor, are considered to be independent of the size of conductor, the most economical conductor and also the most economical line as a whole may be determined by the application of the fundamental principle known as Kelvin's law. This law is based on the condition that the most economical conductor is that for which the sum of the annual fixed charges on the cost of the conductor installed and the cost of energy wasted in the line, I^2R losses, is a minimum. Within certain limits (and this will generally apply for lines of transmission voltage), strict adherence to Kelvin's law will be feasible.

The limits of application of Kelvin's law may be explained by the following: In an attempt to determine the most economical line, covering a range of conductors, for example, No. 4 and 4/0, if the calculations were based on the condition that the cost of poles, insulators, and other material were independent of the actual size of conductor, the result would be considerably in error, since it is obvious that a 4/0 line, with an equivalent factor of safety, would require much heavier construction than would a line using No. 4 conductor. For this reason, unless all items of cost are taken into account, a wide range of conductor sizes should never be contemplated.

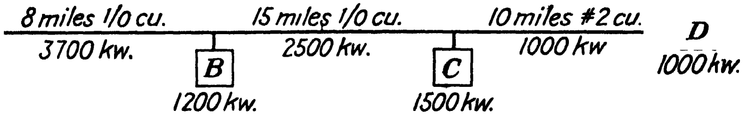

FIG. 15.—Simple transmission system.

The degree of accuracy desired in connection with energy-loss calculations will largely govern the method to be employed. The designer will almost always have to use as a basis information and data applying to conditions that are as far as possible similar to those which may be expected to apply. Unfortunately data applicable to power-loss calculations cannot be shortened to the same extent as can those for use in voltage-drop calculations.

The most accurate method for computing power loss in a transmission circuit is probably from the mean annual current. However, the results derived by this method may be very much in error unless an actual load curve is available to represent a fair average condition from which the mean annual current may be derived.

The equivalent hour method for computing I^2R losses is probably the most direct. The following should be carefully noted in this connection, however:

When maximum load exists for a short period of the day, and the load is steady the remainder of the day, the equivalent hours are approximately proportional to the square of the load factor. On the other hand, when the load is steady for a part of the day and entirely off the remainder of the day, the equivalent hours are directly proportional to the load factor. When essentially maximum load is on a good portion of the day, and comparatively small the remainder of the day (a condition that is more or less typical of certain industrial and transmission loads, similar to the curves in Fig. 16), the equivalent hours are proportional to some value that lies between the load factor and the square of the load factor. From this it will be seen that the equivalent hours are dependent not only on the load factor but also on the shape of the load curve.

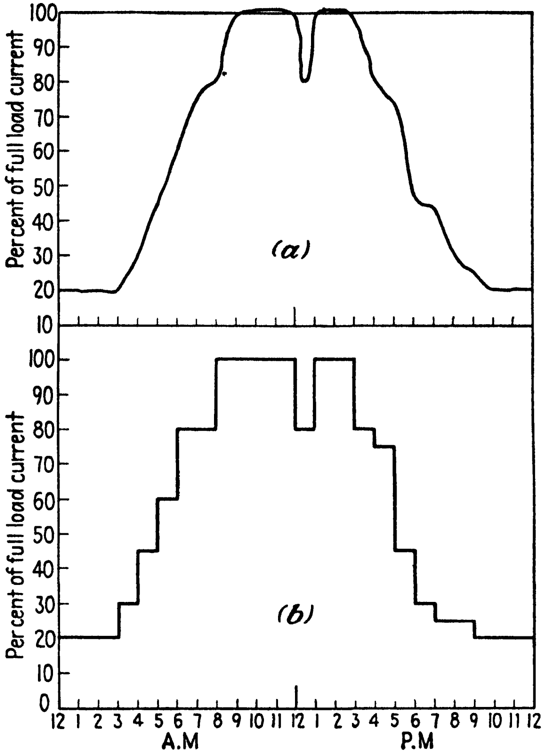

FIG. 16.—Daily load curve.

Refer to Fig. 16, which simulates a typical average transmission daily load curve. Figure 16*b* indicates how the actual load curve *a* is blocked off to facilitate an approximate breakdown into periods of time. Certain percentages of the total load are then carried throughout the day of 24 hours. It is to be noted that instead of actual current values, percentage of full load current is plotted as ordinates. This makes the curve applicable for use regardless of the value of full load current.

The designer cannot take too much for granted in problems of this nature. In one section of the country, the correct size of conductor for a given line may be dictated by one condition. From a purely economical standpoint, an entirely different set of conditions may be the governing factor for a similar problem in another section, or for some other utility. For example, where generating capacity greatly exceeds the demand for power, the wasted energy in the line is of no serious consequence, since it would afford no drain on the available supply. Its value would be equivalent to the cost of generation, and transmission. Under this condition, comparatively high I^2R losses are permissible. A conductor would then be chosen for which the sum of the annual fixed charges on the cost of conductor installed and the cost of wasted energy is a minimum, provided, however, that the voltage drop in the line does not exceed the permissible value.

On the other hand, where the system is fully loaded or short of generating capacity, *i.e.*, where there is a ready sale for all available power, the value of wasted energy would be equivalent to the established sale price per kwh. This would necessitate keeping the I^2R losses at a minimum value and would require a comparatively large and more costly conductor.

Some years ago, F. H. Buller and C. A. Woodrow, engineers of the General Electric Company, developed an approximate method for finding the equivalent hours from the load factor.[1] This equation was developed from a series of representative load curves, and is as follows:

$$F_e = 0.7(LF)^2 + 0.3(LF) \qquad (8)$$

where F_e = multiplying factor.
LF = load factor, expressed as a decimal.

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[1] See *Electrical World*, July 14, 1928, p. 59; also Aug. 18, 1928, p. 320.

Multiplying the total number of hours the line is in service during the year (8,760 hours for a transmission circuit) by the factor F_e will give the approximate equivalent hours. As pointed out by the authors, caution should be exercised in the use of this equation except for load factors between about 35 and 75 percent.

In Example 1, 2/0 copper conductor was arbitrarily chosen and was found to be satisfactory from a standpoint of voltage drop. It now remains to be seen whether the 2/0 copper meets the requirements of economy of operation. For the purpose of demonstrating the procedure, it is assumed that the average annual conditions for the 1,500-kw peak load given in the example are as indicated by the load curve (Fig. 16). The mean annual current method of analysis will be resorted to for the solution, with a check by the equivalent hour method.

An analysis of Fig. 16*b* gives the approximate period during the day that each different value of current, expressed as a percentage of maximum load current, is carried. From that figure, the heating factor can be derived by the following equation, and from this the mean annual current.

$$H_f = \sqrt{I_1^2 \times T_1 + I_2^2 \times T_2 + I_3^2 \times T_3} \tag{9}$$

where H_f = heating factor.

I_1 = maximum current carried for time T_1.

I_2 = current carried for time T_2, etc.

Substituting the values as read from Fig. 15*b* in (9) and for convenience setting the work down as follows, the value of H_f is derived.

Percent I		Percent T		
1.00^2	×	0.25000	=	0.250000
0.80^2	×	0.16667	=	0.106667
0.75^2	×	0.04167	=	0.023437
0.60^2	×	0.04167	=	0.015000
0.45^2	×	0.08333	=	0.016875
0.30^2	×	0.08333	=	0.007500
0.25^2	×	0.08333	=	0.005208
0.20^2	×	0.25000	=	0.010000
		Total		0.434687 $H_f = \sqrt{0.434687} = 0.6593.$

$$\text{Voltage to neutral} = \frac{22}{1.732} = 12.7 \text{ kv}$$

$$\text{Load to neutral} = \frac{1{,}500}{3} = 500 \text{ kw}$$

$$I_{\max} = \frac{500}{(12.7 \times 0.8)} = 49.2 \text{ amp}$$

The mean annual current is equal to the product of the heating factor H_f and peak load current $I = 0.6593 \times 49.2 = 32.44$ amp.

In order to determine which size of conductor actually gives the best economy, three different sizes of copper will be investigated, with the work carried out as in Table 17.

TABLE 17.—COMPARATIVE ANNUAL COST OF CONDUCTORS

Size H.D. copper	1/0	2/0	3/0
Area, cir mils	105,535	133,077	167,806
Resistance of 20 miles one conductor, ohms	11 098	8 806	6 99
Weight of three conductors, lb	103,200	130,200	164,400
Cost of conductor at 20¢ per lb	\$20,648	\$26,040	\$32,880
I^2R losses, kwh	306,898	243,516	193,298
Fixed charges at 10 percent	\$ 2,065	\$ 2,604	\$ 3,288
Cost of I^2R losses at 1¢ per kwh	\$ 3,069	\$ 2,435	\$ 1,933
Total annual costs	\$ 5,134	\$ 5,039	\$ 5,221

This shows the 2/0 copper to give the lowest annual cost. Unless some factor other than economy entered into the problem, this would be the size to use.

Equivalent-hour Method.—By analyzing load curve *b* (Fig. 16), we find the average load to be 28.19 amp, from which the load factor = 28.19/49.2 = 0.5729. Substituting this value in (8) and solving for F_e, we have

$$F_e = 0.7 \times (0.5729)^2 + 0.3 \times 0.5729 = 0.4016$$
$$0.4016 \times 8{,}760 = 3{,}518 \text{ equivalent hours}$$
$$I^2R \text{ loss} = 49.2^2 \times 8.806 \times 3{,}518 \times 3 = 224{,}970 \text{ kwh}$$

From Table 17, the actual I^2R loss for the 2/0 copper, calculated from the mean annual current, is 243,516 kwh. The I^2R loss calculated as above is therefore too low by

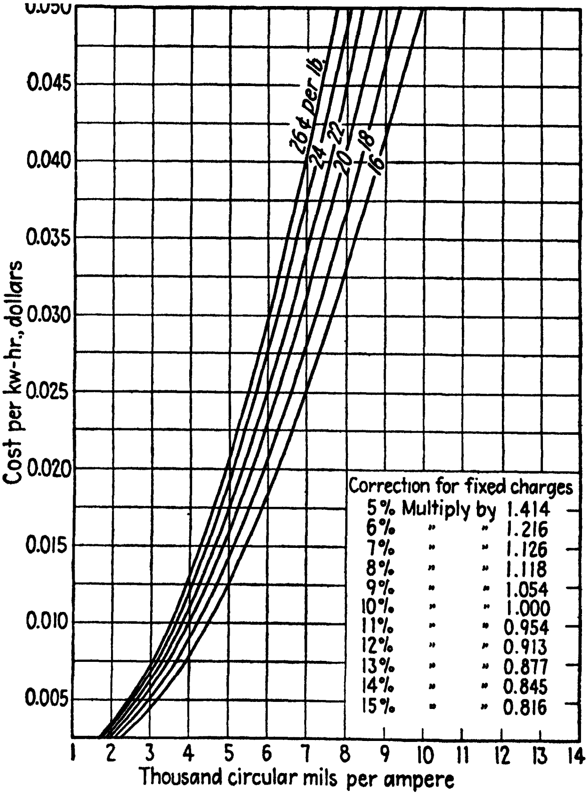
$$1 - (224{,}970/243{,}516) = 7.62 \text{ percent}$$

The correct value of F_e is the square of the heating factor

$H_f = 0.6593^2 = 0.4347$, from which $0.4347 \times 8{,}760 = 3{,}808$ equivalent hours. F_e, in this case, is approximately equal to

$$0.56(LF)^2 + 0.44 \times (LF) = 0.56 \times (0.5729)^2 + 0.44 \times 0.5729 = 0.4358$$

For a typical transmission load curve, this equation should give results within engineering accuracy.



Correction for fixed charges		
5%	Multiply by	1.414
6%	" "	1.216
7%	" "	1.126
8%	" "	1.118
9%	" "	1.054
10%	" "	1.000
11%	" "	0.954
12%	" "	0.913
13%	" "	0.877
14%	" "	0.845
15%	" "	0.816

Fig. 17.—Curves for determining economical size of copper based on mean annual current.

The work can be shortened considerably, after the mean annual current has been determined, by the use of the curves in Figs. 17 to 20. Figure 17 is for use with hard-drawn stranded copper. Figures 18 and 19 are for use with a.c.s.r., and Fig. 20 is for use with copperweld-copper.

The curves in Figs. 17 to 20 were plotted from values derived from the following equations, which are based on the fundamental principle of Kelvin's law.

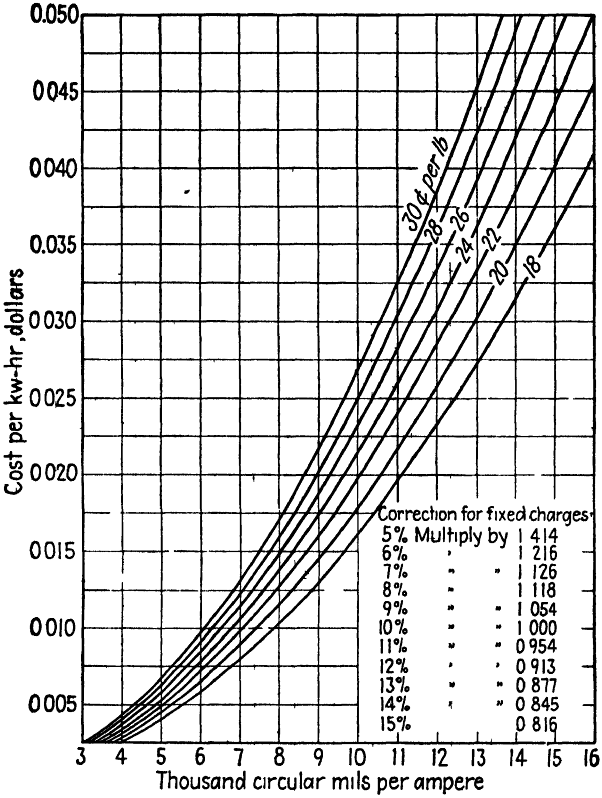


FIG. 18.—Curves for finding most economical a.c.s.r. for 6al/1st and 26al/7st Based on mean annual current

Hard-drawn bare stranded copper

$$\text{Cir mils per amp} = 55{,}867 \times \sqrt{\frac{C_e}{(C_c \times a)}} \tag{10}$$

A.c.s.r. 6al/1st

$$\text{Cir mils per amp} = 105{,}987 \times \sqrt{\frac{C_e}{(C_c \times a)}} \tag{11}$$

A.c.s.r. 26al/7st

$$\text{Cir mils per amp} = 106{,}207 \times \sqrt{\frac{C_e}{(C_c \times a)}} \tag{12}$$

A.c.s.r. 30al/7st

$$\text{Cir mils per amp} = 99{,}656 \times \sqrt{\frac{C_e}{(C_c \times a)}} \qquad (13)$$

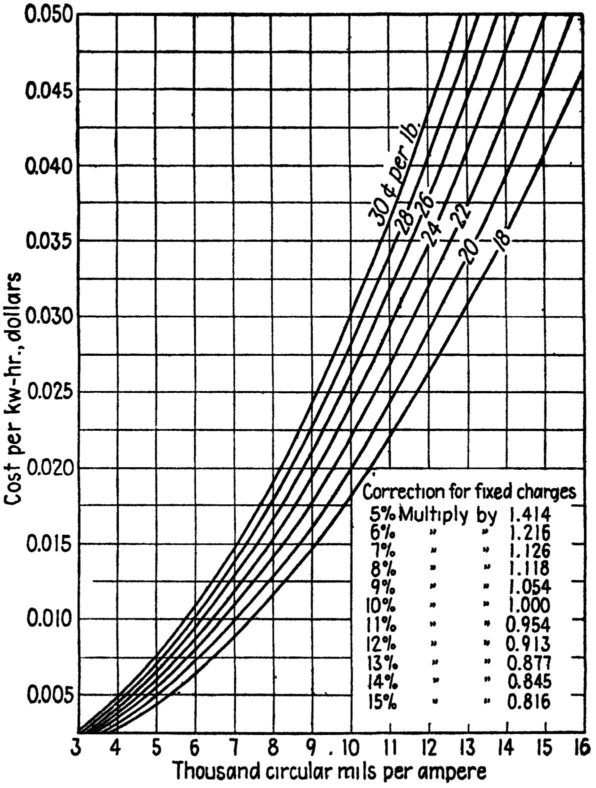
Type F copperweld-copper

$$\text{Cir mils per amp} = 61{,}164 \times \sqrt{\frac{C_e}{(C_c \times a)}} \qquad (14)$$

Type V copperweld-copper

$$\text{Cir mils per amp} = 63{,}233 \times \sqrt{\frac{C_e}{(C_c \times a)}} \qquad (15)$$

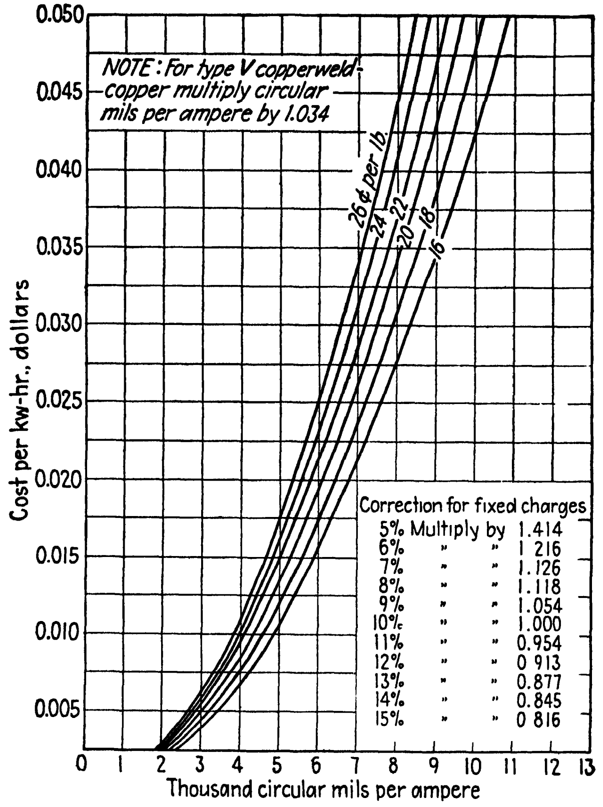
where C_e = cost per kwh, in dollars.
C_c = cost of conductor per pound in place, in dollars.
a = annual fixed charges, percent, expressed as a whole number.



Correction for fixed charges		
5%	Multiply by	1.414
6%	" "	1.216
7%	" "	1.126
8%	" "	1.118
9%	" "	1.054
10%	" "	1.000
11%	" "	0.954
12%	" "	0.913
13%	" "	0.877
14%	" "	0.845
15%	" "	0.816

FIG. 19.—Curves for finding most economical a.c.s.r. for 30al/7st. Based on mean annual current.

The constant of 55,867 in Eq. (10) was derived by Lord Kelvin.[1] The constants in Eqs. (11) to (15) were derived by the author. These constants are based on the effective 60-cycle resistance at a conductor temperature of 25°C. The difference in the constants for 6al/1st and 26al/7st is so small, only approximately 0.7 percent, that Fig. 18 may safely be used for both types of stranding.



NOTE: For type V copperweld-copper multiply circular mils per ampere by 1.034

Correction for fixed charges

5%	Multiply by	1.414
6%	" "	1.216
7%	" "	1.126
8%	" "	1.118
9%	" "	1.054
10%	" "	1.000
11%	" "	0.954
12%	" "	0.913
13%	" "	0.877
14%	" "	0.845
15%	" "	0.816

FIG. 20.—Curves for finding most economical type F copperweld-copper. Based on mean annual current. NOTE: For type V copperweld-copper multiply cir mils per ampere by 1.034.

For convenience, the curves have been plotted from values using as a basis 10 percent annual fixed charges on the cost of the conductor. A small table in the lower right-hand corner of each sheet gives correction factors for other rates of fixed charges, from 5 to 15 percent, inclusive. For example, assume a value of 13 percent fixed charge, 22¢ copper, and an energy rate of $0.0135 per kwh. Following downward in Fig. 17 from the intersection of the 22¢ curve and an energy rate of $0.0135 per kwh, we find at the bottom a value of 4,400 cir mils per amp. The multiplying factor from the table for 13 per cent is seen to be 0.877. Circular mils per ampere at

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[1] See "Standard Handbook for Electrical Engineers," 5th ed., p. 1102.

$$13 \text{ percent} = 4{,}400 \times 0.877 = 3{,}859$$

Other values are found in the same manner.

A study of Eqs. (10) to (15) will reveal the fact that, when the size of conductor is determined by the application of Kelvin's law, it is independent of the voltage and current and therefore bears no definite relation to the length of the line or the amount of power to be transmitted.

The loss factor method of computing annual power losses is somewhat more involved than either the equivalent hour or the mean annual current method. It is generally considered to be more applicable to the determination of losses in a distribution system and will therefore not be considered further.

Losses as Affected by Power Factor.—The advisability of maintaining a reasonably high lagging power factor of loads on a transmission circuit cannot be overemphasized, not only as it affects the area of conductor and power loss, but also from a standpoint of voltage regulation. Another item of considerable importance often overlooked in this connection is the additional station and line capacity made available at the higher power factors. Of course, the power factors of loads supplied over a given transmission line are not always susceptible to control by the electric utility. Many power companies, however, have a clause in their power contracts that penalizes power users for power factors falling below a certain specified level. The penalty imposed, in the form of additional power cost, usually makes it advantageous for comparatively large power users to install power factor corrective equipment.

The curves in Figs. 7 to 14 indicate the ratio of voltage drop at various power factors to the drop at 0.8 power factor. They also demonstrate quite conclusively that no definite relation exists among voltage drop, power factor, and size of conductor. A definite relation does exist, however, among line current,

TABLE 18 —RELATIVE I^2R LOSSES AS AFFECTED BY POWER FACTOR

Pf, cos of angle of lag	Pf, cos of angle of lag														
	0 50	0 55	0 60	0 65	0 70	0 75	0 80	0 85	0 90	0 95	0 96	0 97	0 98	0 99	1 00
0 50	1 000	1 210	1 440	1 690	1 960	2 250	2 560	2 890	3 240	3 160	3 686	3 764	3 842	3 920	4 000
0 55	0 826	1 000	1 190	1 397	1 620	1 860	2 116	2 388	2 678	2 983	3 047	3 110	3 175	3 240	3 306
0 60	0 694	0 840	1 000	1 174	1 360	1 563	1 780	2 007	2 248	2 507	2 560	2 610	2 670	2 723	2 775
0 65	0 565	0 716	0 852	1 000	1 160	1 331	1 515	1 710	1 917	2 136	2 181	2 227	2 273	2 320	2 367
0 70	0 510	0 617	0 735	0 862	1 000	1 148	1 307	1 474	1 652	1 842	1 881	1 920	1 960	2 000	2 040
0 75	0 444	0 538	0 640	0 751	0 871	1 000	1 138	1 284	1 440	1 604	1 638	1 670	1 708	1 742	1 778
0 80	0 396	0 473	0 562	0 660	0 765	0 879	1 000	1 129	1 265	1 410	1 440	1 470	1 501	1 531	1 561
0 85	0 346	0 419	0 498	0 585	0 678	0 779	0 886	1 000	1 121	1 249	1 276	1 302	1 329	1 357	1 384
0 90	0 309	0 373	0 445	0 522	0 605	0 694	0 790	0 892	1 000	1 114	1 138	1 162	1 186	1 210	1 233
0 95	0 277	0 335	0 399	0 468	0 543	0 624	0 709	0 800	0 897	1 000	1 021	1 043	1 064	1 086	1 108
0 96	0 271	0 328	0 391	0 458	0 532	0 610	0 694	0 784	0 879	0 979	1 000	1 021	1 042	1 063	1 085
0 97	0 266	0 321	0 383	0 449	0 521	0 598	0 680	0 768	0 861	0 959	0 979	1 000	1 021	1 042	1 063
0 98	0 260	0 315	0 375	0 440	0 510	0 586	0 666	0 752	0 843	0 940	0 960	0 980	1 000	1 021	1 041
0 99	0 255	0 309	0 367	0 432	0 500	0 574	0 653	0 737	0 826	0 921	0 940	0 960	0 980	1 000	1 020
1 00	0 250	0 303	0 360	0 423	0 490	0 562	0 640	0 723	0 810	0 903	0 922	0 949	0 960	0 980	1 000

I^2R losses, and power factor. For any given load, the current varies inversely as the power factor, while the I^2R loss varies inversely as the square of the power factor. Table 18 has been compiled to facilitate finding the relation between I^2R loss and power factor.

To use Table 18, proceed as follows: Assume that we have an original power factor of 0.75 and want to know the relative I^2R loss at 0.95 power factor. Since the I^2R loss varies as the square of the power factor, $(0.75^2/0.95^2 = 0.624)$. From 0.75 power factor at the top, follow down to the horizontal line corresponding to 0.95 power factor at the left, where we find the value 0.624. If the I^2R loss, for example, is 100 kw at 0.75 power factor, it will be 62.4 kw at 0.95 power factor. Conversely, if the original power factor is 0.95 and the I^2R loss 62.4 kw $(0.95^2/0.75^2 = 1.604)$. $1.604 \times 62.4 = 100$ kw. The relative values for any other condition, within the range of the table, are found in the same manner. For values falling below 0.95 power factor, given in intervals of 5, direct interpolation will be sufficiently accurate for all practical purposes.

For short transmission lines, since the capacitance is so small as to have no appreciable effect on the performance of the circuit, the current for a given load and power factor is uniform throughout the entire length of the line. Under this condition, maximum line efficiency will occur at unity power factor. This is not true, however, as will be shown later, for high-voltage lines of any considerable length.

CHAPTER V

MEDIUM-LENGTH AND LONG TRANSMISSION LINES

Medium-length Lines.—A.c. transmission lines having a length of 50 miles and longer are not susceptible to the same method of analysis, with the same degree of accuracy, as are the lines of 50 miles and less in length treated in Chap. IV. This is because, when a.c. voltage is impressed on the conductors of an overhead transmission circuit, the conductors of which are separated by a dielectric (such as air), the circuit acts to a certain extent as a condenser. It is alternately charged and discharged, producing a current (known as "charging current"), which in a sense passes through the air between the conductors. This charging current is in shunt by 90 deg with the voltage producing it.

The charging current of a transmission circuit, at a given voltage and conductor spacing, varies directly as the voltage, the frequency, and the line length. The charging current flowing over the inductance of the circuit produces a voltage rise along the line which increases in value from the sending end toward the receiving end. The voltage rise produced by the charging current varies directly as the line voltage and the square of the line length. The charging kva of a transmission circuit is equal to three times the product of the charging current and the sending-end voltage to neutral. The charging kva varies as the square of the voltage to neutral and directly as the line length.

In the treatment of lines from 50 to 100 miles in length, which we shall designate as medium-length lines, the calculations are only slightly more involved than are those required for shorter lines. The results derived by the simplified methods are well within engineering accuracy. In Example 3, the value derived by rigorous solution, which takes into account the non-uniform distribution (known as "distribution effect") of the voltage and current along the circuit, is given in parentheses, in addition to the calculated result, for comparison with the results derived from the simplified method of solution for the 75-mile line.

The commonly accepted symbol for charging current is I_c. It would seem to be preferable, however, to use the symbol I_2. For example, in the calculations for long lines, the symbol I_1 is used to designate the receiving-end current modified by the distribution effect of the circuit. I_s, the line current at the sending end of the line, is therefore equal to the vector sum of the receiving-end current I_1 and the line-charging current I_2.

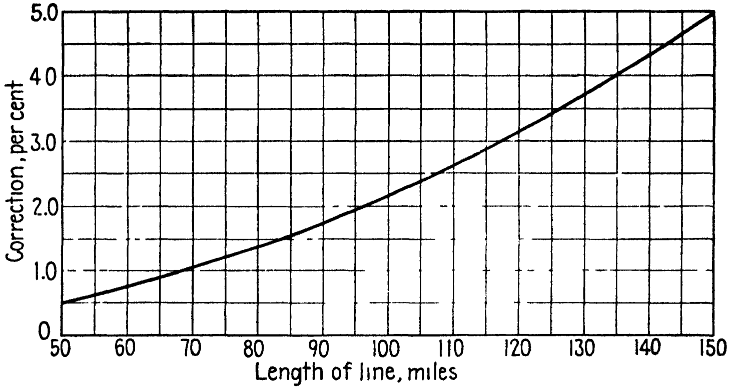

Fig. 21.—Voltage-drop correction for distance. To obtain net drop subtract percent correction from calculated drop.

One of the first items of information to be determined for any transmission-line problem is the size of conductor required for a given load, at the desired or available voltage, and the transmission distance, to ensure that the voltage drop will not exceed a certain permissible value. By the use of Eq. (7) and Fig. 21, the size of conductor for a given voltage drop may be determined almost immediately. Figure 21 has been designed for use in correcting for voltage rise produced by the charging current flowing through the inductance of the circuit. The use of this curve will be demonstrated in Example 3.

Example 3. For a 66-kv 60-cycle three-phase 75-mile line, consisting of 3/0 twelve-strand copper, equivalent spacing of conductors 7 ft, and loaded to 4,200 kw at 0.8 power factor, it is required to find the following:

a. E_s, voltage at the sending end.
b. I_2, line charging current in amperes.
c. I_s, line current at the sending end.
d. $(Pf)_s$, power factor at the sending end.

e. $(Kva)_{sn}$, $(Kw)_{sn}$
f. Phase angles at full load.
Load to neutral = 4,200/(0.8 × 3) = 1,750 kva
$E_{rn} = 66/1.732 = 38.106$ kv
$I_r = 1{,}750/38{,}106 = 45.92$ amp

Solution for a. From Table 16, the drop per kw per mile at 100 volts, for 3/0 twelve-strand copper, is 9.141 volts, from which

$$\frac{4{,}200 \times 75 \times 9.141}{660} = 4{,}363 \text{ volts drop} = 6.61 \text{ percent of } 66{,}000 \text{ volts}$$

From Fig. 21, the correcting factor for 75 miles is seen to be 1.21 percent. Subtracting this from the calculated voltage drop, we have 6.61 − 1.21 = 5.4 percent net drop.

$$E_s = 66{,}000 + (0.054 \times 66{,}000) = 69{,}564 \ (69{,}555) \text{ volts}$$
$$E_{sn} = 69{,}564/1.732 = 40{,}164 \ (40{,}159) \text{ volts to neutral}$$

Solution for b.—From Table 7, for 3/0 twelve-strand copper, the capacity susceptance b is given as 5.779 micromhos per mile of single conductor.

Charging current $I_2 = E_{rn} \times b \times 10^{-6} = 38.106 \times 5.779 \times 10^{-6} = 0.220215$ amp per mile
Total charging current = 75 × 0.220215 = 16.52 (16.45) amp

Solution for c

$$I_s = \sqrt{(I_r \cos \theta)^2 - (I_r \sin \theta - I_2)^2} \tag{16}$$

The values under the radical are 45.92 × 0.8 = 36.74 and (45.92 × 0.6) − 16.52 = 11.04.

From (16) $I_s = \sqrt{(36.74)^2 - (11.04)^2} = 38.36$ (37.92) amp

Solution for d

I_s lags E_{rn} by the angle whose

$$\cos = (I_r \times pf)/I_s = (45.92 \times 0.8)/38.36 = 0.9577$$

The angle = 16°43′. sin 16°43′ = 0.2877.

$$(Pf)_s = \frac{0.9577 + (I_sR)/E_{rn}}{0.2877 + (I_sX)/E_{rn}} \tag{17}$$

From Table 4, the resistance $r = 0.3495$ ohm per mile.

$$R = 0.3495 \times 75 = 26.21 \text{ ohms total resistance}$$

$$x = 0.7411 \text{ ohm per mile reactance.} \quad X = 75 \times 0.7411 = 55.58 \text{ ohms total reactance}$$

$$(I_sR)/E_{rn} = (38.36 \times 26.21)/38{,}106 = 0.0264$$

$$(I_sX)/E_{rn} = (38.36 \times 55.58)/38{,}106 = 0.0559$$

From (17) $(Pf)_s = \dfrac{0.9557 + 0.0264}{0.2877 + 0.0559} = 0.944(0.949)$ pf

Solution for e

$$(Kva)_{sn} = I_s \times E_{sn} = 38.36 \times 40.164 = 1{,}541\ (1{,}523) \text{ kva}$$

$$(Kw)_{sn} = I_s \times E_{sn} \times (pf)_s = 38.36 \times 40.164 \times 0.944 = 1{,}455\ (1{,}445) \text{ kw per phase}$$

Solution for f. The current at the sending end leads the current at the receiving end by 36°52′ − 16°43′ = 20°09′ (20°41′). The voltage at the sending end leads the voltage at the receiving end by 19°15′ − 16°43′ = 2°32′ (2°11′). The current at the sending end lags the voltage at the sending end by

$$2°32' + 16°43' = 19°15' \ (18°22')$$

$(Pf)_s = \cos 19°15' = 0.944$, as above.

In order to visualize the problem as a whole, a vector diagram, such as that shown in Fig. 22, should always be constructed for problems applying to lines of this character.

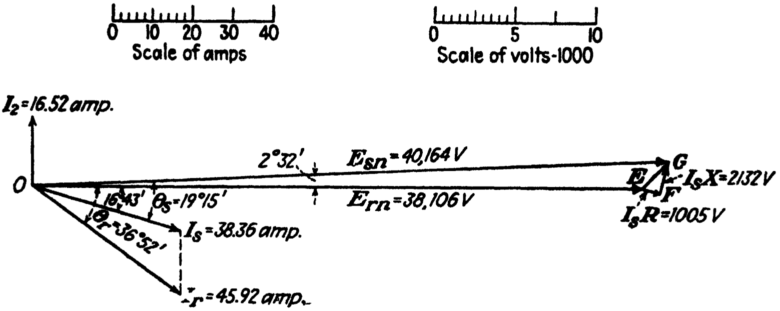

FIG. 22.—Vector diagram for loaded transmission line of Example 3.

To construct the diagram, as in Fig. 22, for which positive or counterclockwise rotation of the vectors is assumed, proceed as follows:

Lay off line OE horizontally to any convenient scale and equal to E_{rn}, $= 38{,}106$ volts. The scale should be such as to permit a fairly accurate construction of the impedance triangle, since the IR and IX drops must be drawn to the same scale as OE. Lay off from O vector I_r, the receiving-end current vector, displaced from OE by 36°52′ (0.8 power factor angle) in a lagging direction and equal to 45.92 amp. Any suitable scale may be used for the current vectors, but they should be of such length as to form a fairly well proportioned diagram. Next, draw from O vector I_s equal to 38.36 amp and lagging E_{rn} by 16°43′. Draw from O vector I_2, the charging current, equal to 16.52 amp and leading E_{rn} by 90 deg. Draw EF parallel with vector I_s and equal to $I_sR = 38.36 \times 26.21 = 1{,}005$ volts to the same scale as OE. Draw FG equal to $I_sX = 38.36 \times 55.58 = 2{,}132$ volts. Now draw in OG to represent the voltage at the sending end of the line $= E_{sn} = 40{,}164$ volts. This completes the diagram and affords a good illustration of the various items of voltage, current, and impedance, in both phase and magnitude.

Care should be exercised in attempting to compute the power loss by the I^2R method when the line is of such length that the electrostatic capacity (capacitance) is of sufficient value to affect the performance of the circuit materially. In this case, the receiving-end current is 45.92 amp, from which I^2R would $= (45.92)^2 \times 26.21 = 55.3$ kw per phase. On the other hand, using the sending-end current $I^2R = (38.36)^2 \times 26.21 = 41.2$ kw per phase. It is evident that the power loss derived by either of the above methods will be considerably in error. By using the average of I_r and I_s as a basis for computing the I^2R losses, for lines not exceeding about 100 miles in length, the results will be sufficiently accurate for all practical purposes. For example,

$$(45.92 + 38.36)/2 = 42.14 \text{ amp.}\quad (42.14)^2 \times 26.21 = 46.5 \text{ kw per phase}$$

This checks within 8.5 kw of the value by the simplified method, and within 1.5 kw of the value by the rigorously calculated method.

Calculations for Long Lines.—Analysis of transmission-line design and performance for a given condition, aside from its other properties, is predicated on the length of the line in miles.

The physical length of a line is primarily of value as a basis for determining its electrical length. The electrical length of a transmission circuit is a function of its impedance and must be known before the performance of the line can be determined. Two lines may be of the same length, measured in feet or miles; but they may differ greatly in their electrical length. A line with a conductor of small cross section would have a greater electrical length than a line otherwise the same with a large conductor.

Complex Algebra.—We have been considering what may be called "medium-length lines," using simplified methods of analysis that have been demonstrated to give, for the most part, a fair degree of accuracy. For the analysis of long lines, *i.e.*, lines having a length in excess of 100 miles, it will be necessary to diverge considerably from the methods previously used and to resort to complete vector analysis, or, in other words, the use of complex algebra. This method of analysis, since it takes into account the distribution effect of the circuit, gives exact results for the known or assumed conditions.

The designer will usually be given to understand that, as transmission lines increase in length and importance, the time element does not enter into the situation to the extent that only roughly approximate results are required. The voltage drop for any line may be determined very quickly but is only one step in the complete analysis that will generally be required.

A working knowledge of the application of the principle of vector analysis by the use of equations in complex form is assumed. The following may be of value, however, in clarifying the subject in the minds of thóse who are not thoroughly familiar with the use of complex quantities as applied to the solution of transmission-line problems.

1. $a + jb$ and $R + jX$ are complex algebraic expressions in terms of rectangular coordinates. a, in the first expression, and R, in the second expression, are taken along the horizontal or axis of reals. jb and jX represent, in mathematical language, the imaginary component taken at right angles to the axis of reals. When a term is prefixed by the letter j, the j quantity is to be taken at right angles to a line called the "vector of reference," either above or below, depending on whether it is preceded by a plus or by a minus sign. As will shortly be

evident, the prefix j bears an important relation to the algebraic operations, such as addition, subtraction, and multiplication.

Let $R + jX = Z\underline{/\theta}$, where $R = 3$ ohms resistance and $X = 4$ ohms reactance. $3 + j4 = 5\underline{/53°08'}$ ohms impedance. $R + jX$, as previously stated, are expressed in rectangular coordinates. $Z\underline{/\theta} = 5\underline{/53°08'}$ is an expression in polar coordinates and indicates a magnitude of 5 ohms having an angular displacement of 53°08′ in a leading direction, *i.e.*, above the reference vector. This is the angle whose tangent $= \frac{4}{3}$, written $\tan^{-1} \frac{4}{3}$. The absolute value of $3 + j4 = \sqrt{3^2 + 4^2} = 5$ ohms.

2. The quantities 3 and $j4$ are the quadrature components, since they are displaced from each other by 90 deg; and the quantity 5 is the vector sum, or resultant of the components 3 and $j4$. This might logically be called "component analysis." In electrical engineering problems, all-vectors are referred to a given line called the "vector of reference." This is usually taken as the receiving-end voltage vector E_{rn}. The j component is leading or lagging, depending on whether it is a plus or a minus quantity. Counterclockwise rotation of the vector is assumed, since this is considered positive and standard for transmission-line vector diagrams.

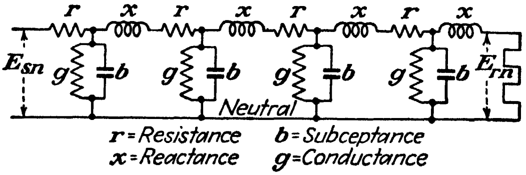

FIG. 23.—Equivalent circuit to neutral.

Figure 23 represents, symbolically, a physical conception of what takes place in a transmission circuit. r and x represent the unit resistance and reactance of the circuit in ohms. b and g represent the unit susceptance and conductance of the circuit in mhos. This is as the constants would appear in an artificial circuit shown lumped at four places along the line.

The conductance g, which represents leakage over the insulators, corona discharge, etc., for a well-designed line, may be taken as zero. The susceptance b, for lines of comparatively high voltage and of any considerable length, is an all-important property because the calculations required in the electrical design of the line are largely involved with the capacity susceptance of the circuit. The capacitance, or condenser effect, produces a charging current which, flowing through the resistance and reactance of the circuit, modifies the linear constants (resistance and reactance) requiring for correct solution the application of what are known as the auxiliary constants A, B, and C. Through the application of the auxiliary constants, the non-uniform distribution of the voltage and current along the line is taken into account. This non-uniform distribution of the voltage and current throughout the length of the line, as previously explained, is known as the "distribution effect" of the circuit.

Auxiliary Constants.—The auxiliary constants A, B, and C of a transmission circuit are functions of the physical properties of the line and the frequency. They are entirely independent of the voltage and current.

$A = a_1 + ja_2$. The auxiliary constants a_1 and a_2 determine the length and position of the sending-end voltage vector at zero load. For all practical purposes, the constant a_1 may be taken as equal to the receiving-end voltage minus the reactance volts drop due to the charging current. Constant a_2 represents the resistance volts drop due to the charging current for each volt at the receiving end of the line. Since the charging current leads the receiving-end voltage by 90 deg, the resistance volts drop produced by the charging current is at right angles to E_{rn} in a leading direction.

$B = b_1 + jb_2$. Constants b_1 and b_2 represent the resistance and reactance in ohms modified by the distribution effect of the circuit. The product of these constants and the receiving-end current I_r will give the voltage consumed by the resistance and by the reactance of the circuit, respectively. Because of the distribution effect, b_1 and b_2 will be somewhat less in value than the linear constants R and X. This difference will increase as the length of the line increases. The impedance triangle for a line of appreciable capacitance will be smaller than for a line with no capacitance.

$C = c_1 + jc_2$. Constants c_1 and c_2 represent the conductance and susceptance of the circuit. The value of c_1, except for lines of great electrical length, may safely be considered negligible. Constant c_2 represents the line charging current for each volt at the receiving end of the line. The product of c_2 and the receiving-end voltage to neutral will give the total line charging current.

There are two practical and comparatively easily applied methods for deriving the auxiliary constants A, B, and C. The auxiliary constants may be derived by the application of the convergent series and also by the application of hyperbolic functions. The solution by convergent series is generally to be preferred, since it embodies only an understanding of the simple rules of algebra. Many engineers have an aversion to the use of complex quantities, but this can usually be traced to their unfamiliarity with the subject. After a little practice with the method, the work becomes quite simple and interesting.

The auxiliary constants A, B, and C may be derived from the following equations:

$$A = \left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{24} + \cdots\right) \tag{18}$$

$$B = Z\left(1 + \frac{YZ}{6} + \frac{Y^2Z^2}{120} + \cdots\right) \tag{19}$$

$$C = Y\left(1 + \frac{YZ}{6} + \frac{Y^2Z^2}{120} + \cdots\right) \tag{20}$$

For the purpose of demonstrating a method of carrying out the design calculations for determination of the complete line performance, the following problem is assumed:

> *Example* 4. 110-kv, 125-mile 60-cycle three-phase transmission line, consisting of 336,400 cm a.c.s.r. (equivalent 4/0 copper) 26al/7st having an equivalent spacing of 15.75 ft, supplying a load of 30,000 kw, 0.85 lagging receiver power factor.
>
> *a.* Calculate the auxiliary constants A, B, and C, for the 125-mile line.
> *b.* Calculate the performance of the line at 100, 75, 50, 25, and 0 percent loads.
> *c.* Construct the performance diagram.

Linear Constants.—If we had elected to have the equivalent spacing of the conductors correspond to those of a given table value, as is rarely the case, it would be a simple matter to read the linear constants directly from tables. However, since the greater value should be obtained from the solution by actually performing the operations, the odd spacing of conductors has purposely been chosen.

$r = 0.278$ ohm resistance per mile from Table 8.
From Table 8 at 15-ft equivalent spacing, $x = 0.779$ ohm inductive reactance
$D'/D = 15.75/15 = 1.05$. From Fig. 5, the adder corresponding to $D'/D = 1.05 = 0.006$
$x = 0.779 + 0.006 = 0.785$ ohm reactance at 15.75-ft spacing in ohm per mile
$z = r + jx = 0.278 + j0.785 = 0.8328$ ohm impedance per mile
$c = (0.038829)/\log_{10}(2D/d)$ capacitance in microfarads per mile
$D = 15.75 \times 2 \times 12 = 378$ in. spacing of conductors
$d = 0.721$ in., diameter of conductor
$2D/d = 378/0.721 = 524.27$. $\log_{10} 524.27 = 2.719555$
$c = 0.038829/2.719555 = 0.014277$ microfarads per mile
$b = 2\pi fc = 377 \times 0.014277 \times 10^{-6} = 5.38255 \times 10^{-6}$ mho susceptance per mile
$g =$ conductance (taken as zero)
$R = rl = 0.278 \times 125 = 34.75$ ohms total resistance
$X = xl = 0.785 \times 125 = 98.125$ ohms total reactance
$Z = R + jX = 34.75 + j98.125 = 104.1$ ohms total impedance
$B = bl \times 10^{-6} = 5.38255 \times 125 \times 10^{-6} = 0.0006728$ mho total susceptance
$G = Gl =$ zero mho conductance
$Y = G + jB = 0 + j0.0006728 = 0.0006728$ mho total admittance

Solution for a. Solution for the Auxiliary Constants A, B, and C.—Multiplication of YZ

$$
\begin{array}{rr@{\;}l}
Y = & 0 & + j0.0006728 \\
Z = & 34.75 & + j98.125 \\
\hline
YZ = & -0.0660185 & + j0.0233798 \\
YZ = & -0.0660185 & + j0.0233798 \\
\hline
 & 0.0043584 & - j0.0015435 \\
 & -0.0005466 & - j0.0015435 \\
\hline
Y^2Z^2 = & 0.0038118 & - j0.0030870
\end{array}
$$

Substituting in (18), setting the work down as follows, and solving for constant A, we have

$$\begin{array}{rl}
 & 1.0000000 + j0.0000000 \\
(YZ)/2 = & -0.0330093 + j0.0116899 \\
(Y^2Z^2)/24 = & 0.0001586 - j0.0001286 \\ \hline
a_1 + ja_2 = & 0.9671496 + j0.0115613 \\
\text{Say} & 0.96715 + j0.01156 \\
= & 0.96722\underline{/0^\circ 41'}
\end{array}$$

Substituting in (19) and solving for constant B

$$\begin{array}{rl}
 & 1.0000000 + j0.0000000 \\
(YZ)/6 = & -0.0110031 + j0.0038966 \\
(Y^2Z^2)/120 = & 0.0000318 - j0.0000257 \\ \hline
 & 0.9890287 + j0.0038709 \\
Z = & 34.75 + j98.125 \\ \hline
 & 34.3687 + j\,0.1345 \\
 & -\ 0.3798 + j97.0484 \\ \hline
b_1 + jb_2 = & 33.9889 + j97.1829 \\
\text{Say} & 33.99 + j97.18 \\
= & 102.95\underline{/70^\circ 43'}
\end{array}$$

Substituting in (20) and solving for constant C

$$\begin{array}{lrl}
\text{From (19)} & & 0.9890287 + j0.0038709 \\
 & Y = & 0 + j0.0006728 \\ \hline
 & c_1 + jc_2 = & -0.0000026 + j0.0006654 \\
 & = & 0.0006654\underline{/90^\circ 14'}
\end{array}$$

Summarizing

$$\begin{aligned}
a_1 + ja_2 &= 0.96715 + j0.01156 \\
A &= 0.96722\underline{/0^\circ 41'} \\
b_1 + jb_2 &= 33.99 + j97.18 \\
B &= 102.95\underline{/70^\circ 43'} \\
c_1 + jc_2 &= -0.0000026 + j0.0006654 \\
C &= 0.0006654\underline{/90^\circ 14'}
\end{aligned}$$

Any desired degree of accuracy may be obtained by extending the series in Eqs. (18) to (20); however, for lines not exceeding 100 to 150 miles in length, two places in the series will give results sufficiently accurate for all practical purposes.

The major difficulty encountered by those who find it necessary to use complex quantities only occasionally is with the signs. It is therefore suggested that the following notations be kept in an available place for ready reference.

$$\begin{aligned} j \times j &= j^2 = -1 \\ j \times -j &= +1 \\ j \times +1 &= +j \\ j \times -1 &= -j \\ -j \times -1 &= +j \\ -j \times -j &= -1 \end{aligned}$$

Exact Equations of the Transmission Circuit

$$\begin{aligned} E_{sn} &= E_{rn} + I_r B \\ &= E_1 + E_2 \end{aligned} \tag{21}$$

$$\begin{aligned} I_s &= I_r A + E_{rn} C \\ &= I_1 + I_2 \end{aligned} \tag{22}$$

where E_{sn} = voltage to neutral at sending end, load conditions.
E_1 = voltage to neutral at sending end, zero load (constant).
E_2 = impedance drop modified by the distribution effect.
I_s = current at the sending end, load conditions.
I_1 = current at receiving end modified by the distribution effect.
I_2 = total line charging current.

The numerical values for substitution in these equations are expressed in complex form.

Solution for b

Voltage to neutral = E_{rn} = 110,000/1.732 = 63,510 volts
Kw to neutral = 30,000/3 = 10,000
Kva to neutral = 30,000/(3 × 0.85) = 11,765

$$\begin{aligned} I_r &= 11{,}765/63.510 = 185.25 \text{ amp} \\ &= 185.25(.85 - j0.5268) = 157.46 - j97.59 \text{ vector amp} \\ E_1 &= E_{rn}(a_1 + ja_2) \\ &= 63{,}510(0.96715 + j0.01156) = 61{,}424 + j\ 734 \\ E_2 &= I_r(b_1 + jb_2) \\ &= (157.46 - j97.59)(33.99 + j97.18) = \underline{14{,}836 + j11{,}985} \end{aligned}$$

From (21) $E_{sn} = E_1 + E_2$ $= 76{,}260 + j12{,}719$
$= 77{,}313/\underline{9°28'}$ volts

By following the practice of expressing the absolute value of voltage and current in polar form, the phase angle of any particular value may be found. For example,

$$12{,}719/76{,}260 = 0.1667 = \tan 9°28'$$

This would be written $\tan^{-1} 12{,}719/76{,}260 = 9°28'$ and signifies that E_{sn} leads E_{rn} by 9°28′. All vectors that lag the reference vector will have the angle prefixed by a minus sign.

$$\begin{aligned}
I_1 &= I_r(a_1 + ja_2) \\
&= (157.46 - j97.59)(0.96715 + j0.01156) = 153.42 - j92.56 \\
I_2 &= E_{rn}(c_1 + jc_2) \\
&= 63{,}510(-0.0000026 + j0.0006654) = \underline{-0.16 + j42.26} \\
\text{From (22), } I_s &= I_1 + I_2 = 153.26 - j50.30 \\
&= 161.3\underline{/-18°10'} \text{ amp}
\end{aligned}$$

$(Kw)_{sn} = (76.260 \times 153.26) + (12.719 \times -50.3)$
$= 11{,}688 - 640 = 11{,}048$ kw per phase at the sending end

$(Kva)_{sn} = 77.313 \times 161.3 = 12{,}471$ kva per phase at the sending end

$(Pf)_s = 11{,}048/12{,}471 = 0.8859$ lagging power factor at the sending end

Power loss $= 11{,}048 - 10{,}000 = 1{,}048$ kw per phase

Transmission efficiency $= 10{,}000/11{,}048 = 0.906 = 90.6$ percent

Phase Angles at 100 *Percent Load.*—The voltage at the sending end leads the voltage at the receiving end by 9°28′. The current at the sending end lags the voltage at the receiving end by 18°10′. The current at the sending end therefore lags the voltage at the sending end by 9°28′ + 18°10′ = 27°38′. The power factor at the sending end is equal to the cos of 27°38′ = 0.8859, as above.

Performance at Zero Load

$$\begin{aligned}
E_1 &= 61{,}424 + j734 \text{ vector volts} \\
E_2 &= \underline{0 \qquad + j0} \\
E_{sn} &= 61{,}424 + j734 = 61{,}428\underline{/0°41'} \text{ volts} \\
I_1 &= 0 \qquad + j0 \\
I_2 &= \underline{-0.16 + j42.26} \\
I_s &= -0.16 + j42.26 = 42.26\underline{/90°14'} \text{ amp}
\end{aligned}$$

$$
\begin{aligned}
(Kw)_{sno} &= 61.424 \times -0.16 + 0.734 \times 42.26 \\
&= -9.83 + 31.02 = 21.19 \text{ kw per phase at zero load} \\
(Kva)_{sno} &= 61.428 \times 42.26 = 2{,}596 \text{ kva per phase at zero load} \\
(Pf)_{so} &= 21.19/2{,}596 = 0.00816. \text{ Say 0.82 percent leading.}
\end{aligned}
$$

TABLE 19.—VOLTAGE AND CURRENT DISTRIBUTION

Per-cent load	E_1 volts	E_2, volts	E_{sn}, volts	I_1, amp	I_2, amp	I_s, amp
100	61,428	19,071	$77{,}313\underline{/9^\circ 28'}$	179.17	42.26	$161.30\underline{/-18^\circ 10'}$
75	61,428	14,303	$73{,}198\underline{/7^\circ 38'}$	133.63	42.26	$117.36\underline{/-13^\circ 11'}$
50	61,428	9,356	$69{,}169\underline{/5^\circ 35'}$	89.59	42.26	$76.19\underline{/-3^\circ 39'}$
25	61,428	4,768	$65{,}420\underline{/3^\circ 17'}$	44.79	42.26	$42.71\underline{/26^\circ 36'}$
0	61,428	0	$61{,}428\underline{/0^\circ 41'}$	0.00	42.26	$42.26\underline{/90^\circ 14'}$

TABLE 20.—TOTAL THREE-PHASE POWER AND POWER LOSS

percent load	$(KW)_s$	$(KW)_r$	Kw loss	$(KVA)_s$	I_r	$(PF)_s$ per cent
100	33,144	30,000	3,144	37,413	185.25	−88.59
75	24,090	22,500	1,590	25,771	138.94	−93.47
50	15,606	15,000	606	15,870	92.63	−98.70
25	7,676	7,500	176	8,362	46.31	91.83
0	64	0	64	7,788	0.00	0.82

Minus sign indicates lagging power factor.

Solution for c. In general, transmission-line performance diagrams leave much to be desired from a standpoint of depicting conditions over the entire range of possible operation. It is important for the operating engineer to know in advance as much as possible about how a given line will behave under any and all possible operating conditions. In order to visualize such information readily, a diagram such as Fig. 24, which shows all values of voltage and current, in both phase and magnitude, from 100 percent to zero load, should be constructed for all important high-voltage lines. The values on this diagram are given in load intervals of 25 per cent. Values of voltage and current applying to other load conditions can be scaled from the diagram, since the scale is such as to permit of fairly accurate reading.

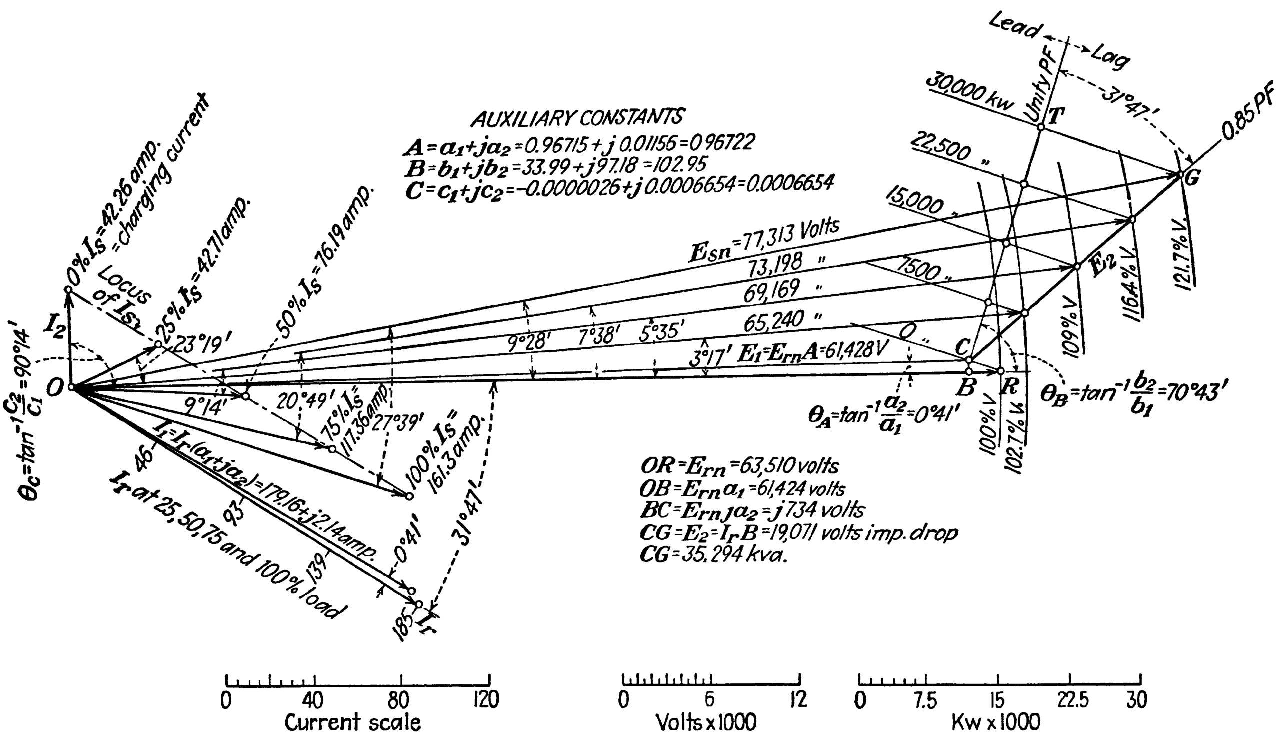

FIG. 24.—Performance diagram for 110-kv 125 mile line

Construction of the Diagram.—Lay off line OR to any convenient scale and equal to $E_{rn} = 63{,}510$ volts. This will be the reference vector. Locate on OR point B a distance from O equal to $E_{rn}a_1 = 63{,}510 \times 0.96715 = 61{,}424$ volts. Draw BC at right angles to OR and equal to

$$E_{rn}ja_2 = 63{,}510 \times j0.01156 = j734 \text{ volts.}$$

Then $OC = E_{rn}A = 63{,}510 \times 0.96722 = 61{,}428$ volts. $OC = E_1$ and determines the length and position of the sending-end voltage vector at zero load.

Next, draw from C the unity power factor line making an angle $\theta_b = 70°43'$ with vector OR as shown. Draw from C line CG, which represents the impedance drop at 100 percent load $= I_r(b_1 + jb_2) = (157.46 - j97.59)(33.99 + j97.18) = 14{,}836 + j11{,}985 = 19{,}071$ volts displaced from OR by angle $\tan^{-1} 11{,}985/14{,}836 = 38°56'$. Vector CG lags the unity power factor line by $70°43' - 38°56' = 31°47'$ (0.85 power factor angle). Line $CG = E_2$ at 100 percent load to the same scale as OR. This line also represents receiver current and receiver kva but of course to a different scale.

Lay off from G to the left a line perpendicular to the unity power factor line at T. This will be the 100 percent load point and is equal to 30,000 kw. Divide line CT into four equal parts as shown, and draw through these points, at right angles to CT, intersecting points on line CG as shown. These points will represent kw on CT and kva on CG. The sending-end voltage vectors, at the given loads, will terminate on line CG at the load points.

Current Diagram.—To construct the current diagram at the left, draw vector OI_r to any convenient scale and equal to 185 amp at 100 percent load, lagging E_{rn} by $31°47'$. Next, draw I_1 equal to

$$I_r(a_1 + ja_2) = 185.25(0.96715 + j0.01156) = 179.17 + j2.14 \text{ vector amp.}$$

I_1 leads vector OI_r by $0°41'$. Draw I_2 to represent the line charging current and equal to 42.26 amp, to the same scale as OI_r, leading vector OR by $\theta_c = 90°14'$. $I_s = I_1 + I_2$ (vectorially) and is equal to 161.3 amp lagging E_{sn} by $27°38'$. The locus of I_s

is a straight line as shown. The current at intermediate loads is taken from Table 19.

Sending-end current vectors are referred to sending-end voltage vectors corresponding to that particular load. This is as it should be, since it facilitates the determination of the sending-end power factor at the given load. I_2 is, however, referred to vector *OR* and is displaced from vector *OC*, the sending-end voltage vector at zero load, by $90°14' - 0°41' = 89°33'$.

The value of 30,000 kw is given as 100 percent load. It is obvious, however, that 30,000 kw cannot be transmitted over this line at 0.85 lagging power factor without excessive voltage drop. By referring to the voltage arcs, it will be seen that the ratio of $E_{sn}/E_{rn} = 121.7$ percent at 100 percent load, or a voltage drop, in terms of E_{rn} of 21.7 percent.

$$\text{Kva} = 30{,}000/0.85 = 33{,}144.$$

This line should operate satisfactorily up to about 20,000 kw at 0.85 lagging power factor. It is not to be inferred that this line should normally be operated at a load of 30,000 kw. This comparatively high load was purposely assumed in order that the vectors would be of appreciable length and that the values for the various loads could be more clearly shown on the diagram.

If the line were without capacitance, the impedance triangle would have as origin the point *R*, the end of the receiver voltage vector. Obviously, since the circuit under discussion contains considerable capacitance, the impedance triangle, of which line *CG* forms the hypotenuse, must be raised and rotated to the left to the point *C*, which is the terminus of the sending-end voltage vector at zero load.

The complete calculations and construction of the diagram involve considerable time and effort. However, in comparison with the importance of the problem and the amount of investment required for a line of this character, quite often amounting to several hundred thousand dollars, the time required for the complete solution is negligible. The important thing to bear in mind is that it is too late after the line is constructed to make corrections if it fails to meet the expected requirements.

This problem is set up and the calculations are carried through to demonstrate the step-by-step procedure for deriving exact results, limited only by the number of terms in the series and by the completeness and accuracy of the work. No attempt is made to enter into the voltage- and phase-control principles of the problem at this time, for that would serve only to confuse the issue in this particular solution. Those principles of the work will be taken up in due course.

Figure 20, as previously stated, is designed to correct for voltage rise toward the receiving end of a transmission circuit, as affected by the charging current. The auxiliary constant A is practically constant for any given line length, regardless of the size or type of conductor. The correction for voltage rise as read from the curve is equal to $(1/A - 1) \times 100$ percent for any given line length. For example,

$$(1/0.96722 - 1) \times 100 = 3.39 \text{ percent.}$$

If the entire load of 30,000 kw is dropped and the sending-end voltage to neutral remains constant at 77,313 volts, the voltage rise will be $77{,}313 \times 0.0339 = 2{,}620$ volts. The correction as read from the curve for a line length of 125 miles is approximately 3.43 percent, the difference being because the curve is based on the use of copper. The error, however, when using equivalent a.c.s.r. is only $3.43 - 3.39 = 0.04$ percent.

For all practical purposes, we may assume the unit line charging current c_2 to flow through one-half the reactance of the circuit. For example, $a_1 = (1 - 0.0006654 \times 97.18/2) = 0.96767$, as against the rigorously calculated value of 0.96715. Also, if we assume the unit charging current to flow through one-half the resistance of the circuit, we have

$$j0.0006654 \times 33.99/2 = 0.01131$$

for the value of a_2, as against 0.01156 calculated rigorously.

CHAPTER VI

MECHANICAL DESIGN OF TRANSMISSION LINES

The mechanical and electrical design of a transmission line should be governed largely by the character and permanency of the service to be rendered. If the proposed line is to be the only source of supply for a city or industrial establishment, where continuity of service is paramount, the designer should make use of every facility for making the line automatic and as far as possible lightningproof.

Three items are of major importance, and each must be given careful consideration.

1. The structures must have ample strength to withstand the stresses imposed on their component parts by the line itself. These include stresses set up by the tension in conductors at dead-end points, compression stresses due to guy tension, transverse loads due to angles in the line, and vertical stresses due to the weight of conductors and the vertical component of conductor tension.
2. Both structure and conductors must have sufficient strength, with a predetermined factor of safety, to withstand the loads due to the line itself and the stresses imposed by wind and ice loads.
3. The structures must have sufficient height, and be so located, taking into account the topography of the country traversed by the line, to provide adequate clearance from ground at either maximum load or maximum temperature. The conductor clearance from the ground, the tracks of railroads, and wire line crossings, as well as from buildings and other objects, must meet the requirements of the National Electrical Safety Code (N.E.S.C.).

The designer should strive to use materials to the best advantage, keeping in mind the fact that the construction of a proposed line must be justified economically. When a proposed line, for example, is to be located in a heavy loading district, it should be designed and constructed to withstand the stresses imposed by the loading per lineal foot given in the table for heavy loading that applies to the conductor that is to be used on the line. The structures and conductors must withstand such loads without impairment of their strength or weakening in any manner. Overdesign should be avoided, *i.e.*, a class 3 pole should not be used where a class 4 pole will adequately meet the requirements. Similarly, a pole band is not needed to support the crossarm when a thru-bolt will have ample strength. The qualifications of a designer are best exemplified by his ability to obtain maximum results at minimum cost.

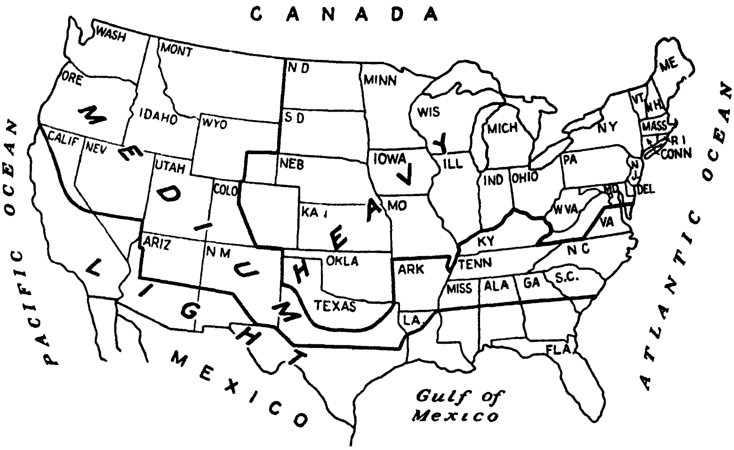

FIG. 25.—District loading map.

The Revised Safety Code.—Revision of the National Electrical Safety Code, under the procedure of the American Standards Association and sponsorship of the National Bureau of Standards, was initiated early in 1937. The work of revision was completed in 1941. Part 2 of the revised edition, which applies to the installation and maintenance of overhead and underground electric supply and communication circuits, may now be obtained from the Superintendent of Documents, Washington, D.C.

Many transmission and distribution engineers throughout the country have recognized for some time that the loading districts, as indicated on the loading map in the fourth edition of the code, did not fairly depict the severity or non-severity of conductor loading in certain sections of the country. A long-time study by the several committees of engineers and operators engaged in this work revealed that the loading map should be revised. The revised loading map is shown by Fig. 25.

A 10-year study of glaze storms, or ice deposits on wires and cables, by engineers representing power, railroad, and communication companies revealed that in only 1 percent of glaze storms did the wind pressure equal 8 lb per sq ft (the assumed figure in the old code) and in only 10 percent of the glaze storms did the wind pressure exceed 4 lb per sq ft.[1] A value of 4 lb per sq ft was therefore adopted as reasonable for the horizontal wind pressure on the ice-covered conductor in the heavy and medium loading districts. A value of 9 lb per sq ft upon conductors without ice covering was adopted for the light loading district. The assumption in the old code was 12 lb.

Analysis of the data pertaining to conductor loading indicated that the resultant loading assumptions in the old code were fairly representative. In order to assume the same resultant load with 4 lb per sq ft wind pressure as under the old code assumptions of 8 lb, the vertical loading would have to be greater than that assumed in the old code.

The committee no doubt gave careful consideration to the fact that exceptionally good results have been experienced from designs based on loading assumptions specified in the old code and that it would be undesirable at this time to promulgate a set of rules that would make it necessary to alter the basis of sag and tension calculations. Accordingly, a series of constants was applied to the various conductor materials, which, when added directly to the resultant load per lineal foot, would give a conductor loading approximately equivalent to that prescribed in the old code. This made it possible to continue the use of existing conductor sag and tension charts based on the old code. The loading and constants applying to various conductor materials for the loading districts are given in the table on page 79. They were taken from Part 2 of the revised code.

It has been stated that the constants, when added directly to the resultant, will give a conductor loading equivalent to that prescribed in the old code. However, a comparison of the resultant values, plus the constant, in the new tables, with corresponding resultant values in the loading tables based on the old code reveals that, for many of the conductor sizes, there is an appreciable difference between the new and the old values. This is especially true at light and medium loading. For hard-drawn copper and copperweld-copper, the new values are lower than those in the old tables at light loading but higher than the old values at medium and heavy loading. For a.c.s.r., the new values are appreciably lower at light and medium loading. The new values at heavy loading are lower than corresponding values in the old tables, but the difference is appreciably less than the difference at light and medium loading. The constants as prescribed in the new edition of the code are as follows:

---

[1] See *Electrical World*, Sept. 6, 1941.

	Loading districts		
	Light	Medium	Heavy
Radial thickness of ice, in	0.0	0.25	0.50
Horizontal wind pressure, lb per sq ft.	9	4	4
Temperature (deg F)	+30	+15	0
Constant to be added to the resultant in lb per ft:			
For bare conductors of copper, steel, copper alloy, copper-covered steel, and combinations thereof	0.05	0.19	0.29
For bare conductors of aluminum (with or without steel reinforcement)	0.05	0.22	0.31
For weatherproof and similar covered conductors (all materials)	0.05	0.22	0.31

Since the loading districts are national in scope, there will obviously be local areas in each of the loading districts in which the loading will be either more or less severe than for the district as a whole. Accordingly, when the construction of an important line is under consideration and no definite information is readily available as to the ice and wind loading in the local area, past records of the local weather bureau should be studied. Where the actual loadings are found to differ materially from the code assumptions, the line should be designed and constructed to meet such conditions.

Many designing engineers have an inclination to design all lines, regardless of type, that are in close proximity to the border line between medium and heavy loading districts, for medium loading. For the following reasons, the author believes this is not the correct procedure for lines using comparatively large high-strength conductors.

TABLE 21.—MECHANICAL PROPERTIES OF HARD-DRAWN COPPER WIRES AND CABLES

Cir mils	AWG	No. of wires	Wire diam., in.	Cable diam., in.	Area, sq in.	Lb per ft	Lb per mile 1-wire	Lb per mile 3-wire	Min. breaking strength
26,251	6	1	0 1620	0 1620	0 02062	0 0795	419 5	1,259	1,280
41,741	4	1	0 2043	0 2043	0 03278	0 1264	667 4	2,002	1,970
41,741	4	7	0 0772	0 2320	0 03278	0 1289	680 6	2,042	1,938
66,371	2	1	0.2576	0 2576	0 05213	0 2009	1,060 8	3,182	3,003
66,371	2	7	0 0974	0 2920	0 05213	0 2049	1,082 0	3,246	3,045
83,693	1	1	0 2893	0 2893	0.06573	0 2533	1,337	4,011	3,688
83,693	1	3	0 1670	0 3600	0 06573	0 2559	1,351	4,053	3,620
83,693	1	7	0 1093	0 3280	0 06573	0 2584	1,364	4,092	3,804
105,535	1/0	7	0 1228	0.3680	0 08286	0 3257	1,720	5,160	4,750
133 077	2/0	7	0 1379	0 4140	0 10450	0 4109	2,170	6,510	5,927
167,806	3/0	7	0 1548	0 464	0 1318	0 5181	2,736	8,208	7,366
167,806	3/0	12	0 1183	0 492	0 1318	0 5181	2,736	8,208	7,556
211,600	4/0	7	0 1739	0 522	0 1662	0 6533	3,449	10,347	9,154
211,600	4/0	12	0 1328	0 552	0 1662	0 6533	3,449	10,347	9,483
211,600	4/0	19	0 1055	0 528	0 1662	0 6533	3,449	10,347	9,617
250,000		12	0 1443	0 600	0 1964	0 7719	4,076	12,228	11,130
250,000		19	0 1147	0 574	0 1964	0 7719	4,076	12,228	11,360
300,000		12	0 1581	0 657	0 2356	0 9263	4,891	14,673	13,170
300,000		19	0 1257	0 629	0.2356	0 9263	4,891	14,673	13,510
350,000		12	0 1708	0.710	0 2749	1 0810	5,708	17,124	15,140
350,000		19	0 1357	0.679	0.2749	1 081	5,708	17,124	15,590
400,000		19	0 1451	0 726	0 3142	1 235	6 521	19,563	17,560
450,000		19	0.1539	0 770	0 3534	1 389	7,334	22,002	19,750
450,000		37	0.1103	0.772	0 3534	1 389	7,334	22,002	20,450
500,000		19	0 1622	0 811	0 3927	1 544	8,152	24,456	21,950
500,000		37	0 1162	0 814	0 3927	1 544	8,152	24,456	22,510

Modulus of elasticity, 17,000,000 lb per sq in.
Coefficient of linear expansion, 0.000,009,4 per deg. F.

A line designed and constructed for heavy loading may reach its maximum design load only comparatively few times during its entire useful life. Accordingly, since there is such wide range between stresses at maximum and those at normal load, the normal stresses are extremely low as compared with the stresses at maximum design load. On the other hand, if a line is designed and constructed for medium loading, even with the stresses at maximum loading somewhat less than those for a similar line designed for heavy loading, the normal stresses will be appreciably higher in the medium loading line. For this reason, a line constructed for heavy loading may have a longer life and require less maintenance than a similar line designed and constructed for medium loading. The normal, or 60 deg, stress for hard-drawn copper conductors should, in general, not exceed about 15,000 lb per sq in.

TABLE 22.—LOADING TABLE FOR HARD-DRAWN COPPER WIRES AND CABLES, LB PER LIN FT

AWG or MCM	No. of wires	Light loading			Medium loading			Heavy loading		
		Cable + 0 in. ice	Wind 9, lb per sq ft	Resultant + 0.05	Cable + ¼ in. ice	Wind 4, lb per sq ft	Resultant + 0.19	Cable + ½ in. ice	Wind 4, lb per sq ft	Resultant + 0.29
6	1	0.0795	0.1215	0.1952	0.2075	0.2207	0.4929	0.4911	0.3873	0.9154
4	1	0.1264	0.1532	0.2486	0.2676	0.2348	0.5460	0.5643	0.4014	0.9825
4	7	0.1289	0.1740	0.2666	0.2788	0.2440	0.5605	0.5838	0.4107	1.0038
2	1	0.2009	0.1932	0.3287	0.3587	0.2525	0.6287	0.6720	0.4192	1.0821
2	7	0.2049	0.2190	0.3500	0.3734	0.2640	0.6473	0.6973	0.4307	1.1096
1	1	0.2533	0.2170	0.3835	0.4210	0.2631	0.6865	0.7441	0.4298	1.149
1	3	0.2559	0.2700	0.4220	0.4455	0.2867	0.7198	0.7906	0.4533	1.201
1	7	0.2584	0.2460	0.4068	0.4381	0.2760	0.7078	0.7732	0.4427	1.181
1/0	7	0.3257	0.2760	0.4769	0.5178	0.2893	0.7831	0.8654	0.4560	1.268
2/0	7	0.4109	0.3105	0.5650	0.6173	0.3047	0.8784	0.9792	0.4713	1.377
3/0	7	0.5181	0.3480	0.6741	0.7401	0.3213	0.9968	1.117	0.4880	1.509
3/0	12	0.5181	0.3690	0.6861	0.7488	0.3307	1.0086	1.135	0.4973	1.529
4/0	7	0.6533	0.3915	0.8116	0.8933	0.3407	1.1461	1.289	0.5076	1.675
4/0	12	0.6533	0.4140	0.8234	0.9026	0.3507	1.1583	1.307	0.5173	1.696
4/0	19	0.6533	0.3960	0.8139	0.8952	0.3427	1.1486	1.292	0.5093	1.679
250	12	0.7719	0.4500	0.9435	1.036	0.3667	1.289	1.456	0.5333	1.841
250	19	0.7719	0.4305	0.9338	1.028	0.3580	1.279	1.440	0.5247	1.823
300	12	0.9263	0.4928	1.0992	1.208	0.3857	1.458	1.646	0.5523	2.026
300	19	0.9263	0.4718	1.0895	1.200	0.3763	1.448	1.628	0.5430	2.006
350	12	1.0810	0.5325	1.2550	1.379	0.4033	1.627	1.833	0.5700	2.210
350	19	1.081	0.5093	1.245	1.370	0.3930	1.615	1.814	0.5597	2.188
400	19	1.235	0.5445	1.400	1.538	0.4087	1.782	1.997	0.5753	2.368
450	19	1.389	0.5775	1.554	1.706	0.4233	1.948	2.179	0.5900	2.547
450	37	1.389	0.5790	1.555	1.707	0.4240	1.948	2.180	0.5907	2.548
500	19	1.544	0.6083	1.709	1.874	0.4370	2.114	2.359	0.6037	2.725
500	37	1.544	0.6105	1.710	1.874	0.4380	2.115	2.360	0.6047	2.726

Table 21 gives the mechanical properties of hard-drawn copper wire and cables. For convenience in finding quickly the weight per line mile of three-phase circuits, a column has been included to give the total weight per mile for three conductors. Table 22 gives the loading in pounds per lineal foot for hard-drawn copper wires and cables. The column under Resultant for light loading gives the resultant of the vertical weight of the bare conductor and the horizontal wind pressure of 9 lb per sq ft on the bare conductor plus a constant of 0.05, as specified in the new code. For medium loading, the constant of 0.19 is added directly to the resultant of the vertical weight of conductor plus ¼ in. ice and 4 lb per sq ft wind pressure on the ice-covered conductor. For heavy loading, the constant of 0.29 has been added directly to the resultant of the vertical weight of conductor

TABLE 23.—MECHANICAL PROPERTIES OF A.C.S.R.

Cir mils	AWG	No. of wires	Diam., in.	Area, sq in.		Lb per ft	Lb per mile 1-cond.	Lb per mile 3-cond.	Min. breaking strength
				Al	Total				
41,741	4	6/1	0.250	0.0328	0.0383	0.0576	304	912	1,830
66,371	2	6/1	0.316	0.0521	0.0608	0.0916	484	1,452	2,790
83,693	1	6/1	0.355	0.0657	0.0767	0.1155	610	1,830	3,480
105,535	1/0	6/1	0.398	0.0829	0.0967	0.1456	769	2,307	4,280
133,077	2/0	6/1	0.447	0.1045	0.1219	0.1837	970	2,910	5,345
167,806	3/0	6/1	0.502	0.1318	0.1538	0.2316	1,223	3,669	6,675
211,600	4/0	6/1	0.563	0.1662	0.1939	0.2921	1,542	4,626	8,420
266,800		6/7	0.633	0.2095	0.2367	0.3413	1,802	5,406	9,645
266,800		26/7	0.642	0.2095	0.2436	0.3656	1,930	5,790	11,250
336,400		26/7	0.721	0.2642	0.3072	0.4624	2,442	7,326	14,050
336,400		30/7	0.741	0.2642	0.3259	0.5253	2,774	8,322	17,040
397,500		26/7	0.783	0.3122	0.3630	0.5464	2,885	8,655	16,190
397,500		30/7	0.806	0.3122	0.3850	0.6206	3,277	9,831	19,980
477,000		26/7	0.858	0.3746	0.4356	0.6557	3,462	10,386	19,430
477,000		30/7	0.883	0.3746	0.4620	0.7448	3,933	11,799	23,300
556,500		26/7	0.927	0.4371	0.5083	0.7650	4,039	12,117	22,400
556,500		30/7	0.953	0.4371	0.5391	0.8690	4,588	13,764	27,200
636,000		26/7	0.990	0.4995	0.5809	0.874	4,616	13,848	25,000
636,000		30/7	1.019	0.4995	0.6134	0.988	5,213	15,639	31,500
715,500		26/7	1.051	0.5620	0.6535	0.984	5,193	15,579	28,100
715,500		30/19	1,081	0.5620	0.6901	1.111	5,865	17,595	34,600
795,000		26/7	1.108	0.6244	0.7261	1.093	5,770	17,310	31,200
795,000		30/19	1.140	0.6244	0.7668	1.234	6,517	19,551	38,400

plus ½ in. ice and 4 lb per sq ft wind pressure on the ice-covered conductor.

TABLE 24.—LOADING TABLE FOR A.C.S.R.. LB PER LIN FT

AWG or MCM	No. of wires	Light loading			Medium loading			Heavy loading		
		Cable + 0 in. ice	Wind 9, lb per sq ft	Resultant + 0.05	Cable + ¼ in. ice	Wind 4, lb per sq ft	Resultant + 0.22	Cable + ½ in. ice	Wind 4, lb per sq ft	Resultant + 0.31
4	6/1	0.0576	0 1875	0 2461	0 2131	0.2500	0 5485	0.5241	0.4168	0.9796
2	6/1	0 0916	0 2370	0 3041	0 2676	0 2719	0 6015	0.5992	0 4385	1.0525
1	6/1	0 1155	0 2662	0 3401	0 3037	0 2849	0 6364	0 6473	0 4515	1 0992
1/0	6/1	0 1456	0 2985	0 3821	0 3471	0 2994	0 6783	0 7042	0 4660	1.1544
2/0	6/1	0 1837	0 3352	0 4322	0 4005	0 3157	0 7210	0.7727	0.4823	1.2209
3/0	6/1	0 2316	0 3765	0 4920	0 4655	0 3339	0 7929	0.8548	0 5005	1 300
4/0	6/1	0 2921	0 4222	0 5634	0 5449	0 3545	0.8700	0 9533	0.5210	1 396
266 8	6/7	0 3413	0 4748	0.6347	0.6159	0 3775	0 9425	1.0460	0.5445	1 489
266 8	26/7	0 3668	0 4815	0 6553	0.6442	0.3805	0 9682	1.0770	0.5470	1 518
336 4	26/7	0.4624	0.5408	0 7616	0 7644	0 4069	1 0860	1.2220	0.5735	1.660
336 4	30/7	0.5253	0 5558	0 8148	0 8335	0 4137	1 1505	1 2970	0.5805	1.731
397 5	26/7	0 5464	0 5873	0 8522	0 8677	0 4275	1.187	1 344	0.5940	1.779
397 5	30/7	0 6206	0 6045	0 9164	0.9490	0 4352	1 264	1.433	0.6020	1 864
477 0	26/7	0.6557	0 6435	0 9687	1 0000	0.4527	1 318	1 500	0.6195	1.933
477.0	30/7	0 7448	0.6623	1 0467	1.0970	0.4609	1.410	1 605	0.6275	2.033
556 5	26/7	0 7650	0.6953	1.0839	1 1310	0 4756	1 447	1.653	0.6420	2.083
556 5	30/7	0 8690	0 7148	1 1752	1.2430	0 4844	1 554	1 773	0.6510	2 199
636 0	26/7	0.874	0 7425	1.197	1 262	0 4968	1 576	1 801	0.6635	2 229
636 0	30/7	0 988	0 7643	1 299	1 383	0 5065	1 693	1 933	0 6730	2.357
715 5	26/7	0 984	0.7883	1.311	1 389	0.5170	1 702	1.949	0.6835	2 375
715 5	30/19	1 111	0 8108	1.425	1 525	0 5270	1 834	2 094	0 6935	2 516
795.0	26/7	1.093	0 8310	1 423	1 515	0 5360	1 827	2.093	0.7025	2.518
795 0	30/19	1 234	0.8550	1 551	1 666	0 5465	1 973	2 254	0 7130	2 674

Table 23 gives the mechanical properties of a.c.s.r. Table 24 gives the loading per lineal foot at light, medium, and heavy loading. The resultant values in this table were derived in the same manner as described for Table 22, except that the constants to be added to the resultant are 0.22 and 0.31 for medium and heavy loading.

Table 25 gives the mechanical properties of copperweld-copper transmission-line conductors. Table 26 gives the loading per lineal foot for copperweld-copper. The resultant values plus the constants were derived and compiled in the same manner as described for Table 22.

TABLE 25.—MECHANICAL PROPERTIES OF COPPERWELD-COPPER

Equiv. AWG or MCM	Type	No. of wires	Cable diam., in.	Area, sq. in.	Lb per ft	Lb per mile, 1-cond	Lb per mile, 3-cond	Breaking load, lb
4	A	2/1	0 290	0.04276	0.1615	852.8	2,558	3,938
2	A	2/1	0 366	0.06799	0.2568	1,356	4,068	5,876
2	F	6/1	0.308	0.05792	0.2228	1,176	3,528	4,233
1	F	6/1	0 346	0.07303	0.2809	1,483	4,449	5,266
1/0	F	6/1	0.388	0.09207	0.3541	1,870	5,610	6,536
2/0	F	6/1	0 436	0.11620	0 4468	2,359	7,077	8,094
3/0	F	6/1	0.490	0.14640	0.5632	2,974	8,922	9,980
4/0	F	6/1	0 550	0 1847	0.7102	3,750	11,250	12,290
2/0	S	12/2	0.502	0.1297	0.5016	2,648	7,944	10,560
3/0	S	12/2	0.563	0.1635	0.6323	3,339	10,017	12,970
4/0	S	12/2	0.633	0.2061	0.7974	4,210	12,630	15,860
2/0	V	9/3	0.465	0.1238	0.4738	2,502	7,506	9,846
3/0	V	9/3	0.522	0.1560	0.5973	3,154	9,462	12,220
4/0	V	9/3	0.586	0.1968	0.7532	3,977	11,931	15,000
250	V	9/3	0.637	0.2325	0.8899	4,699	14,097	17,420
300	V	9/3	0.698	0.2790	1.0680	5,639	16,917	20,730
350	V	9/3	0.754	0.3254	1.2460	6,578	19,734	23,480
3/0	E	12/7	0.545	0.1776	0.6727	3,552	10,656	16,800
4/0	E	12/7	0.613	0.2239	0.8483	4,479	13,437	20,730
3/0	EK	15/4	0.509	0.1546	0.5935	3,134	9,402	12,370
4/0	EK	15/4	0 571	0.1949	0.7484	3,951	11,853	15,368
250	EK	15/4	0.621	0.2303	0.8842	4,669	14,007	17,841
300	EK	15/4	0.680	0.2763	1.0610	5,602	16,806	20,957
350	EK	15/4	0.735	0.3224	1.2380	6,536	19,608	23,847

In the upper part of Table 27, the mechanical properties of galvanized-steel strands are given for sizes most commonly used in transmission-line construction. The loading per lineal foot for the various loading districts is given in the lower part of the table.

Wind Velocity.—The relation of indicated to actual wind velocity is shown by the two curves in Fig. 26. It is evident from an inspection of the two curves that considerable difference exists between indicated and actual wind velocity. Wind velocity is measured by an anemometer. As far as the author knows, no instrument has been produced that will indicate or record true wind velocity. It should be noted that a wind pressure of 4 lb per sq ft on a cylinder, such as a wire or cable, requires an actual wind velocity of approximately 40 mph. A wind pressure of 9 lb per sq ft requires a wind velocity of approximately 59.5 mph.

TABLE 26.—LOADING TABLE FOR COPPERWELD-COPPER, LB PER LIN FT

Equiv. Cu AWG or MCM	Type	No. of wires	Light loading			Medium loading			Heavy loading		
			Cable + 0 in. ice	Wind 9, lb per sq ft	Resultant + 0.05	Cable + ¼ in. ice	Wind 4, lb per sq ft	Resultant + 0.19	Cable + ½ in. ice	Wind 4, lb per sq ft	Resultant + 0.29
4	A	2/1	0.1615	0 2175	0 3209	0.3294	0.2634	0.6117	0.6527	0.4300	1.072
2	A	2/1	0 2568	0.2745	0 4259	0 4483	0.2887	0 7232	0.7953	0 4554	1.206
2	F	6/1	0.2228	0.2310	0.3709	0.3963	0 2694	0.6692	0.7252	0 4360	1.136
1	F	6/1	0.2809	0 2595	).4324	0 4662	0 2820	0.7349	0 8069	0 4487	1.213
1/0	F	6/1	0 3541	0 2910	0.5083	0.5524	0 2960	0.8167	0.9062	0 4627	1 307
2/0	F	6/1	0.4468	0.3270	0.6037	0.6601	0.3120	0.9201	1 0290	0 4787	1.425
3/0	F	6/1	0.5632	0 3675	0.7225	0.7933	0.3300	1.0492	1 1790	0.4967	1.569
4/0	F	6/1	0 7102	0.4125	0 9713	0.9589	0.3500	1 2107	1.3630	0.5165	1.748
2/0	S	12/2	0 5016	0 3765	0 6772	0 7354	0 3340	0 9977	1.125	0 5005	1.521
3/0	S	12/2	0 6323	0.4223	0 8103	0 8851	0 3544	1 143	1 293	0.5210	1.684
4/0	S	12/2	0 7974	0.4748	0 9780	1.0720	0.3777	1 327	1.502	0.5445	1.888
2/0	V	9/3	0 4738	0 3488	0 6383	0.6961	0 3217	0 9568	1 074	0 4884	1 470
3/0	V	9/3	0.5973	0.3915	0 7642	0.8373	0.3407	1.094	1 233	0 5075	1.623
4/0	V	9/3	0 7532	0 4395	0.9220	1 013	0 3620	1.266	1.428	0 5285	1.813
250	V	9/3	0 8899	0.4778	1.060	1.166	0 3790	1 416	1.597	0.5455	1.978
300	V	9/3	1.068	0 5235	1 239	1.363	0.3994	1.610	1.813	0.5660	2.189
350	V	9/3	1.246	0 5655	1.418	1.558	0.4180	1.803	2.026	0.5845	2.399
3/0	E	12/7	0.6727	0 4088	0 8371	0.9199	0.3484	1.174	1.322	0.5150	1.709
4/0	E	12/7	0.8483	0.4598	1.015	1.117	0.3710	1.367	1.540	0.5375	1.921
3/0	EK	15/4	0.5935	0.3818	0 7557	0.8295	0.3364	1.085	1.221	0.5030	1.610
4/0	EK	15/4	0.7484	0 4283	0.9123	1 004	0.3570	1.255	1.414	0.5235	1.798
250	EK	15/4	0.8842	0.4658	1.049	1.155	0.3737	1.404	1.581	0 5405	1.961
300	EK	15/4	1.061	0.5100	1.227	1.350	0 3934	1.596	1 795	0.5600	2 170
350	EK	15/4	1.238	0 5513	1.405	1.544	0.4117	1.788	2.006	0 5785	2.378

The method of calculating wind pressure on wires and cables, which is now considered standard practice, was developed by H. W. Buck and presented for the first time in a paper read at the St. Louis Louisiana Purchase Exposition in 1904. The equation for finding the wind pressure per square foot on cylindrical wires and cables is as follows:

TABLE 27.—MECHANICAL PROPERTIES OF GALVANIZED-STEEL STRANDS

Nominal diam., in.	No. of wires	Actual diam., in.	Area, sq in.	Lb per ft	Lb per mile	Breaking strength, lb			
						Common	Siemens Martin	High strength	Extra high strength
1/4	7	0.240	0.03519	0.121	639	1,900	3,150	4,750	6,650
5/16	7	0.312	0.05946	0.205	1,082	3,200	5,350	8,000	11,200
3/8	7	0.360	0.07917	0.273	1,441	4,250	6,950	10,800	15,400
7/16	7	0.435	0.1156	0.399	2,107	5,700	9,350	14,500	20,800
1/2	7	0.495	0.1497	0.517	2,730	7,400	12,100	18,800	26,900
1/2	19	0.500	0.1492	0.504	3,543	7,620	12,700	19,100	26,700
5/8	7	0.621	0.2356	0.813	4,293	11,600	19,100	29,600	42,400
5/8	19	0.625	0.2332	0.796	4,203	11,000	18,100	28,100	40,200

Nominal diam., in.	No. of wires	Light loading			Medium loading			Heavy loading		
		Cable + 0 in ice	Wind 9, lb per sq ft	Resultant + 0.05	Cable + ¼ in ice	Wind 4, lb per sq ft	Resultant + 0.19	Cable + ½ in ice	Wind 4, lb per sq ft	Resultant + 0.29
1/4	7	0.121	0.1800	0.2669	0.274	0.2467	0.5586	0.581	0.4135	1.003
5/16	7	0.205	0.2340	0.3610	0.380	0.2755	0.6594	0.710	0.4375	1.124
3/8	7	0.273	0.2700	0.4340	0.463	0.2865	0.7345	0.807	0.4535	1.216
7/16	7	0.399	0.3263	0.5654	0.612	0.3115	0.8767	0.981	0.4785	1.381
1/2	7	0.517	0.3713	0.6865	0.784	0.3315	1.008	1.136	0.4985	1.530
1/2	19	0.504	0.3750	0.6782	0.738	0.3335	0.9999	1.126	0.5000	1.522
5/8	7	0.813	0.4658	0.9870	1.084	0.3735	1.337	1.510	0.5405	1.894
5/8	19	0.796	0.4688	0.9728	1.068	0.3750	1.322	1.496	0.5415	1.881

The actual diameters, correct to three decimal places, were derived as follows:

$$D = \sqrt{[\text{cir mils}(1.3333 - \tfrac{1}{3N})]}\ 10^{-3}$$

where D = diameter of complete cable, in.
Cir mils = 1,273,200 × sq in.
N = number of wires.

$$P = 0.0025 \times V^2 \qquad (23)$$

The following equation for finding the wind pressure on small flat surfaces, such as steel towers, was first proposed by Prof. C.F. Marvin, of the U.S. Weather Bureau:

$$P = 0.004 \times \frac{B}{30} \times V^2 \qquad (24)$$

where P = pressure, lb per sq ft.
B = barometric pressure, in. Hg.
V = wind velocity, mph.

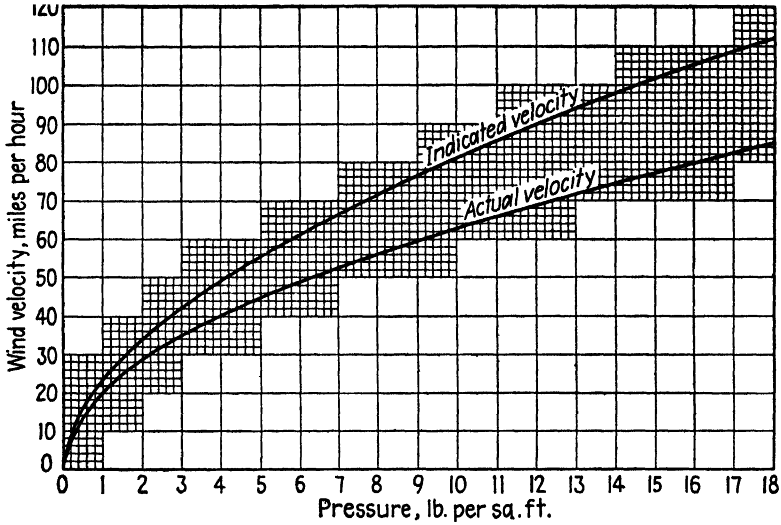

FIG. 26.—Relation between indicated wind velocity, actual velocity, and pressure on wires and cables.

Since $B/30$ in general approximates unity, this term may be omitted. The error in the result will be negligible when (24) is written $P = 0.004 \times V^2$.

In the absence of tables from which the vertical and horizontal components of load due to ice and wind may be read, the values may be derived as follows:

$$W_v = W_{vo} + 1.243(dt + t^2) \quad \text{lb per ft} \qquad (25)$$

$$W_h = \frac{W_c \times 12(d + 2t)}{144} \quad \text{lb per ft} \qquad (26)$$

where W_v = vertical weight of cable and ice load, lb per ft.
W_{vo} = vertical weight of bare cable, lb per ft.
d = diameter of bare cable, in.
t = radial thickness of ice, in.

W_h = horizontal component of load on the ice-covered cable due to wind pressure, lb per ft.

W_c = wind pressure on ice-covered cable, lb per ft.

Wood Transmission Poles.—Of the five species of timber that are used throughout the United States in the construction of overhead transmission lines, two predominate, namely, southern yellow pine and western red cedar. Northern white cedar, douglas fir, and lodgepole pine are used in sections of the country where they grow.

TABLE 28.—DIMENSIONS OF CREOSOTED SOUTHERN PINE POLES

Class	1		2		3		4		5	
	Circ.	Diam.	Circ.	Diam.	Circ.	Diam.	Circ.	Diam.	Circ.	Diam.
Minimum top dimension, in.	27	8.6	25	8.0	23	7.3	21	6.7	19	6.0
Length of pole, ft	Minimum dimensions 6 ft from butt, in.									
30	37.5	11.9	35.0	11.1	32.5	10.3	30.0	9.5	28.0	8.9
35	40.0	12.7	37.5	11.9	35.0	11.1	32.0	10.2	30.0	9.5
40	42.0	13.4	39.5	12.6	37.0	11.8	34.0	10.8	31.5	10.0
45	44.0	14.0	41.5	13.2	38.5	12.3	36.0	11.5	33.0	10.5
50	46.0	14.6	43.0	13.7	40.0	12.7	37.5	11.9	34.5	11.0
55	47.5	15.1	44.5	14.2	41.5	13.2	39.0	12.4	36.0	11.5
60	49.5	15.8	46.0	14.6	43.0	13.7	40.0	12.7	37.0	11.8
65	51.0	16.2	47.5	15.1	44.5	14.2	41.5	13.2	38.5	12.3
70	52.5	16.7	49.0	15.6	46.0	14.6	42.5	13.5	39.5	12.6
75	54.0	17.2	50.5	16.1	47.0	15.0	44.0	14.0		
80	55.0	17.5	51.5	16.4	48.5	15.4	45.0	14.3		
85	56.5	18.0	53.0	16.9	49.5	15.8				
90	57.5	18.3	54.0	17.2	50.5	16.1				

Tables 28 to 32 give the dimensions of poles of 30 ft and longer and for classes from 1 to 5, inclusive. Poles shorter than 30 ft or smaller than class 5 should generally not be used for lines of transmission voltage. However, poles smaller than class 5 are used quite extensively in the construction of rural lines where strength requirements are not of prime importance. The American Standards Association has set up certain requirements, or recommendations, for minimum top dimensions and minimum requirements 6 ft from the butt, specifying such requirements in terms of circumference. In calculating pole-strength requirements for a given condition, however, it is necessary to convert a dimension in terms of circumference to one in terms of diameter. For this reason, the diameter, to the nearest tenth of an inch, for each length and class of pole, as well as the American Standards Association circumference values, are included in the tables.

TABLE 29.—DIMENSIONS OF LODGEPOLE PINE POLES

Class	1		2		3		4		5	
	Circ.	Diam.	Circ.	Diam.	Circ.	Diam.	Circ.	Diam.	Circ.	Diam.
Minimum top dimension, in.	27	8.6	25	8.0	23	7.3	21	6.7	19	6.0
Length of pole, ft	Minimum dimension 6 ft from butt, in.									
30	39.0	12.4	36.5	11.6	34.0	10.8	31.5	10.0	29.0	9.2
35	41.5	13.2	38.5	12.3	36.0	11.5	33.5	10.7	31.0	9.9
40	44.0	14.0	41.0	13.1	38.0	12.1	35.5	11.3	33.0	10.5
45	46.0	14.6	43.0	13.7	40.0	12.7	37.0	11.8	34.5	11.0
50	48.0	15.3	45.0	14.3	42.0	13.4	39.0	12.4	36.0	11.5
55	49.5	15.8	46.5	14.8	43.5	13.8	40.5	12.9	37.5	11.9
60	51.5	16.4	48.0	15.3	45.0	14.3	42.0	13.4	38.5	12.3
65	53.0	16.9	49.5	15.8	46.0	14.6	43.0	13.7		
70	54.5	17.3	51.0	16.2	47.5	15.1				
75	56.0	17.8	52.5	16.7						

TABLE 30.—DIMENSIONS OF NORTHERN WHITE CEDAR POLES

Length of pole, ft	Minimum dimension 6 ft from butt, in.									
30	47.5	15.1	44.5	14.2	41.5	13.2	38.5	12.3	35.5	11.3
35	50.5	16.1	47.5	15.1	44.0	14.0	41.0	13.1	38.0	12.1
40	53.5	17.0	50.0	15.9	46.5	14.8	43.5	13.8	40.0	12.7
45	56.0	17.8	52.5	16.7	49.0	15.6	45.5	14.5	42.0	13.4
50	58.5	18.6	55.0	17.5	51.5	16.4	47.5	15.1	44.0	14.0
55	61.0	19.4	57.5	18.3	53.5	17.0	49.5	15.8	46.0	14.6
60	63.5	20.2	59.5	18.9	55.5	17.7	51.5	16.4		

Minimum top dimensions are the same as in Table 29.

Regardless of the kind of timber, the minimum top dimension is the same for each particular class of pole. The American Standards Association has attempted to set up dimension requirements, taking into account, of course, the ultimate strength, so that poles of different length within the same class will have approximately the same breaking strength. The sizes are such that, if a load is applied 2 ft from the top, regardless of the length, failure will occur at the ground line. In making pole loading calculations in connection with important transmission lines, it is advisable to make certain allowances for reduction of strength over the years from decay, insects, etc. Creosoted pine poles are more susceptible to failure from decay near the top. Cedar poles, unless exceptionally well treated with preservative, are more susceptible to decay and failure at the ground line.

TABLE 31.—DIMENSIONS OF WESTERN RED CEDAR POLES

Class	1		2		3		4		5	
	Circ	Diam	Circ	Diam	Circ	Diam	Circ	Diam	Circ	Diam
Minimum top dimension, in	27	8.6	25	8.0	23	7.3	21	6.7	19	6.0
Length of pole, ft	Minimum dimension 6 ft from butt, in									
30	41.0	13.1	38.5	12.3	35.5	11.3	33.0	10.5	30.5	9.7
35	43.5	13.8	41.0	13.1	38.0	12.1	35.5	11.3	32.5	10.3
40	46.0	14.6	43.5	13.8	40.5	12.9	37.5	11.9	34.5	11.0
45	48.5	15.4	45.5	14.5	42.5	13.5	39.5	12.6	36.5	11.6
50	50.5	16.1	47.5	15.1	44.5	14.2	41.0	13.1	38.0	12.1
55	52.5	16.7	49.5	15.8	46.0	14.6	42.5	13.5	39.5	12.6
60	54.5	17.3	51.0	16.2	47.5	15.1	44.0	14.0		
65	56.0	17.8	52.5	16.7	49.0	15.6	45.5	14.5		
70	57.5	18.3	54.0	17.2	50.5	16.1	47.0	15.0		
75	59.5	18.9	55.5	17.7	52.0	16.6	48.5	15.4		
80	61.0	19.4	57.0	18.1	53.5	17.0	49.5	15.8		
85	62.5	19.9	58.5	18.6	54.5	17.3				
90	63.5	20.2	60.0	19.1	56.0	17.8				

Approximate breaking loads, applied 2 ft from the top, for the five classes included in the tables, are as follows:

Class of pole	1	2	3	4	5
Breaking load, lb	4,500	3,700	3,000	2,400	1,900

See 'Standard Handbook for Electrical Engineers," 7th ed , Sec 4, Par 846

The ultimate fiber stresses, approved by the American Standards Association, made on full-size poles tested to failure for the species of timber given in the tables, are as follows:

Timber	Stress, Lb per Sq In.
Creosoted southern yellow pine	7,400
Lodgepole pine	6,600
Northern white cedar	3,600
Western red cedar	5,600
Douglas fir	7,400

TABLE 32 —DIMENSIONS OF CREOSOTED DOUGLAS FIR POLES

Class	1		2		3		4		5	
	Circ	Diam	Circ	Diam	Circ	Diam	Circ	Diam	Circ	Diam.
Minimum top dimension in	27	8 6	25	8 0	23	7 3	21	6 7	19	6 0
Length of pole ft	Minimum dimension 6 ft from butt, in									
30	37 5	11 9	35 0	11 1	32 5	10 3	30 0	9 5	28 0	8 9
35	40 0	12 7	37 5	11 9	35 0	11 1	32 0	10 2	30 0	9 5
40	42 0	13 4	39 5	12 6	37 0	11 8	34 0	10 8	31 5	10 0
45	44 0	14 0	41 5	13 2	38 5	12 3	36 0	11 5	33 0	10 5
50	46 0	14 6	43 0	13 7	40 0	12 7	37 5	11 9	34 5	11 0
55	47 5	15 1	44 5	14 2	41 5	13 2	39 0	12 4	36 0	11 5
60	49 5	15 8	46 0	14 6	43 0	13 7	40 0	12 7	37 0	11 8
65	51 0	16 2	47 5	15 1	44 5	14 2	41 5	13 2	38 5	12 3
70	52 5	17 0	49 0	15 6	46 0	14 6	42 5	13 5	39 5	12 6
75	54 0	17 2	50 5	16 1	47 0	15 0	44 0	14 0		
80	55 0	17 5	51 5	16 4	48 5	15 4	45 0	14 3		
85	56 0	17 8	53 0	16 9	49 5	15 8				
90	57 5	18 3	54 0	17 2	50 5	16 1				

CHAPTER VII

MECHANICAL CALCULATIONS

Various types of pole framing are used in overhead line construction. The sketch (Fig. 27) shows a scheme used quite extensively for single-pole pin-type construction. One conductor is installed on a pole-top pin, and the other two are installed on a single arm. This sketch indicates what is called "triangular construction" and includes the information required to compute the stresses produced by transverse wind loads on the pole and conductors. To determine the length of unguyed span for a given height, kind, and class of pole, assuming the poles to be set in firm soil, it is required to calculate the bending moment of

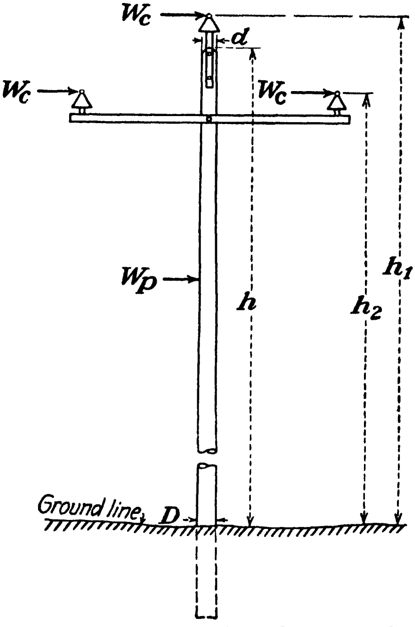

Fig. 27.—Schematic diagram of pole and conductors to facilitate pole loading calculations. Not to scale.

the pole at the ground line, which will usually be the point of failure. From that figure, the fiber stress is determined.

As indicated in Fig. 27

h = height of pole above ground, ft.
h_1 = height of top conductor above ground, ft.
h_2 = height of lower conductors above ground, ft.
d = top diameter of pole, in.
D = ground-line diameter of pole, in.
W_p = wind pressure on pole, lb per sq ft.
W_c = wind pressure on the ice-covered conductors, lb per lin ft.

Pole loading calculations can at best give only roughly approximate results, since there will usually be a slight movement of the pole at the ground line, as a result of which the calculated fiber stress may differ appreciably from the actual value.

The calculations may be performed by means of the following equations:

$$M_p = \frac{W_p \times h^2(D + 2d)}{72} \quad \text{ft-lb} \tag{27}$$

$$\begin{aligned} M_{c1} &= W_c \times h_1 \times \text{horizontal span} \quad \text{ft-lb} \\ M_{c2} &= W_c \times 2 \times h_2 \times \text{horizontal span} \quad \text{ft-lb} \end{aligned} \tag{28}$$

where M_p = bending moment at ground line due to wind pressure on the pole, ft-lb.
M_{c1} = bending moment at ground line due to wind pressure on top conductor, ft-lb.
M_{c2} = bending moment at ground line due to wind pressure on two lower conductors, ft-lb.

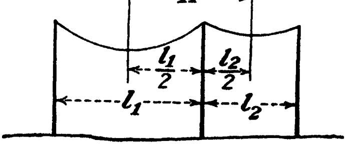

FIG. 28.—Pole loading diagram.

The horizontal span is represented by H in Fig. 28 and is equal to one-half the length of the two adjacent spans l_1 and l_2. It is readily apparent that the transverse load on the horizontal span H must be supported by the intermediate pole. Let us assume l_1 to be 600 ft and l_2 to be 400 ft. The horizontal span H is then equal to $(600 + 400)/2 = 500$ ft.

For example, refer to Fig. 27, and assume the following conditions to apply: 45-ft class 4 creosoted pine pole, set 6 ft in the ground. Three conductors of 3/0 twelve-strand copper, loaded with ½-in. radial thickness of ice. Over-all diameter 1.492 in.

$h = 39$ ft
$h_1 = 40$ ft
$h_2 = 37$ ft
$d = 6.7$ in. from Table 28
$D = 11.5$ in. from Table 28
$W_p = 4$ lb per sq ft
$W_c = 0.4973$ lb per lin ft, from Table 24.

Substituting the assumed values in Eqs. (27) and (28) and solving for the bending moment

$$\begin{array}{lll} M_p = \dfrac{4 \times 39^2(11.5 + 2 \times 6.7)}{72} & = & 2{,}104 \text{ ft-lb} \\ M_{c1} = 0.4973 \times 40 \times 500 & = & 9{,}946 \\ M_{c2} = 0.4973 \times 2 \times 37 \times 500 & = & \underline{18{,}400} \\ \quad\text{Total} & & 30{,}450 \text{ ft-lb} \end{array}$$

$$30{,}450 \times 12 = 365{,}400 \text{ in.-lb.}$$

The section modulus of the pole at the ground line is equal to

$$\frac{\pi \times D^3}{32} = \frac{3.1416 \times 11.5^3}{32} = 149.31$$

Fiber stress = 365,400/149,31 = 2,447 lb per sq in.

The ultimate fiber stress is given as 7,400 lb per sq in., from which the factor of safety = 7,400/2.447 = 3.02.

It is generally considered good engineering practice to design transmission lines with a factor of safety in the order of 2. This is no doubt justifiable for conductors, guys, and hardware. For poles and crossarms, however, the author believes that a factor of safety of not less than 2.5 should be adhered to. As far as transverse loading is concerned, under present code loading assumptions, the horizontal span could be made considerably in excess of 500 ft. In any case, after the first set of calculations have been made, the approximate horizontal span for a given factor of safety may readily be determined.

The bending moment at the ground line due to wind pressure on the three cables is 30,450 − 2,104 = 28,346 ft-lb. The moment due to wind pressure on the top conductor = 9,946/28,346 = 35 percent of the moment due to wind pressure on the three conductors. For a factor of safety of 2.5, the fiber stress = 0.4 × 7,400 = 2,960 lb per sq in. Accordingly, we have (2,960 × 149.31)/12 − 2,104 = 34,726 ft-lb from the wind pressure on the conductors. The moment due to wind load on the top conductor = 0.35 × 34,726 = 12,154 ft-lb. From (28), the horizontal span = 12,154/(0.4973 × 40) = 610 ft.

It will therefore be seen that, under present code loading requirements, the horizontal span will be limited by factors other than that due to transverse loading.

The great majority of wood crossarms used in the United States are of two kinds of timber, creosoted pine and treated fir. The dimensions of wood crossarms for single-pole overhead line construction are now so well standardized that, for a given service and medium-length spans, it is a simple matter to select an arm for a given line with the full assurance that it will have ample strength for the duty required. On the other hand, for long spans and heavy conductors, calculations may be required in order to determine the fiber stress in the arm at the most critical load point.

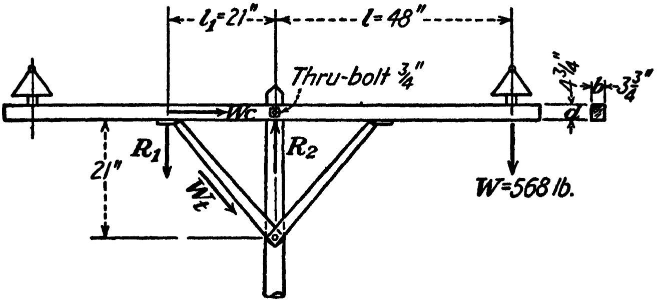

FIG. 29.—Crossarm loading.

The use of wood crossarm braces has increased materially in the last few years. They are likely to replace metal braces almost entirely for overhead lines of transmission voltage up to about 66 kv. Wood crossarm braces, when properly treated or impregnated with preservative, will have many years of useful life. The wood brace has distinct advantages over the metal brace from an operating standpoint, since it permits taking full advantage of the insulating qualities of the wood crossarm. Also, where suspension insulators are used, in a suspended position, with appreciable insulator swing from transverse wind loads on the conductor, there is a decided increase in the flashover value where wood braces are used instead of metal braces.

Figure 29 is an illustration of pole-top construction using a two-piece crossarm brace. The maximum fiber stress in the crossarm with this type of construction, occurs at the thru-bolt and is due to the combined bending moment and the direct stress parallel with the crossarm.

Assume, for example, the 500-ft span and 3/0 twelve-strand copper, which we have discussed. The unit weight of the conductor plus ½ in. radial thickness of ice, from Table 22, is 1.135 lb per ft. 500 × 1.135 = 568 lb vertical load 6 in. from the end of the crossarm, which does not take into account the weight of the crossarm and insulator.

$$R_1 = \frac{Wl}{l_1} \tag{29}$$

$$s = \frac{W_c}{b(d - q)} + \frac{6Md}{b(d^3 - q^3)} \tag{30}$$

where W, l, l_1, b, and d are as shown in Fig. 29.

q = diameter of thru-bolt hole = 0.8125 in.
R_1 = vertical reaction at the brace bolt, lb.
R_2 = vertical reaction at the thru-bolt, lb.
M = bending moment, in.-lb.
s = fiber stress in the arm, lb per sq in.

From (29) $$R_1 = \frac{(48 \times 568)}{21} = 1{,}298 \text{ lb}$$

$$M = 48 \times 568 = 27{,}264 \text{ in.-lb.}$$

Refer to Fig. 29 and Eq. (30). Since the brace makes an angle of 45 deg with the crossarm

$$\begin{aligned}
W_c &= R_1 = 1{,}298 \text{ lb} \\
6Md &= 6 \times 27{,}264 \times 4{,}75 = 777{,}024 \\
b(d - q) &= 3.75(4.75 - 0.8125) = 14.76 \\
b(d^3 - q^3) &= 3.75(107.17 - 0.54) = 400 \\
s &= \frac{1{,}298}{14.76} + \frac{777{,}024}{400} \\
&= 88 + 1{,}943 = 2{,}031 \text{ lb per sq in.} \\
R_2 &= W + R_1 = 568 + 1{,}298 = 1{,}866 \text{ lb}
\end{aligned}$$

The total pressure on the thru-bolt, W_{c2}', is the resultant of the horizontal stress W_c and the vertical reaction R_2.

$$W_{c2} = \sqrt{(1{,}298)^2 + (1{,}866)^2} = 2{,}273 \text{ lb.}$$

The factor of safety for the crossarm = 7,400/2,031 = 3.64. It will therefore be seen that a 3¾″ × 4¾″—9′ creosoted pine crossarm will have a slightly higher factor of safety for the 500-ft span than the 45-ft class 4 pole.

The factor of safety of both the pole and crossarm are based on the ultimate fiber stress of 7,400 lb per sq in. In actual practice, to allow for decay and weakening of the pole and crossarm over the years, a value in the order of 80 to 90 percent of the ultimate fiber stress should be used.

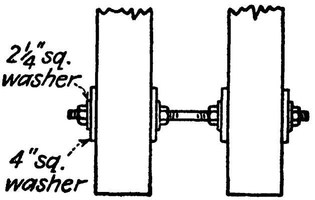

FIG. 30.

Dead-ending on Crossarms.—Where it is at all feasible, vertical dead-ending is used, especially with comparatively large conductors, *i.e.*, the conductors are dead-ended directly to the pole. When this is not possible, crossarm dead-ending is required. Three-wire circuits should generally have the middle conductor attached to the thru-bolt. This gives symmetrical and balanced crossarm loading.

Two values are given for the strength of double arms, depending on the method of installation.[1] For example, where double arming bolts are used, the combined strength of the two arms is given as 30 percent, and where spacer blocks are used, 40 percent of the strength of the two arms when connected as a rigid truss. The writer is of the opinion that the 40 percent value can safely be used with double arming bolts. The installation is made by inserting a 4″ × 4″ × 3⁄16″ square washer between each pair of 2¼ in. square washers on the double arming bolt and the crossarm (see Fig. 30). This makes a very rigid connection, regardless of the spacing between the arms, and requires only standard stock material. Spacer blocks are too often not of the proper length and do not provide the rigidity expected of them.

Let it be required to determine the fiber stress and factor of safety, in each case, with the conductors attached to the arms as indicated by a and b in Fig. 31 for the 3/0 copper conductor.

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[1] See "Overhead Systems Reference Book," p. 358, N.E.L.A.

$$\text{Section modulus} = \frac{(d - q)[(2b + p)^3 - p^3]}{6(2b + p)} \tag{31}$$

$b = 3\frac{3}{4}$ in. $d = 4\frac{3}{4}$ in. $p = 8$ in. $q = 1\frac{3}{16}$ in.
$= 0.8125$ in.

$$d - q = 4.75 - 0.8125 = 3.9575 \text{ in.}$$
$$(2b + p)^3 - p^3 = 15.5^3 - 8^3 = 3{,}725 - 512 = 3{,}213$$
$$6(2b + p) = 6 \times 15.5 = 93$$

From (31) $$\text{Section modulus} = \frac{3.9575 \times 3{,}213}{93} = 136.7$$

Condition a.—The ultimate fiber stress of the crossarm is taken as 7,400 lb per sq in., from which the total resistance moment = 136.7 × 7,400 = 1,011,580 in.-lb. Assuming the strength of the two arms to be 40 percent of that when connected as a rigid truss, we have 0.4 × 1,011,580 = 404,630 in.-lb. The bending moment due to the load of 3,778 lb applied 48 in. from the center of the pole is equal to 48 × 3,778 = 181,340 in.-lb.

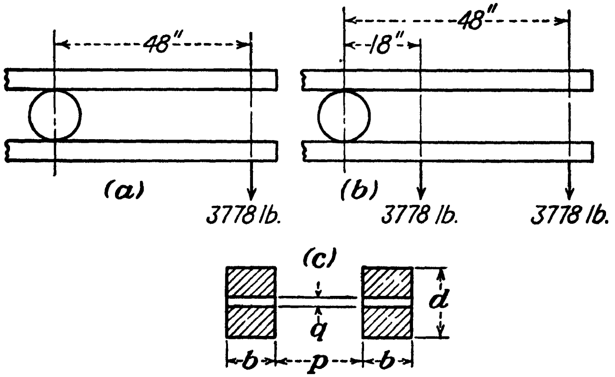

Fig. 31.

$$\text{Factor of safety} = \frac{404{,}630}{181{,}340} = 2.23$$

$$\text{Fiber stress} = \frac{181{,}340}{(0.4 \times 136.7)} = 3{,}318 \text{ lb per sq in.}$$

Condition b

$$\begin{aligned} M_1 &= 48 \times 3{,}778 = 181{,}340 \text{ in.-lb} \\ M_2 &= 18 \times 3{,}778 = \underline{\ 68{,}000} \\ M &= \phantom{18 \times 3{,}778 =\ } 249{,}340 \text{ in.-lb} \end{aligned}$$

$$\text{Fiber stress} = \frac{249{,}340}{(0.4 \times 136.7)} = 4{,}560 \text{ lb per sq in.}$$

$$\text{Factor of safety} = \frac{7{,}400}{4{,}560} = 1.62.$$

Opinions are likely to vary among designers as to the factor of safety at assumed maximum loading that should be adhered to for crossarm dead-end loading.

A full dead end should be considered as different from a double dead end. With a double dead end, there is usually very little unbalanced longitudinal load and only sufficient strength to avoid failure of the arms in case a broken conductor is required. With full dead end, normal as well as maximum load is applied in one direction and if the arms do not fail, they have a tendency to bow and become unsightly. Good engineering would certainly dictate a factor of safety of not less than 3 for full dead end on crossarms.

A convenient and satisfactory method of reinforcing crossarms where dead-ending is required is to back up the arms with a piece of channel iron, as illustrated by Fig. 32. A piece of channel iron that will fit over the arm on the back side of the pole is drilled to take the double arming and thru-bolts. It will generally provide the required additional strength.

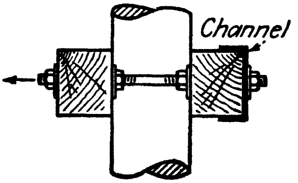

Fig. 32.

The resistance moment of a set of double arms is appreciably affected by the spacing between the arms. The strength will be increased in the order of 10 percent for each inch increase in the spacing from 6 to 10 in.

Stresses Due to Angles in the Line.—It is practically impossible to build a transmission line of any appreciable length without several angles, which may range in magnitude from only a few minutes to 90 deg or more.

Small angles may be turned on single-pin insulators or on a single string of insulators in a suspended position. The latter type of construction is shown by Fig. 33*A*. The maximum angle that can be turned on suspension insulators will depend on the ratio of horizontal tension in the conductor to the weight of the conductor on the insulator string. In general, angles in excess of about 5 deg cannot be turned on insulators in a suspended position without producing excessive insulator swing, thus cutting the clearance between the conductor and the crossarm dangerously close.

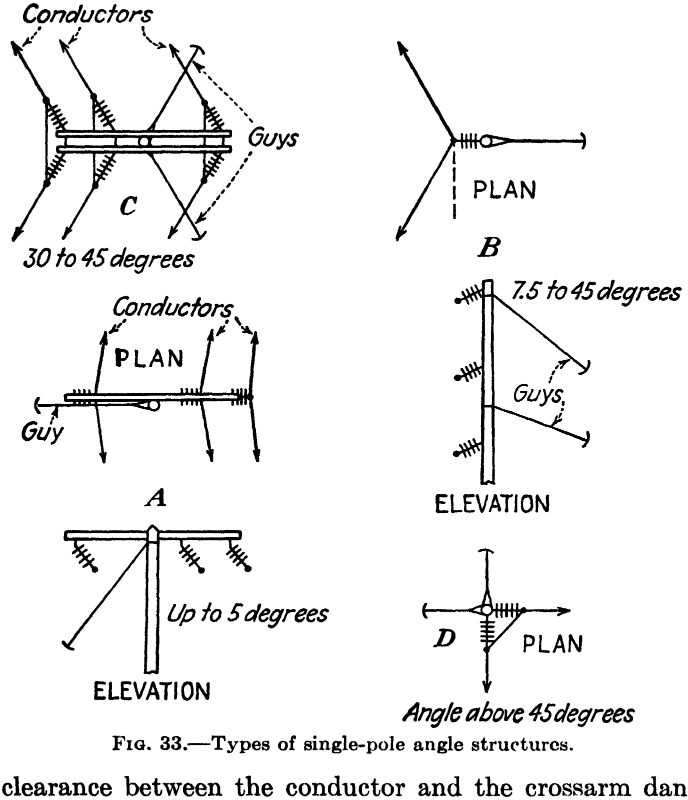

FIG. 33.—Types of single-pole angle structures.

Angles in the range between 10 and 45 deg may be turned on what is called a "vertical angle," shown in Fig. 33*B*. Angles in excess of about 45 deg should not in general be turned on this type of construction, since unequal stresses will be set up in the conductor where it attaches to the suspension clamp. Obviously the sharper the angle, or bend in the conductor at the clamp, the more unequal the stresses will be.

Angles up to about 45 deg in the line may be turned on a double-arm double-tension structure, similar to Fig. 33*C*. The angle that can be turned on this type of construction is limited to about 45 deg. For greater angles, there may be insufficient clearance between phase wires near the structure.

Angles of any magnitude above about 30 deg may be turned on the vertical dead-end type of structure, indicated by Fig. 33*D*. For angles of less than 30 deg turned on this type of structure, the jumpers may swing too close to the pole.

At each angle in the line, the structure and its guying layout must have sufficient strength to withstand the total stresses set up in its component parts by the transverse load. The transverse load, or pull, due to the angle in the line, will depend on the tension in the wire or cable, the number of wires or cables, and the magnitude of the angle. For large angles in important lines, it may be, and often is, necessary to install specially designed angle structures. In general, however, for single-pole construction it is necessary only to guy the pole, using one or the other of the types of construction shown in Fig. 33. In order to design a guying layout properly, it is necessary to know the approximate side pull, or resultant load on the structure, under maximum loading conditions.

The curve in Fig. 34 has been constructed to facilitate the determination of side pull due to angles in a line. By use of this curve, calculations are eliminated almost entirely. Values may be read from Fig. 34 to an accuracy well within engineering requirements. The values used for plotting the curve were derived from the following equation (see Fig. 34*A*):

$$R = 2 \times \sin \tfrac{1}{2}\theta \times H \qquad (32)$$

where R = resultant, or side pull due to the angle, lb.

H = horizontal tension in the cable, lb.

N = number of cables.

θ = angle, deg.

Let us assume, for example, we have at a certain pole in the line an angle of 25 deg. $\sin \frac{25}{2} = \sin 12°30' = 0.216$. Then

$$R = 2 \times 0.216 \times 3{,}778 \times 3 = 4{,}896 \text{ lb}$$

Referring to Fig. 34, we read for an angle of 25 deg a value of about 0.431, from which $0.431 \times 3{,}778 \times 3 = 4{,}885$ lb. The values in the left-hand margin of Fig. 34 are two times the sin of one-half the angle θ multiplied by the tension in the conductor and by the number of conductors.

A curve similar to Fig. 34 is easily prepared and should always be available to the designer, since it is a timesaver and makes errors in the results improbable.

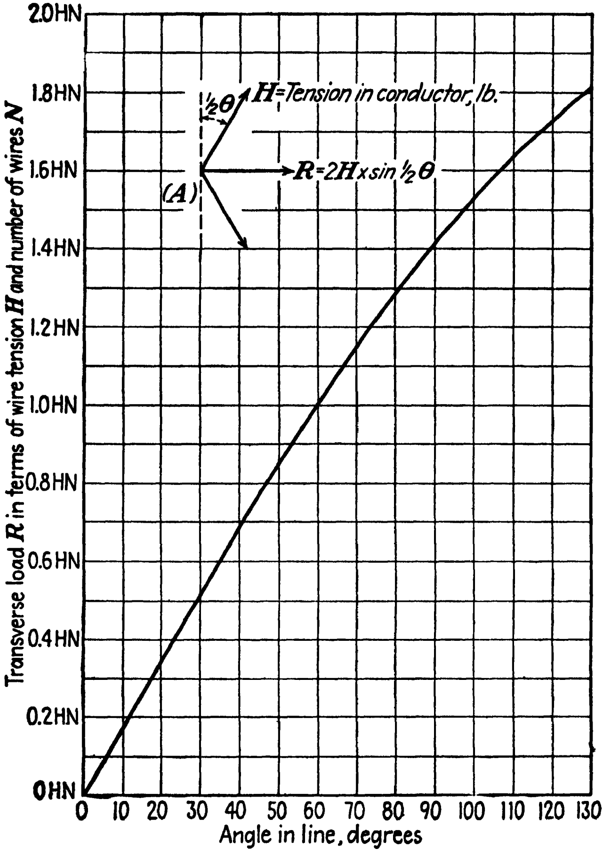

FIG. 34.—Curve for finding transverse load due to angle in line.

Calculating Tension in Guys.—Figure 35*a* shows both plan and elevation of a double dead-end structure at an angle in the line of 25 deg. The side pull, previously determined, on the structure is 4,896 lb at this angle. The side guy would normally be installed with a vertical angle of 45 deg, as shown. The tension in the guy is equal to the horizontal component of load due to tension in the conductors, divided by the sine of the vertical angle. sin 45° is 0.7071, from which the tension in the guy, T_g, = 4,896/0.7071 = 6,924 lb. Generally, as in this case, the guy attachment, for all practical purposes, is at the same elevation as are the conductors at the pole. Under these conditions, the overturning moment on the pole will be negligible.

If the line is in such a location that the coincident ice and wind load are known to be quite severe and to occur quite frequently, then the guying should be designed to have a factor of safety in the order of 2. This would call for two $\frac{3}{8}$-in. Siemens Martin steel strands, or one $\frac{7}{16}$-in. high-strength steel strand.[1] On the other hand, if maximum load is likely to occur only a comparatively few times during the life of the line, a guy strand that will give a factor of safety in the order of 1.5 may be used. This will call for one $\frac{3}{8}$-in. high-strength steel strand.

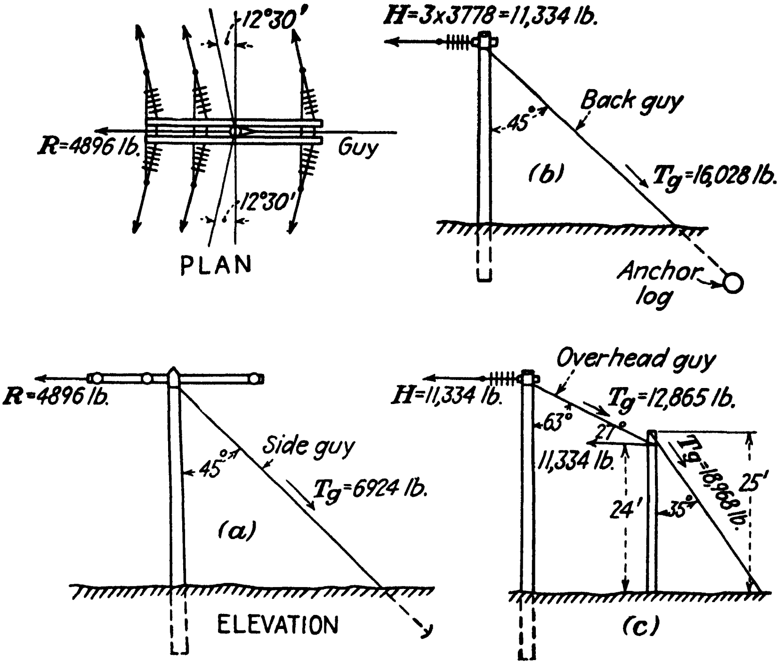

Fig. 35.—Guy tensions at angle and tension structures.

Figure 35*b* shows an elevation of a full dead-end structure. The vertical angle is 45 deg. The horizontal load at the conductor attachment due to the tension in the conductors is 3 × 3,778 = 11,334 lb. The guy tension

---

[1] Steel cable is usually referred to as strand.

$$T_g = \frac{11,334}{0.7071} = 16,028 \text{ lb.}$$

For a factor of safety of 1.5, the required guy strength must be 1.5 × 16,028 = 24,024 lb. This may be made up of two $\frac{5}{16}$ in. extra-high-strength, or one $\frac{7}{16}$-in. extra-high-strength strand.

Figure 35*c* is a full dead-end structure on which an overhead guy is required. The overhead guy makes an angle with the pole of 63 deg, the sine of which is 0.891. The tension T_g = 11,334/0.891 = 12,865 lb. The overhead guy makes an angle with the horizontal at the guy stub of 90 − 63 = 27 deg. The horizontal component of tension at the guy attachment at the stub 12,865 × 0.891 = 11,334 lb, the same as at the line pole.

Since there is approximately 12 in. difference in elevation of the overhead guy and the down guy at the stub, the overturning moment on the guy stub should be computed. The guy tension is derived from that. The bending moment at the ground line is 24 × 11,334 = 272,016 ft-lb. Dividing this value by the height of the down guy will give the horizontal component of tension in the down guy. This is equal to 272,016/25 = 10,880 lb. sin 35° is 0.5736, from which T_g = 10,880/0.5736 = 18,968 lb.

H-frame Lines.—Until some fifteen years ago, the wood pole did not come into its own as a supporting structure for high-voltage transmission lines. It was the general practice to build lines of 110 kv and above of steel towers. During the intervening years, however, a decided change has developed, and steel towers for 66- to 154-kv lines have largely been superseded by wood-pole H-frame lines.

The change from steel to wood for transmission lines of this class has been brought about by lower first cost; improved operating characteristics; susceptibility to protection from lightning disturbances; and the fact that the wood crossarm and pole may be utilized to augment the insulation, thus tending to reduce line outages.

For high-voltage lines, for economic reasons, longer spans than are permissible with single-pole construction are required. For spans exceeding about 500 ft in length, to provide the required mechanical strength and conductor spacing, it is generally necessary to employ H-frame construction.

A lengthy discussion of the merits or demerits of any particular type of H-frame structure or method of design would be out of place here. Each designer usually has his own ideas of what constitutes good engineering practice, having in mind the conductor and structure loading applying to the section of the country in which he is primarily interested. Good engineering practice, however, will eliminate fittings and gadgets that are not absolutely essential to the strength and life of the structure. It should always be kept in mind that no more energy can be delivered over a line costing $7,500 per mile than over a line with the same size of conductor costing $5,000 per mile, but that the fixed charges will be one and one-half times as much, reducing earning capacity appreciably.

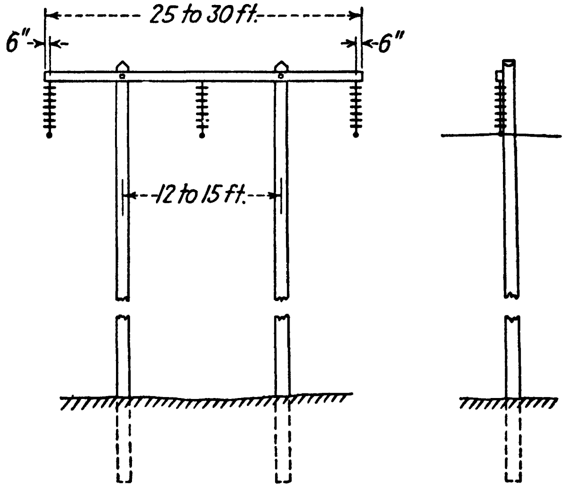

FIG. 36.—Typical 110-kv H-frame.

It is obvious that a transmission line of such design and construction as to withstand the stresses imposed by the severe wind and ice loads that frequently occur in some of the northern states could not be considered justifiable in some of the southern states. A line that would require a class 2 pole and special crossarm construction in one section of the country may require only a class 5 pole and a single crossarm to give the same factor of safety for a given span in another section where ice and wind loads are considerably less severe.

Figure 36 is a sketch of a typical 110-kv H-frame tangent structure without overhead ground wires. Since poles are usually not provided with a supporting medium other than the earth itself, the H-frame is not a rigid structure, and the laws for the calculation of stresses in a rigid structure will not apply. In fact, there is no definite rule by which the stresses set up in the component parts of an H-frame can be determined with any degree of accuracy. The fact that the two poles in a single structure will rarely, if ever, be of the same dimensions and strength adds further to the unreliability of the calculated results. It is therefore necessary to compromise on some basis of design that will take into consideration the observed results of such lines over a long period of time.

Engineers differ greatly as to the relative strength of an H-frame and a single pole. Some contend that an H-frame is three or more times as strong as a single pole of like size and kind. Others maintain that an H-frame is only twice as strong as a single pole. The author is of the opinion, based on many years of observation, that the average strength of an H-frame, in general, will have somewhat more than twice the strength of a single pole, assuming in all cases that the single pole is similar in size and kind to each of the poles combined in the H-frame structure.

If two poles of equal strength were tied together near the top in such manner as not to restrict the deflection of the poles due to the application of load in a direction transverse to the structure, the author believes that the strength of the two poles would be just double that of one similar pole. It should be pointed out, however, that H-frame lines are not so built in practice and that actual conditions differ materially from this theoretical consideration.

Actually the two poles are connected with a crossarm that is bolted securely to each pole with several square inches of surface contact. This creates considerable resistance to the movement of the poles in a transverse direction, thus increasing the rigidity of the structure.

No definite rule can be established for the design of H-frame lines, for local conditions will largely govern the type of structure to be used for a given line. The following simple method for determining the maximum span length for unguyed, unbraced structures may be resorted to; the results will prove to be extremely satisfactory.

No attempt is made to calculate the fiber stress in the H-frame as such. It is only necessary to calculate the fiber stress in a single pole of the class and kind to be used (see pages 93 and 94). Assume a span, which can be taken as the ruling span of the proposed H-frame line if desired, and calculate the fiber stress for this span length and the size of conductor to be used for the unguyed pole. The fiber stress, so derived, usually at the ground line, divided into the ultimate fiber stress of the pole will give the factor of safety of the pole.

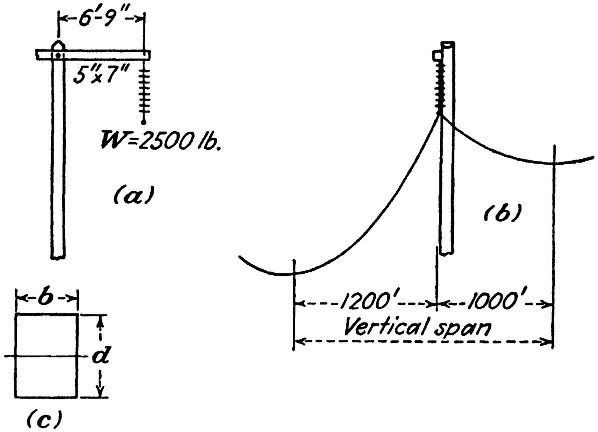

FIG. 37.—Crossarm loading.

If the line under consideration supplies extremely important loads where continuity of service is paramount, the factor of safety as derived above for the single pole may be multiplied by 2. On the other hand, for lines of lesser importance, a multiplying factor of approximately 2½ may logically be used. Once the factor of safety of the H frame is derived in this manner, the maximum span for any given factor is readily determined.

H-frame Crossarms.—Since crossarm braces are not generally used on H-frame structures, the point of maximum stress in the arm will be at the thru-bolt. Where conductor loading is comparatively heavy, as in long hillside spans, the crossarm design should be given careful consideration. For lines of any considerable length with only a comparatively few points where crossarm loading is likely to be severe, it is not advisable to purchase crossarms heavier than those needed for practically the entire line in order to provide an arm of sufficient strength for a relatively small number of the total number of structures.

When the effective vertical span, illustrated in Fig. 37, is such that maximum stress in the arm exceeds about 50 percent of its ultimate strength, good engineering practice is to double-arm that particular structure, using a double-arming plate from which to suspend the insulator assembly. By double-arming structures at points where extra crossarm strength is required, rather than installing one heavier arm, it is possible to purchase arms of the same size for all lines of the same voltage and similar type on the entire system. This is a decided advantage from a maintenance standpoint, as well as requiring only a minimum number of arms to be carried in stock.

a shows the end of crossarm with its load.

b indicates that the total weight of cable plus its ice load between the low points in the adjacent span is imposed on the one crossarm.

c is an enlarged view of the end of the crossarm.

Weight of end of crossarm = 70 lb
Weight of insulator assembly = 100
Weight of cable and ice = 2,500
Total weight 2,670 lb

$$2{,}670 \times 81 = 216{,}270 \text{ in.-lb.}$$

$$\text{Section modulus} = \frac{bd^2}{6} = \frac{(5 \times 7^2)}{6} = 40.83$$

$$\text{Fiber stress} = \frac{216{,}270}{40.83} = 5{,}297 \text{ lb per sq in.}$$

$$\text{Factor of safety} = \frac{7{,}400}{5{,}297} = 1.4$$

This will require double arming, for which the factor of safety will be 2.8.

Where crossarms of an appreciable size are used, since the thru-bolt hole is in the neutral field, it is permissible to disregard the material cut away in the hole in calculating the strength of the arm.

The trend in H-frame construction in certain parts of the country is toward the use of the double plank arm, which provides excessive crossarm strength at many structures in order to obtain adequate strength at other structures. This, of course, has the advantage of standardization and is no doubt justifiable in sections of the country where the profile is comparatively level and permits long-span construction throughout. The line must be justified economically, however; and, when the profile is such that many comparatively short spans are required, the design should be made to fit the conditions.

CHAPTER VIII

SAG AND TENSION CALCULATIONS

The subject matter dealing with determination of sag in overhead transmission-line conductors is divided into two parts. The first part, covered in this chapter, is restricted to use with short and medium length spans, *i.e.*, spans having a length in the order of 1,000 ft and less. The second part deals with and develops a new method for calculating sag in overhead conductors which is adaptable for use with long spans, *i.e.*, spans having a length in the order of 1,000 ft and more. It is discussed in Chap. IX.

The calculation of sag in transmission-line conductors consisting of a homogeneous material, such as all copper or all steel, resolves itself into a comparatively simple problem. On the other hand, when the conductor consists of a composite material, such as a.c.s.r. or copperweld-copper, sag calculations become more involved because the tensile strengths and temperature coefficients are different for the different kinds of material of which the cable is composed. For such conductors, it is well to consult the manufacturer of the particular type of material to be used. Manufacturers will furnish stringing sag and tension curves or charts from which they may be derived.

Several methods are available for computing sag in overhead line conductors. All require considerable work when loading and temperature changes are taken into account, an essential consideration for any complete solution to sag problems. For spans having a length not exceeding about 1,000 ft, the most practicable solution is by the application of what is known as the "parabola method."

The work is comparatively simple when the conductor loading and temperature remain unchanged. When changes in loading and temperature occur, however, the sag and tension for the changing conditions are also readily determined but require the plotting of two sets of curves, as will be shown.

When the wire or cable is assumed to conform to the curve of the parabola, its weight is assumed to be uniformly distributed along its horizontal projection (see Fig. 38). Definite loadings of ice and wind are assumed, but the actual loading may be quite different from assumed values. Properties of the conductors, such as elasticity and coefficient of expansion, may vary considerably. These values are rarely the same for two pieces of wire, the average of which, when combined into a cable of several wires, may differ appreciably from that of a single wire. Also, owing to the sun's heat, absorption and reflection may make the temperature of the conductor differ from that of the thermometer.

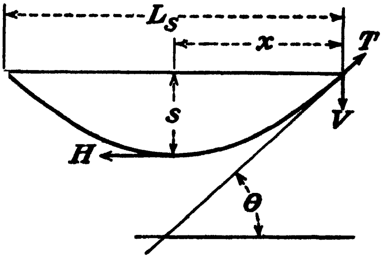

FIG. 38.—Parabola.

Sag Calculations.—In order to demonstrate the method of performing the work, the following conditions are assumed to apply:

500-ft ruling span
3/0 twelve-strand hard-drawn copper cable
w_r = unit resultant weight of the cable loaded with ½-in. radial thickness of ice and 8-lb wind at 0°F = 1.5091 lb per ft
w_v = unit weight of the ice-covered cable = 1.135 lb per ft
w_{vo} = unit weight of the bare cable = 0.5181 lb per ft
Coefficient of linear expansion = 0.0000094 ft per deg F
A = area of cable = 0.1318 sq in.
M = modulus of elasticity = 17,000,000 lb per sq in.
MA = 2,240,600
Ultimate, or breaking, strength = 7,556 lb

Assumed maximum design load is taken as 50 percent of the ultimate strength of the cable = 3,778 lb at 0°F, ½-in. ice 8 lb per sq ft wind pressure.

EQUATIONS OF THE PARABOLA

$$S = \frac{wL_s^2}{8H} = \text{sag at center of span} \qquad (33)$$

$$L = L_s + \frac{8 \times S^2}{3 \times L_s} = \text{length of conductor in span} \qquad (34)$$

$$\tan \theta = \frac{(2 \times S)}{x} \text{ for level span}$$

$$T = \frac{H}{\cos \theta} = \text{tension in conductor at the support} \qquad (35)$$

$$V = H \times \tan \theta = \text{vertical component of tension at the support} \qquad (36)$$

From (33) $S = \dfrac{1.5091 \times 500^2}{8 \times 3{,}778} = 12.48$ ft

From (34) $L = 500 + \dfrac{8 \times 12.48^2}{3 \times 500} = 500.831$ ft

$$\tan \theta = \frac{2 \times 12.48}{250} = 0.09984 \quad \theta = 5°42'$$

$$\cos \theta = 0.99505$$

From (35) $T = \dfrac{3{,}778}{0.99505} = 3{,}796.8$ lb

From (36) $V = 3{,}778 \times 0.09984 = 377.2$ lb

In the calculation of sags, it is required to compute the unstressed length of the wire or cable. This assumes that all tension is removed from the cable and that it could be suspended from the supports and have tension applied at will. The unstressed length of the 3/0 copper cable may be determined as follows:

$$L_u = \frac{L}{1 + (H/MA)} = \text{unstressed length of cable in span} \qquad (37)$$

$L = 500.831$ ft (previously determined)
$H = 3{,}778$ lb
$MA = 2{,}240{,}600$

From (37) $L_u = \dfrac{500.831}{1 + 3{,}778/2{,}240{,}600} = 499.988$ ft at 0°F

$$e = \frac{H \times L_u}{M \times A} = \text{stretch of cable due to applied tension}$$

The change in length for an increasing temperature, hard-drawn copper $= L_u(1 + 0.0000094 \times t)$. For a decreasing temperature, the change in length $= L_u(1 - 0.0000094 \times t)$, where t represents the increase or decrease in degrees Fahrenheit.

UNSTRESSED LENGTHS IN THE 500-FT SPAN

−20° $L_u = 499.988(1 - 0.0000094 \times 20) = 499.894$ ft
0° $L_u = 499.988(1 + 0.0000094 \times 0) = 499.988$ ft
30° $L_u = 499.988(1 + 0.0000094 \times 30) = 500.129$ ft
32° $L_u = 499.988(1 + 0.0000094 \times 32) = 500.1384$ ft
60° $L_u = 499.988(1 + 0.0000094 \times 60) = 500.270$ ft
90° $L_u = 499.988(1 + 0.0000094 \times 90) = 500.411$ ft
120° $L_u = 499.988(1 + 0.0000094 \times 120) = 500.552$ ft

When tension is applied to an unstressed cable, the elongation varies directly as the applied tension within its elastic limit. For example, the unstressed length of the 3/0 copper in the 500-ft span is computed to be 499.988 ft at 0°F. If we apply tension in increments of 1,000 lb, we have

$$\text{Elongation for 1,000 lb} = \frac{1{,}000 \times 499.988}{2{,}240{,}600} = 0.223149 \text{ ft}$$

Adding the stretch for each 1,000 lb to the length of the unstressed cable at 0°F, the following values are obtained:

$$0° \; L_u = 499.988 \text{ ft}$$

H
1,000 $L = 499.9880 + 0.2231 = 500.2111$ ft
1,500 $L = 499.9880 + 0.3347 = 500.3227$ ft
2,000 $L = 499.9880 + 0.4463 = 500.4343$ ft
2,500 $L = 499.9880 + 0.5579 = 500.5459$ ft
3,000 $L = 499.9880 + 0.6694 = 500.6574$ ft
3,500 $L = 499.9880 + 0.7810 = 500.7690$ ft
4,000 $L = 499.9880 + 0.8926 = 500.8806$ ft

$$S = 0.5\sqrt{\frac{3L_s(L - L_s)}{2}} = \text{sag in terms of cable length} \qquad (38)$$

For the 500-ft span, (38) may be reduced to

$$S = 0.5\sqrt{750(L - L_s)}$$

From (38), the sag from the cable length corresponding to the given values of tension *H* is as follows:

H

1,000 $S = 0.5\sqrt{750 \times 0.2111} = 6.29$ ft
1,500 $S = 0.5\sqrt{750 \times 0.3227} = 7.78$ ft
2,000 $S = 0.5\sqrt{750 \times 0.4343} = 9.02$ ft
2,500 $S = 0.5\sqrt{750 \times 0.5459} = 10.12$ ft
3.000 $S = 0.5\sqrt{750 \times 0.6574} = 11.10$ ft
3,500 $S = 0.5\sqrt{750 \times 0.7690} = 12.00$ ft
4,000 $S = 0.5\sqrt{750 \times 0.8806} = 12.85$ ft

In order to determine the final results for the 500-ft span, proceed as follows:

1. On a sheet of graph paper, as indicated by Fig. 39, enter in the vertical margin values of tension embracing the required range for the given conductor. Enter sags, in feet, at the bottom of the sheet. Use a scale for both sag and tension that may be

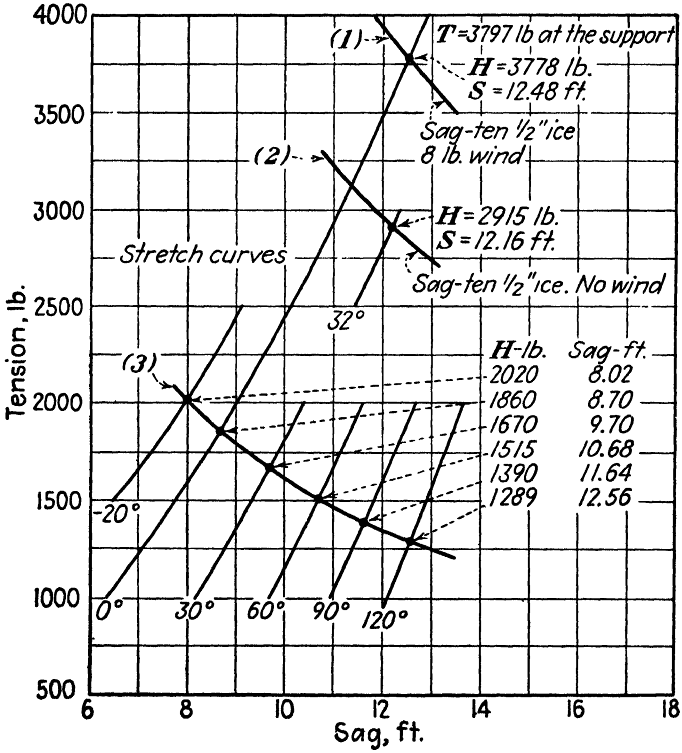

FIG. 39.—Sag-tension curves 500-ft span 3/0 twelve-strand copper.

read with a fair degree of accuracy. Since the maximum design tension for the 3/0 copper cable is less than 4,000 lb, assume tensions of 4,000, 3,750, 3,250, and 3,000 lb. From (33), using a resultant weight of 1.5091 lb per ft, compute the sag for each given value of tension. Plot the values as curve 1. The numerator constant of (33) $= 1.5091 \times 500^2 = 377{,}275$.

H	
4,000	$S = 377{,}275/32{,}000 = 11.79$ ft
3,750	$S = 377{,}275/30{,}000 = 12.57$ ft
3,500	$S = 377{,}275/28{,}000 = 13.47$ ft
3,250	$S = 377{,}275/26{,}000 = 14.51$ ft

2. Assume the wind load to be removed from the cable. Then the vertical weight of the cable and ice is 1.135 lb per ft. Using this weight and Eq. (33), compute the sag for 3,250, 3,000, and 2,750 lb. Plot the values as curve 2. This curve is plotted for the purpose of determining whether the sag is greater at 32 deg with the ice-covered cable or at maximum temperature. The numerator of (33) for this loading $= 1.135 \times 500^2 = 283{,}750$.

H	
3,250	$S = 283{,}750/26{,}000 = 10.91$ ft
3,000	$S = 283{,}750/24{,}000 = 11.82$ ft
2,750	$S = 283{,}750/22{,}000 = 12.80$ ft

3. Assume both wind and ice load to be removed from the cable, and the weight $w_{vo} = 0.5181$ lb per ft. From this weight and (33), compute the sag for a tension of 2,200, 2,000, 1,800, 1,600, 1,400, and 1,200 lb. Plot the values as curve 3. Since the intersections on this curve indicate sag and tension for the unloaded cable at various temperatures, it is important that care be exercised in its construction. The numerator of (33) $= 0.5181 \times 500^2 = 129{,}525$.

H	
2,200	$S = 129{,}525/17{,}600 = 7.36$ ft
2,000	$S = 129{,}525/16{,}000 = 8.10$ ft
1,800	$S = 129{,}525/14{,}400 = 8.99$ ft
1,600	$S = 129{,}525/12{,}800 = 10.12$ ft
1,400	$S = 129{,}525/11{,}200 = 11.56$ ft
1,200	$S = 129{,}525/9{,}600 = 13.49$ ft

Curves 1, 2, and 3 give sag in terms of weight and tension. These curves are of no particular value except as a means to an end. This will be readily understood as the work progresses.

TABLE 33.—SAG AND LENGTH AS AFFECTED BY TEMPERATURE AND TENSION

Tension, lb	Temp., deg F	Length, $L_u + e$ ft	Sag in terms of L and L_s
1,000	−20	500.1117	4.15
1,500	−20	500.2287	6.55
2,000	−20	500.3402	7.99
1,000	0	500.2111	6.29
1,500	0	500.3227	7.78
2,000	0	500.4343	9.02
2,500	0	500.5459	10.12
3,000	0	500.6574	11.10
3,500	0	500.7690	12.00
4,000	0	500.8806	12.85
1,000	30	500.3521	8.13
1,500	30	500.4637	9.32
2,000	30	500.5752	10.39
2,500	32	500.6962	11.43
2,750	32	500.7521	11.88
3,000	32	500.8078	12.31
1,000	60	500.4931	9.62
1,500	60	500.6047	10.65
2,000	60	500.7163	11.59
1,000	90	500.6341	10.90
1,500	90	500.7457	11.82
2,000	90	500.8573	12.68
1,000	120	500.7751	12.06
1,500	120	500.8867	12.89
2,000	120	500.9983	13.68

In order to derive the cable length at the various temperatures, add the stretch for each given tension to the unstressed length of cable at the given temperature, using the stretch of 0.223149 ft per 1,000 lb. Then, from the lengths so obtained, calculate the sag from (38). It is sufficient, however, to calculate the sag for only three points on the curve except at zero degrees. Compile the values as in Table 33.

From the sag in the last column of Table 33, plot the hypothetical, or stretch, curves for −20, 0, 30, 32, 60, 90, and 120°F. These curves are of no particular value in themselves but must be used in conjunction with curves 1, 2, and 3. The only rational points are at the intersections of the two sets of curves. It should be particularly noted that the 0° curve intersects curve 1 at approximately 3,778 lb. This is the point of assumed maximum tension and checks the work and method of analysis. If there is an error in the work, the two curves will not cross at the point of assumed maximum tension. The only rational and usable values derived from the sag-tension or stretch curves, occur when both equations give the same value.

TABLE 34.—VALUES FOR PLOTTING SAG TEMPLATE CURVES

Span, ft	Sag, ft	
	−20 deg	120 deg
100	0.32	0.50
200	1.28	2.00
300	2.89	4 52
400	5.13	8 03
500 (ruling span)	8 02	12.56
600	11 55	18.09
700	15.72	24.62
800	20.53	32.15
900	25.98	40.69
1,000	32.08	50.24

Since the sag at 32° for the ice-covered cable (see curves 2 and 3, Fig. 39) is less than the 120° sag for the bare cable, we are now concerned only with the points of intersection on curve 3, which are given in Fig. 39.

For the parabola solution, it is safe to assume that the sag will vary as the square of the span length for spans at least double the length of the ruling span. With a series of undead-ended spans of unequal length, the cable will assume curves such that the horizontal component of tension will be the same in all spans between dead-end points.

For the purpose of spotting structures on the profile, it is necessary to construct, usually of transparent celluloid, a sag template, which should include the curve for minimum and maximum temperatures. The values from which the template is to be cut may be computed by proportion. For example, the −20° sag is read from Fig. 39 to be 8.02 ft in the 500-ft span.

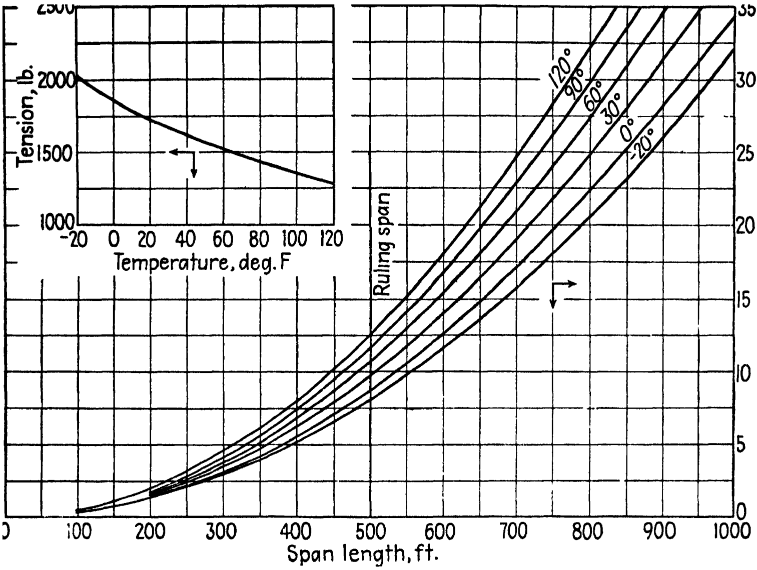

Fig 40.—Stringing sag and tension curves for 3/0 twelve-strand copper.

The 120° sag is read as 12.56 ft. The values at other span lengths are proportional to the square of the span length and are compiled in Table 34.

In addition to the −20 and 120 deg sags, as given in Table 34, the sags at the intermediate temperatures are computed by proportion from the values in Fig. 39 and are plotted as shown in Fig. 40 for field use. Where practicable, the cable should be strung to tension, corresponding to the temperature, by dynamometer, and the sag should be checked by targets. Usually each pull will consist of ½ to 1 mile or more. If possible, a level span should be chosen for checking the sag. A good grade of thermometer should be used, and it should be checked periodically with a thermometer that is known to be approximately correct.

This method of calculating sag in overhead conductors should not be used in its entirety for spans in excess of about 1,000 ft or where the sag greatly exceeds 5 percent of the span length. The 120-deg sag, from Table 34, is given as 50.24 ft in the 1,000-ft span. The catenary method gives this as 50.42 ft, or a difference of only 2.16 in. The difference for a 1,500-ft span between the parabola and catenary solutions is in the order of 1 ft. Thus it will be seen that, as the span length increases appreciably over 1,000 ft, the error by the parabola method increases quite rapidly. Since the sag by the parabola method is less than by the catenary, for the same conditions, the factor of safety from a standpoint of ground clearance is reduced.

CHAPTER IX

NEW METHOD OF CALCULATING SAG IN LONG SPANS

MEAN LENGTH METHOD

Many large cities and industrial centers depend entirely on transmission lines for their electric service. Many comparatively large power-consuming areas are supplied over a single circuit line and have become accustomed to nearly 100 percent service.

The quality and continuity of electric service supplied over a transmission line depend largely on whether the conductors have been properly installed. The designer must determine in advance the amount of sag and tension to be given the wires or cables of a particular transmission line at a given temperature. This should not be taken to mean that the sag and tension shall only roughly approximate a given value. They should be calculated to a high degree of accuracy, and the field work must conform quite closely to the calculated values.

Three hazardous elements must be taken into account in the calculation of conductor sags, namely, ice, wind, and low temperature. Maximum stress in the conductor, as is well known, occurs when the coincident wind and ice loads occur at low temperatures. It is therefore the problem of the designer to determine the sag to be given a particular conductor at stringing temperature such that the stress under the most severe assumed loading conditions will not exceed a certain prescribed value.

In the early stages of transmission-line construction, when a span in the order of 200 ft was considered long, there was not a great deal of need for refinement in the design of conductor sags. However, as time went on and higher and higher voltages were required to keep pace with the ever-increasing demand for greater transmission distances, it became apparent that something would have to be done to keep down the cost of line insulation. Obviously the only way this could be accomplished was by resorting to longer spans, thus reducing the number of structures.

Designing engineers throughout the country began clamoring for a simple and direct method of calculating sag in overhead conductors, which could be relied upon to give results with a better than fair degree of accuracy for long spans. Spans in the order of 1,000 ft in length are now considered long.

Probably the most widely used method up to the present time for calculating sag in spans of an appreciable length is that developed by Percy H. Thomas, entitled "Sag Calculations for Suspended Wires," which was presented at the twenty-eighth annual convention of the American Institute of Electrical Engineers in Chicago in 1911.

The Thomas method provides for a semigraphical solution and is based on the assumption that a wire suspended between two supports at equal elevations will conform to the curve of a catenary. Thomas first conceived the plan of reducing the span in length, without altering the shape of the curve, until the span is 1 ft long. The calculations are then made for the 1-ft span, after which it is a simple matter to convert the values into corresponding quantities for the actual span.

Although the Thomas method is not difficult of solution, it requires the use of specially prepared charts that must be constructed with a great deal of care. Further, a chart that is applicable for use with comparatively long spans is not applicable for use, with the same degree of accuracy, for short spans, and conversely. At least three charts are needed to cover the range of spans commonly met with in practice, and they should be of such size that the scale will permit of fairly accurate reading. Three chart readings are also required for each value of sag and conductor length. A considerable amount of work is also required to determine the effect of temperature changes on the sag and tension.

Other methods have been evolved for computing sag in overhead conductors, probably the most popular of which is the tables that were compiled by James S. Martin and published by the Copperweld Steel Company in 1931. The table values were derived from the equations of the catenary curve and are quite simple to use for a given condition. When changes in temperature occur, which must be taken into account in all sag problems, the Martin tables require a method of trial and error.

The simple and direct method of calculating sag in overhead transmission-line conductors, as described in the following discussion, was developed partly by Edwin S. Parker and partly by the author. Several years ago, Mr. Parker discovered that taking the sag derived from the equations of the parabola, multiplying it by the unit weight of cable, span length, and the mean of the cable and span length, then dividing this product by eight times the horizontal component of tension, would give a sag corresponding almost exactly to that derived from the catenary solution.

Mr. Parker prepared a paper entitled "Sag Tension Curves," which was published by the American Society of Civil Engineers in December, 1926. Parker's discovery has been little used, largely because it left much to be desired in clarity and directness. For example, although the method remained susceptible to solution from the equations of the parabola, slightly modified, which make use of the horizontal component of tension, an attempt was made to use as a basic assumed starting point the tension at the support, and an involved quadratic equation was required to determine the horizontal component of tension.

By delving into the problem, the author has made further discoveries, one of which is that, by plotting two sag-tension curves, one from values derived by application of the equations of the parabola and the other from values derived by application of the modified equations, and plotting the two curves on the same sheet, a horizontal line drawn from a given tension on the lower curve (from the parabola) will intersect the upper curve (from the modified equations) at a sag that will be comparable with that derived from the catenary solution. This will be made clear in what follows.

Two methods are involved: the strictly parabola method, which will be indicated throughout the discussion as (parabola), and the new method, which will be known as the "mean length method" and will be indicated as (mean length method).

In the development of a new method for performing the operations in connection with a problem, regardless of its nature, it is essential that no information necessary for a full and complete understanding of the subject be omitted. For this reason, an attempt has been made to give complete information. All data that are pertinent to the subject are included in the details, and more work is therefore involved than would actually be required in practice.

The profile over which a given line is to be constructed is often of such nature that adjacent spans in certain sections of the line must vary greatly in length. This may, and often does, make it necessary to dead-end some of the longer spans in the line. To make sure that the wire or cable will not be overstressed or that adequate ground clearance at maximum sag will be maintained, it is often necessary to calculate the sag and tension in the long dead-ended spans separately from the calculations made for the ruling span.

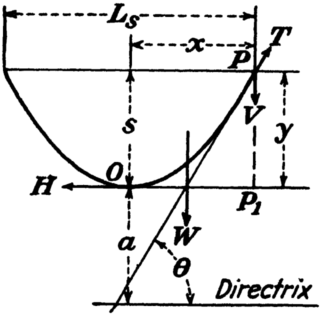

Fig. 41.—Parabola.

Considerable work is required for a final analysis in any sag problem. The two theories involved give approximately equal results for spans up to about 1,000 ft in length. One theory (parabola) is based on the assumption that a wire or cable suspended between two supports at equal elevations will conform to the curve of the parabola. The other theory (catenary) is based on the assumption that a wire or cable suspended between two supports at equal elevations will conform to the curve of the catenary.

To emphasize the need for a simple and direct method of computing sag in long spans which will give results on a par with the sag derived from the catenary solution, a brief discussion of the properties of the two curves should be of interest.

It will be seen from Fig. 41 that the lowest point of the cable at O is the origin of coordinates x and y. There are three forces acting on the cable: the horizontal tension H at O, the tension T along the tangent at P, and the weight of the cable W between O and P. The weight of the cable is assumed to be uniformly distributed along its horizontal projection OP_1. The weight of cable in the half span $= W = wx$ and bisects OP_1. w represents the unit weight of the cable. Since the origin of coordinates x and y is at O, $y = S$.

It is obvious that, for the above assumptions to be warranted, the sag in proportion to the span length must be relatively small for a span of an appreciable length.

$$a = \frac{H}{2w} \qquad y = \frac{x^2}{4a} \qquad \tan\theta = \frac{2y}{x}$$

Resolving tangentially $T = \frac{H}{\cos\theta}$

Resolving vertically $V = H \times \tan\theta$

Refer to the catenary (Fig. 42). If a heavy string that is perfectly flexible is suspended between two supports at equal elevations and in equilibrium in a vertical plane, the curve made by the string will conform to the curve of the catenary. The weight of the wire or cable is assumed to be uniformly distributed along the arc *DP*. In this case, we have the horizontal tension acting on the cable at *D*, the tangential tension *T* at *P*, and the weight of the cable *wL* acting at the center of gravity *G* of the arc *DP*.

$$a = \frac{H}{w} \qquad y = a\cosh\frac{x}{a} \qquad \tan\theta = \frac{L}{a}$$

Resolving horizontally $H = T\cos\theta$
Resolving vertically $V = T\sin\theta$

The criterion for conforming to the curve of the catenary is perfect flexibility. It is therefore apparent that, since a metallic conductor, with either solid or standard stranding, is far from perfectly flexible, it does not altogether meet the requirements. For long spans with deep sags, however, a metallic conductor conforms quite closely to the curve of the catenary. The degree of conformity is of course a matter for conjecture.

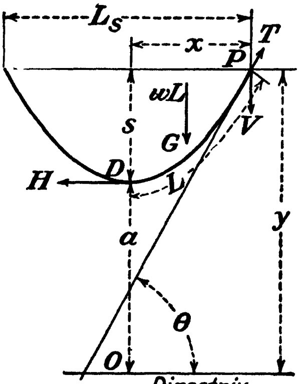

Fig. 42.—Catenary.

When the horizontal tension is the same, the radius of curvature at the lowest point of the cable is the same for both the parabola and the catenary, but the outlines of the two curves are different at all points between the lowest point of the curve and the point of support. As the span length and sag are increased, the difference in outline of the two curves becomes more pronounced. Changes in the loading will produce different changes in the length of the two curves to make the sags different.

Since the sag by the catenary solution is greater than the sag by the parabola solution for the same horizontal tension, the angle θ will be greater; and the vertical component of tension will therefore be greater for the catenary than for the parabola. This difference will increase along the curve toward the support, becoming maximum at the support.

The calculations are comparatively simple, regardless of the method employed, as long as the conductor loading and temperature remain unchanged. When changes in loading and temperature occur, however, the parabola solution simplifies the calculations. As far as the writer knows, no method has previously been developed by which direct application can be made to compensate for the effect of temperature changes on the sag and tension when the catenary solution is applied. When the parabola solution is employed, no specially prepared charts or tables of hyperbolic functions are required. The mean length method, as previously mentioned, retains the simplicity of the parabola solution but gives results to a degree of accuracy comparable with those derived by the catenary solution.

The problem is best exemplified by the actual solution of a given example. In order to demonstrate the ease and simplicity with which the work may be carried out, the following conditions are assumed to apply. The span chosen for this example is not an exceedingly long span but will be found to comply quite closely with a condition that is not at all out of the ordinary, especially where mountainous profiles are encountered. The work is carried out in considerable detail.

Conditions

2,000-ft level span
3/0 twelve strand copper cable, hard drawn
w_r = resultant weight of cable + ½ in. radial thickness of ice + 8 lb per sq ft wind pressure = 1.5091 lb per ft
w_v = weight of cable +½-in. ice = 1.135 lb per ft
w_{vo} = weight of the bare cable = 0.5181 lb per ft
A = cross-sectional area of cable = 0.1318 sq in.
M = modulus of elasticity = 17,000,000 lb per sq in.

$MA = 17{,}000{,}000 \times 0.1318 = 2{,}240{,}600$

Coefficient of linear expansion = 0.0000094 ft per deg F

H = horizontal component of tension = 3,778 lb at maximum design load = 50 percent of the breaking strength

T = tension at the support, lb

V = vertical component of tension at the support, lb

L_s = actual span length for level span, ft

S = sag at center of span, ft (parabola)

L = length of cable in span, ft (parabola)

L_m = mean length of cable and span, ft = $\frac{L + L_s}{2}$

S_c = sag at center of span, ft (mean length method)

L_c = length of cable in span, ft (mean length method)

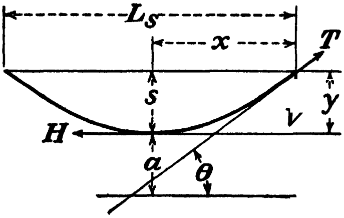

Fig. 43.—Parabola.

Copper cable has been selected for the 2,000-ft span, since its excessive sag better serves to indicate any inequalities in the results of the mean length and the catenary solutions.

$$y = \frac{x^2}{4a} \tag{39}$$

$$a = \frac{H}{2w} = \text{length of cable whose weight is equal to one-half the horizontal tension} \tag{40}$$

$$S = \frac{wL_s^2}{8H} = \text{sag at center of level span} \tag{41}$$

$$L = L_s + \frac{8S^2}{3L_s} = \text{length of cable in span} \tag{42}$$

$$\tan \theta = \frac{2y}{x} \tag{43}$$

$$T = \frac{H}{\cos \theta} = \text{tension in cable at the support} \tag{44}$$

$$V = H \tan \theta = \text{vertical component of tension at support} \tag{45}$$

MODIFIED EQUATIONS OF THE PARABOLA

$$S_c = \frac{w \times L_s \times L_m}{8 \times H} = \text{sag at center of span (mean length method)} \tag{46}$$

$$L_c = L_s + \frac{8S_c^2}{3L_s} = \text{length of cable in span (mean length method)} \qquad (47)$$

Equations (42) and (47) for the 2,000-ft span may be reduced to the following form:

$$L = L_s + S^2 \times 0.001333 \qquad L_c = L_s + S_c^2 \times 0.001333$$

In problems such as this, in which a considerable amount of calculation is required, it is advisable to take advantage of all the short cuts that can logically be resorted to. In most cases, certain constants may be set up that will have the effect not only of shortening the work but of reducing the liability of error in the final results to a minimum. For example:

The following values constitute the numerator of Eq. (41)

$$w_r \times L_s^2 = 1.5091 \times 2{,}000^2 = 6{,}036{,}400$$
$$w_v \times L_s^2 = 1.1350 \times 2{,}000^2 = 4{,}540{,}000$$
$$w_{vo} \times L_s^2 = 0.5181 \times 2{,}000^2 = 2{,}072{,}400$$

The following values are for substitution in the numerator of Eq. (46)

$$w_r \times L_s = 1.5091 \times 2{,}000 = 3{,}018.2$$
$$w_v \times L_s = 1.1350 \times 2{,}000 = 2{,}270.0$$
$$w_{vo} \times L_s = 0.5181 \times 2{,}000 = 1{,}036.2$$

Solution a

From (39) and (40) $x = 1{,}000$ ft

$$a = \frac{3{,}778}{2 \times 1.5091} = 1{,}251.74$$

$$y = \frac{10^6}{4 \times 1{,}251.74} = 199.72 \text{ ft}$$

From (41) $S = \dfrac{1.5091 \times 2{,}000^2}{8 \times 3{,}778} = 199.72$ ft

From (42) $L = 2{,}000 + \dfrac{8 \times 199.72^2}{3 \times 2{,}000} = 2{,}053.1852$ ft

From (43) $\tan\theta = \dfrac{2 \times 199.72}{1{,}000} = 0.399444$

$$\theta = 21°46' \qquad \cos\theta = 0.92866$$

From (44) $T = \dfrac{3{,}778}{0.92866} = 4{,}068.22$ lb

From (45) $V = 3{,}778 \times 0.399444 = 1{,}509.1$ lb

$$L_m = \frac{2{,}000 + 2{,}053.1852}{2} = 2{,}026.5926 \text{ ft}$$

From (46) $$S_c = \frac{1.5091 \times 2{,}000 \times 2{,}026.5926}{8 \times 3{,}778} = 202.38 \text{ ft}$$

From (47) $$L_c = 2{,}000 + \frac{8 \times 202.38^2}{3 \times 2{,}000} = 2{,}054.6089 \text{ ft}$$

To complete the problem, two distinct sets of curves are required, namely, a set of sag-tension curves and a set of stretch curves. Sag-tension curves should be plotted for the cable loaded with ice and wind, for the cable loaded with ice only, and for the cable with no ice or wind. The stretch curves, which give sag in terms of cable length, are plotted from values that take into account the elongation of the cable due to the application of tension as well as the change in length due to variations in temperature.

Assume tensions both above and below the assumed maximum tension of 3,778 lb, say, 3,900, 3,800, 3,700, and 3,600 lb. Compute the sag for each given tension. Use a numerator of 6,036,400, and multiply the tension by 8. Set the work down as follows:

H Parabola

3,900 $S = 6{,}036{,}400/31{,}200 = 193.47$ ft $L = 2{,}049.9098$ ft

By inspection $L_m = 2{,}024.9549$ ft

3,800 $S = 6{,}036{,}400/30{,}400 = 198.57$ ft $L = 2{,}052.5712$ ft

By inspection $L_m = 2{,}026.2856$ ft

3,700 $S = 6{,}036{,}400/29{,}600 = 203.93$ ft $L = 2{,}055.4512$ ft

By inspection $L_m = 2{,}027.7256$ ft

3,600 $S = 6{,}036{,}400/28{,}800 = 209.60$ ft $L = 2{,}058.5747$ ft

By inspection $L_m = 2{,}029.2874$ ft

H Mean Length Method

3,900 $S_c = (3{,}018.2 \times 2{,}024.9549)/31{,}200 = 195.89$ ft

3,800 $S_c = (3{,}018.2 \times 2{,}026.2856)/30{,}400 = 201.18$ ft

3,700 $S_c = (3{,}018.2 \times 2{,}027.7256)/29{,}600 = 206.76$ ft

3,600 $S_c = (3{,}018.2 \times 2{,}029.2874)/28{,}800 = 212.67$ ft

Assume now that the wind load has been removed from the cable but that the temperature has not risen sufficiently for the ice to fall off. As the wind load is removed, the tension in the cable will be reduced. Assume tensions of 3,000, 2,900, 2,800, and 2,700 lb. Use a numerator constant of 4,540,000, and compute the sag S, length L, and mean length L_m.

Calculate S_c, sag by the mean length method, in the same manner as above, but use a numerator of 2,270 × L_m, instead of 3,018.2 × L_m.

Finally, assume both the ice and wind loads to be removed from the cable. We now have the unloaded cable, for which the tension over the temperature range should lie between 1,500 and 1,000 lb.

TABLE 35.—VALUES FOR SAG-TENSON CURVES AT VARIOUS CABLE LOADINGS

Horizontal ten., lb	Parabola		Mean length method	
	Sag S, ft	Length L, ft	Length L_m, ft	Sag S_c, ft
	Cable loaded with ice and wind			
3,900	193.47	2,049.9098	2,024.9549	195.89
3,800	198.57	2,052.5712	2,026.2856	201.18
3,700	203.93	2,055.4512	2,027.7256	206.76
3,600	209.60	2,058.5747	2,029.2874	212.67
	Cable loaded with ice only			
3,000	189.17	2,047.7120	2,023.8560	191.42
2,900	195.69	2,051.0593	2,025.5297	198.19
2,800	202.68	2,054.7715	2,027.3858	205.45
2,700	210.19	2,058.9037	2,029.4519	213.28
	Unloaded cable			
1,400	185.04	2,045.6510	2,022.8255	187.15
1,350	191.89	2,049.0951	2,024.5476	194.24
1,300	199.27	2,052.9443	2,026.4722	201.91
1,250	207.24	2,057.2645	2,028.6322	210.21

Assume tensions of 1,400, 1,350, 1,300, and 1,250 lb. Use a numerator constant of 2,072,400, and compute the sag for each given tension. Compute the value of L and L_m, as before. From a numerator value of 1,036.2, calculate S_c for each given tension. Compile the values in tabular form for convenience in plotting.

Solution b. The next step in the solution is to compute the unstressed length of the cable at the various temperatures. When the line is designed for heavy loading, as in this case, it is customary to assume a conductor temperature range of −20 to 120°F. The calculations are usually made at 20 or 30° intervals. It is required to know the unstressed length of the cable at each given temperature. This assumes that the cable could be placed on its supports and have the force of gravity applied at will. This is of course only a theoretical consideration, but it is nevertheless an essential consideration in connection with sag problems. The unstressed length of the cable L_u may be derived from the following equation:

$$L_u = \frac{L}{1 + (H/MA)} \tag{48}$$

Since the maximum design load of 3,778 lb horizontal tension is taken at 0°F, the unstressed length of cable in the 2,000-ft span

$$L_u = \frac{2{,}053.1852}{1 + (3{,}778/2{,}240{,}600)} = \frac{2.053.1852}{1.00168615} = 2{,}049.729 \text{ ft}$$

The unstressed cable will change 0.0000094 ft per ft of length for each change of 1°F temperature. Accordingly, the following tabulations give the unstressed length of cable in the 2,000-ft span at each given value of temperature.

The change in length for an increasing temperature for hard-drawn copper $= L_u(1 + 0.0000094 \times t)$ and for a decreasing temperature $= L_u(1 - 0.0000094 \times t)$. t represents the increase or decrease in temperature, degrees Fahrenheit.

$$\begin{aligned}
-20°L_u &= 2{,}049.729(1 - 0.0000094 \times 20) = 2{,}049.3437 \text{ ft}\\
0°L_u &= 2{,}049.729(1 + 0.0000094 \times 0) = 2{,}049.7290 \text{ ft}\\
32°L_u &= 2{,}049.729(1 + 0.0000094 \times 32) = 2{,}050.3455 \text{ ft}\\
60°L_u &= 2{,}049.729(1 + 0.0000094 \times 60) = 2{,}050.8850 \text{ ft}\\
90°L_u &= 2{,}049.729(1 + 0.0000094 \times 90) = 2{,}051.4631 \text{ ft}\\
120°L_u &= 2{,}049.729(1 + 0.0000094 \times 120) = 2{,}052.0411 \text{ ft}
\end{aligned}$$

Solution c. When tension is applied to an unstressed cable, since the material of which the cable is composed will have a certain amount of elasticity, it will elongate, or stretch, an amount that will be directly proportional to the applied tension, within the elastic limit of the cable. The elongation may be determined as follows:

$$e = \frac{H \times L_u}{MA} \tag{49}$$

$$\text{Elongation for 1,000 lb} = \frac{1{,}000 \times 2{,}049.729}{2{,}240{,}600} = 0.91481 \text{ ft}$$

Adding the stretch for each 1,000 lb, or fractional part thereof, to the unstressed cable at 0°F, the length L at any given load is obtained.

$$0°L_u = 2{,}049.729 \text{ ft}$$

H	
1,000	$L = 2{,}049.7290 + 0.9148 = 2{,}050.6438$ ft
2,000	$L = 2{,}049.7290 + 1.8296 = 2{,}051.5586$ ft
3,000	$L = 2{,}049.7290 + 2.7444 = 2{,}052.4734$ ft
3,778	$L = 2{,}049.7290 + 3.4562 = 2{,}053.1852$ ft
4,000	$L = 2{,}049.7290 + 3.6593 = 2{,}053.3883$ ft

$$S = 0.5\sqrt{\frac{3L_s(L - L_s)}{2}} = \text{sag in terms of length (parabola)} \tag{50}$$

$$S_c = 0.5\sqrt{\frac{3L_s(L_c - L_s)}{2}} = \text{sag in terms of length (mean length method)} \tag{51}$$

$3L_s/2 = 3{,}000$, from which the above two equations may be reduced to the following form for the 2,000-ft span:

$$S = 0.5\sqrt{3{,}000(L - L_s)} \qquad S_c = 0.5\sqrt{3{,}000(L_c - L_s)}$$

To illustrate: At maximum load,

$$L - L_s = 2{,}053.1852 - 2{,}000 = 53.1852 \text{ ft.}$$

$$S = 0.5\sqrt{3{,}000 \times 53.1852} = 199.72 \text{ ft.}$$

Calculating the points for the stretch curve at 0°F, we have

H	Parabola
1,000	$S = 0.5\sqrt{3{,}000 \times 50.6438} = 194.89$ ft
2,000	$S = 0.5\sqrt{3{,}000 \times 51.5586} = 196.64$ ft
3,000	$S = 0.5\sqrt{3{,}000 \times 52.4734} = 198.38$ ft
3,778	$S = 0.5\sqrt{4{,}000 \times 53.1852} \times 199.72$ ft
4,000	$S = 0.5\sqrt{3{,}000 \times 53.3883} = 200.10$

At maximum load of 3,778 lb, L_c was previously computed to be

2,054.6089 ft and the sag S_c, in terms of weight and tension, was 202.38 ft. The sag in terms of length

$$S_c = 0.5\sqrt{3{,}000 \times 54.6089} = 202.38 \text{ ft}$$

The catenary gives a sag of 202.4 ft, or a difference of only 0.02 ft.

The difference in L and L_u at maximum load is equal to $2{,}053.1852 - 2{,}049.7290 = 3.4562$ ft. Accordingly, the length at zero load

(mean length method) $= 2{,}054.6089 - 3.4562 = 2{,}051.1527$ ft.

H	0°F	Mean Length Method
0	$L_c = 2{,}051.1527$	
1,000	$L_c = 2{,}051.1527 + 0.9148 = 2{,}052.0675$ ft	$S_c = 197.61$ ft
2,000	$L_c = 2{,}051.1527 + 1.8296 = 2{,}052.9823$ ft	$S_c = 199.34$ ft
3,000	$L_c = 2{,}051.1527 + 2.7444 = 2{,}053.8971$ ft	$S_c = 201.05$ ft
3,778	$L_c = 2{,}051.1527 + 3.4562 = 2{,}054.6089$ ft	$S_c = 202.38$ ft
4,000	$L_c = 2{,}051.1527 + 3.6593 = 2{,}054.8120$ ft	$S_c = 202.75$ ft

The length L_c at 3,778 lb is computed to be 2,054.6089 ft. Then, since the elongation for 1,000-lb tension is 0.9148 ft, $2{,}054.6089 - (3.778 \times 0.9148) = 2{,}051.1527$ ft at zero load. Adding the stretch for each 1,000 lb to the length at zero load, the above tabulations are obtained.

The sag-tension curves plotted from values derived under solution *a* and the stretch curves derived from calculations under solution *c* are shown in Fig. 44. Sag-tension curves from the parabola solution are indicated by the dash lines 1, 2, and 3. The stretch curve from the parabola solution is indicated by 7. Sag-tension curves 4, 5, and 6 and stretch curve 8 are from the mean length method. Stretch curve 7 (parabola) is seen to intersect curve 1 at a tension of 3,778 lb and a sag of 199.72 ft. This serves as a definite check on the work and provides conclusive evidence not only that the work is correct, within the accuracy of construction of the curves, but also that the method is rational.

Stretch curve 8, plotted from the values above, intersects curve 4 at a tension of 3,778 lb and a sag of 202.38 ft. The tension at the support $T = 3{,}778 + 202.38 \times 1.5091 = 4{,}083.41$ lb. This

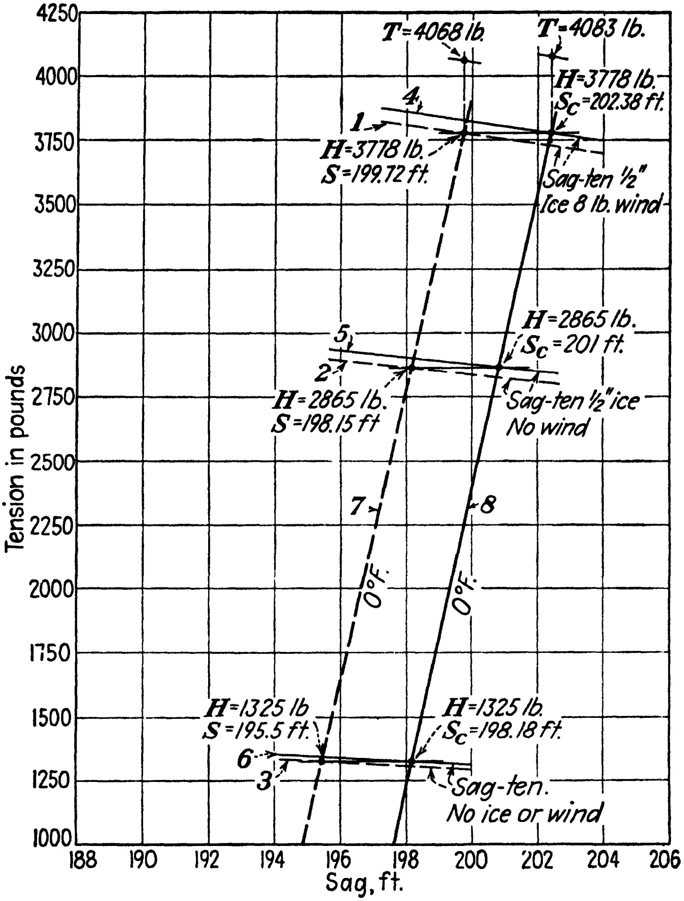

FIG. 44.—Sag-tension curves illustrating comparative values derived from the parabola and mean length methods. A 2,000-ft-level span. Sag-tension and stretch curves for 3/0, twelve-strand copper wire. Curves 1, 2, and 3, sag tension (parabola); 7, stretch curve 0°F (parabola); 4, 5, and 6, sag-tension curves, and 8, stretch curve—mean length method.

gives the same value for T as that derived from the catenary solution, but $4{,}083.41 - 4{,}068.22 = 15.19$ lb more than that given by the parabola method.

To facilitate reading sag and tension at the curve intersections Fig. 44 is constructed to a scale that will permit reading to a high degree of accuracy. Stretch curve 8 intersects sag-tension curves 4, 5, and 6 at the same tension as that at which stretch curve 7 intersects sag-tension curves 1, 2, and 3. This is clearly indicated by the horizontal projections from the dash-line intersections. In practice, curve 8 should be plotted since it facilitates the reading of the sag where the horizontal projections intersect the full-line sag-tension curves. This sag is comparable with that derived from the catenary solution.

Solution d. The values from which the two stretch curves 7 and 8 in Fig. 44 were plotted are based on the cable loading at a conductor temperature of 0°F. Assume, for example, that the temperature has risen from 0 to 32°. The 32° curve is for the purpose of determining whether the sag at 32° for the ice-covered cable is more than the 120° sag for the bare cable. The 0° stretch curve 8 intersects sag-tension curves as follows: curve 6 at a tension of 1,325 lb and a sag of 198.18 ft; curve 5 at a tension of 2,865 lb and a sag of 200.8 ft; and curve 4 at a tension of 3,778 lb and a sag of 202.38 ft.

$$32°L_u = 2{,}050.3456 \text{ ft}$$

H Parabola

1,000 $L = 2{,}050.3456 + 0.9148 = 2{,}051.2604$ ft
$S = 196.07$ ft

1,250 $L = 2{,}050.3456 + 1.1435 = 2{,}051.4891$ ft
$S = 196.51$ ft

1,400 $L = 2{,}050.3456 + 1.2807 = 2{,}051.6263$ ft
$S = 196.77$ ft

1,500 $L = 2{,}050.3456 + 1.3722 = 2{,}051.7178$ ft
$S = 196.95$ ft

2,000 $L = 2{,}050.3456 + 1.8296 = 2{,}052.1752$ ft
$S = 197.82$ ft

3,000 $L = 2{,}050.3456 + 2.7444 = 2{,}053.0900$ ft
$S = 199.54$ ft

The 32° stretch curve intersects sag-tension curve 2, Chart 1, at a tension of 2,850 lb and a sag of 199.12 ft, from which the following values are computed:

$$L = 2{,}000 + 199.12^2 \times 0.001333 = 2{,}052.8660 \text{ ft}$$
$$L_m = (2{,}000 + 2{,}052.8660)0.5 = 2{,}026.4330 \text{ ft}$$

$$S_e = \frac{2{,}270 \times 2{,}026.4330}{8 \times 2{,}850} = 201.75 \text{ ft} \qquad L_c = 2{,}054.2732 \text{ ft}$$

The length at

zero load = 2,054.2732 − (0.9148 × 2.85) = 2,051.6660 ft,

from which

H	Mean Length Method	
0	L_c = 2,051.6660 ft	
1,250	L_c = 2,052.8095 ft	S_c = 199.02 ft
1,400	L_c = 2,052.9467 ft	S_c = 199.27 ft
1,500	L_c = 2,053.0382 ft	S_c = 199.45 ft
2,000	L_c = 2,053.4956 ft	S_c = 200.30 ft
2,500	L_c = 2,053.9530 ft	S_c = 201.16 ft
2,850	L_c = 2,054.2732 ft	S_c = 201.75 ft
3,000	L_c = 2,054.4104 ft	S_c = 202.00 ft

The sag and length at −20, 60, 90, and 120 degrees may be obtained for both the parabola and mean length methods by referring to the unstressed length values, previously given, for each temperature, using the unstressed length at each desired temperature as a starting point and performing the operations as demonstrated for the 0 and 32° values. For convenience in plotting, compile the values at all temperatures as in Table 36.

Refer to Chart 1, in which the sag-tension curves from values in Table 34 and stretch curves from values in Table 36 have been plotted.

a and *b* in Chart 1 give results, in graphical form, for the complete solution. *a* indicates values computed from equations of the parabola. *b* indicates values computed from the modified equations (mean length method). The sags indicated in *b* are comparable with those derived from the catenary solution for the same span and cable loading. To avoid possible confusion, *a* and *b* are drawn separately. However, the points of intersection in *b* will project horizontally from corresponding points of tension in *a*.

c is an enlarged view of the lower parts of *a* and *b* and indicates that the accuracy to which the values can be read depends only upon the scale to which the chart is constructed.

The sag-tension and stretch curves are of no particular value in themselves but must be used in conjunction with each other. The only rational values are at the points of intersection, which is the point at which the computation for sag in terms of weight and tension gives the same value as does the computation for

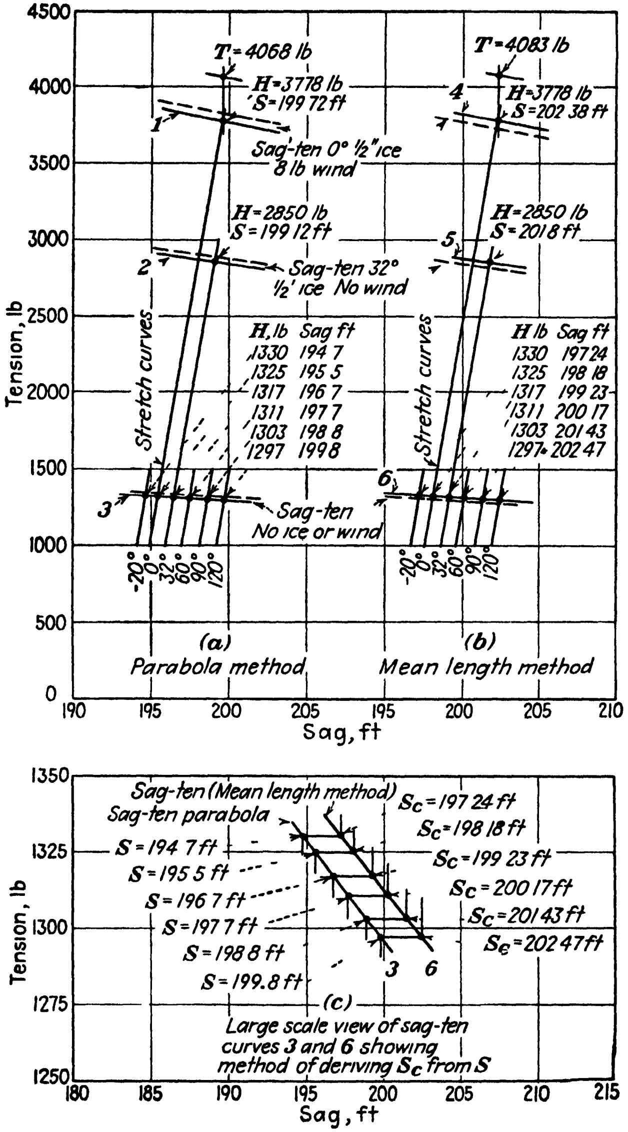

CHART 1 —Sag-tension chart for 3/0, twelve-strand hard-drawn copper, 2,000-ft-level span (*a*) Sag-tension and stretch curves from equations of the parabola (*b*) Sag-tension and stretch curves from the modified equations—mean length method (*c*) Large-scale view of sag-tension curves 3 and 6, (*a*) and (*b*), showing that horizontal projections from a given sag and tension on curve 3 intersects curve 6 at the sag derived from the mean length method.
Horizontal scale the same for (*a*), (*b*), and (*c*).

TABLE 36.—COMPARISON OF SAG AND LENGTH, PARABOLA AND MEAN LENGTH METHOD, AS AFFECTED BY TEMPERATURE

Parabola				Mean length method	
H	Temp., deg F	Length $L_u + e$ ft	Sag in terms of L and L_s	Length $L_{ue} + e$ ft	Sag in terms of L_c and L_s
1,000	−20	2,050 2586	194.15	2,051.5677	196.66
1,250	−20	2,050 4872	194.59	2,051.7964	197.10
1,500	−20	2,050 7159	195.03	2,052.0251	197.53
1,000	0	2,050.6438	194.89	2,052.0675	197.61
2,000	0	2,051.5586	196.64	2,052.9823	199.34
3,000	0	2,052.4734	198.38	2,053.8971	201.05
4,000	0	2,053 3883	200.10	2,054.8120	202 75
1,000	32	2,051.2604	196.07	2,052.5808	198.58
2,000	32	2,052.1752	197.82	2,053.4956	200.30
3,000	32	2,053.0900	199.54	2,054.4104	202.00
1,000	60	2,051.7998	197.10	2,053.1397	199.64
1,250	60	2,052 0285	197.54	2,053.3684	200.07
1,500	60	2,052.2572	197.97	2,053.5971	200.49
1,000	90	2,052.3779	198.20	2,053.8213	200.91
1,250	90	2,052.6066	198.63	2,054.0500	201.34
1,500	90	2,052.8353	199.06	2,054.2787	201.75
1,000	120	2,052.9559	199.29	2,054.3855	201.96
1,250	120	2,053.1846	199.72	2,054.6142	202.39
1,500	120	2,053.4133	200.15	2,054.8429	202.81

TABLE 37.—COMPARATIVE VALUES OF SAG AND TENSION AT VARIOUS TEMPERATURES

Temp., deg F	Parabola		Mean length method		Catenary	
	Sag, ft	Ten., lb	Sag, ft	Ten., lb	Sag, ft	Ten., lb
−20	194.7	1,330	197.24	1,330	197.2	1,331
0	195.5	1,325	198.18	1,325	198.2	1,325
32	196.7	1,317	199.23	1,317	199.1	1,319
60	197.7	1,311	200.17	1,311	200.1	1,311
90	198.8	1,303	201.43	1,303	201.4	1,304
120	199.8	1,297	202.47	1,297	202.5	1,297

sag in terms of length. A knowledge of this theory is essential to a clear understanding of sag problems.

Table 37 gives the comparison of values for sag derived from the parabola, from the mean length method, and from the catenary solutions, at various temperatures.

The tabulated values for the parabola and mean length methods were read from *c* in Chart 1. The values for the catenary were derived by the Thomas method, using a chart large enough for accurate reading. Previous calculations have demonstrated that the mean length method gives results that compare with results by the catenary solution at maximum load. It can be seen from Table 37 that the values are comparable with those derived from the catenary solution over the entire temperature range.

TABLE 38.—COMPARATIVE VALUES OF SAG AND CABLE LENGTH

Span, ft	Mean length method			Catenary		
	Sag, ft	Percent of span	Length, ft	Sag, ft	Percent of span	Length, ft
1,000	50.10	5.01	1,003.32	50.10	5.01	1,006.67
1,250	78.42	6.27	1,260.99	78.42	6.27	1,263.03
1,500	113.18	7.55	1,522.77	113.19	7.55	1,522.54
1,750	154.47	8.83	1,786.36	154.48	8.83	1,785.85
2,000	202.38	10.12	2,054.61	202.40	10.12	2,053.61
2,250	257.03	11.42	2,328.28	257.16	11.42	2,326.49
2,500	318.54	12.74	2,608.23	318.61	12.74	2,605.19
2,750	387.09	14.08	2,895.30	387.20	14.08	2,890.37
3,000	462.82	15.43	3,190.40	462.99	15.43	3,182.76
3,250	545.91	16.80	3,501.85	546.23	16.81	3,483.09
3,500	636.56	18.19	3,808.73	636.98	18.20	3,792.10

The mean length method would be of little value if it were not universally applicable as far as span lengths are concerned. Accordingly, Table 38 gives the comparative values of the mean length and catenary solutions for spans from 1,000 to 3,500 ft, inclusive. The values give sag in feet and percentage of span length and cable length in percentage of span length for the

3/0 twelve-strand copper at a horizontal tension of 3,778 lb. The important factor is the sag in percentage of span length. The sag will rarely exceed about 15 percent of the span; therefore, no information of value would be obtained by extending the table to include longer spans.

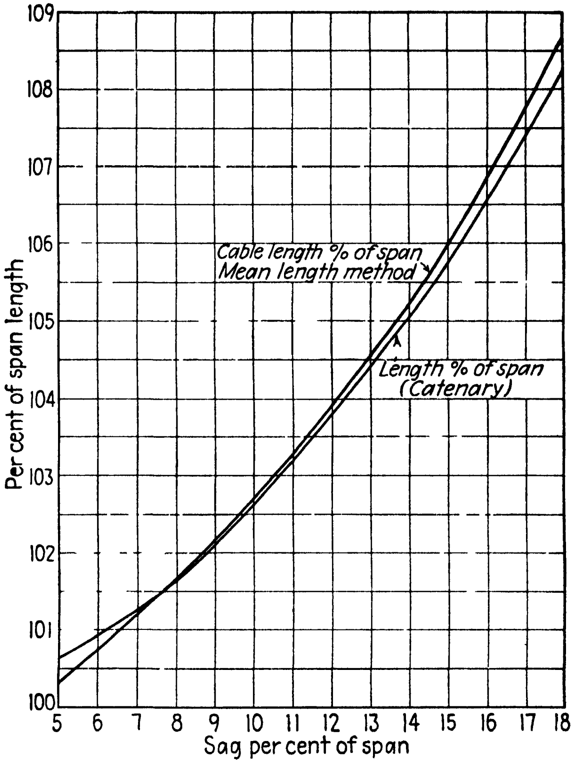

FIG. 45.—Comparison of conductor length in percent of span mean length and catenary plotted against percent sag.

The theory behind the mean length method lies in Table 38, better visualized in Fig. 45. It will be seen from the table that the sag by the mean length method follows an exponential law that is almost exactly equivalent to the hyperbolic theory of the catenary solution. For example, the sag by the catenary solution $S = (y - a)$. Or $S = a(\cosh x/a - 1)$. $\cosh x/a = \frac{1}{2}(e^{x/a} + e^{-x/a})$, where e is the base of the natural logarithm system.

$$S = \frac{w \times L_s^2}{8H} \qquad (41)^*$$

$$S_c = \frac{w \times L_s \times L_m}{8 \times H} \qquad (46)^*$$

Equations (41) and (46), which are now quite familiar, are both of the same form, the only difference being that the product of $L_s \times L_m$, in (46) gives a somewhat larger value than L_s^2 in (41). As the span length L_s is made longer, $L_s \times L_m$ becomes increasingly larger than L_s^2. From this, it is obvious that S_c will also become increasingly larger than S.

$$L = L_s + \frac{8S^2}{3L_s} \qquad (42)^*$$

$$L_c = L_s + \frac{8S_c^2}{3L_s} \qquad (47)^*$$

Since S_c, as explained above, becomes increasingly larger than S as the span L_s is made longer, the length L_c will increase more rapidly than the length L (see Fig. 45).

The length of cable in the span derived from the catenary solution = $2a(\sinh x/a)$. For the same span length, this equation will give a cable length in excess of the length given by (47) (Mean length method) for sags up to approximately 7.5 percent of the span. For sags in excess of 7.5 percent of the span length, (47) will give a cable length in excess of that given by the catenary solution. This is clearly indicated by the two length curves in Fig. 45. The length curves are given in percentage of span length and are plotted against the percentage of sag.

Since sag is a function of cable length, L_c from (47), which is a modified equation of the parabola, will increase at such a rate that the sag from the mean length method may be said to equal the sag from the catenary solution. The difference in sag derived from the two methods is less than 0.1 of 1 percent for a sag equivalent to 20 percent of the span length.

Transmission engineers and manufacturers are well aware that commercial cable cannot be manufactured with properties exacting enough so that such comparisons may be expected to obtain in practice. For example, no two pieces of cable manufactured at different times, and, for that matter, no two wires in the same cable will have exactly the same properties. For a sag in the order of 20 percent of the span length, the catenary solution will give a sag approximately 0.1 of 1 percent more than that given by the mean length method. In practice, the sag may be less for the cable for which the catenary solution is used than for the cable for which the mean length method was used. It cannot then be said that one method will give results that are more accurate than the other.

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* Equations repeated from pp. 126 and 127.

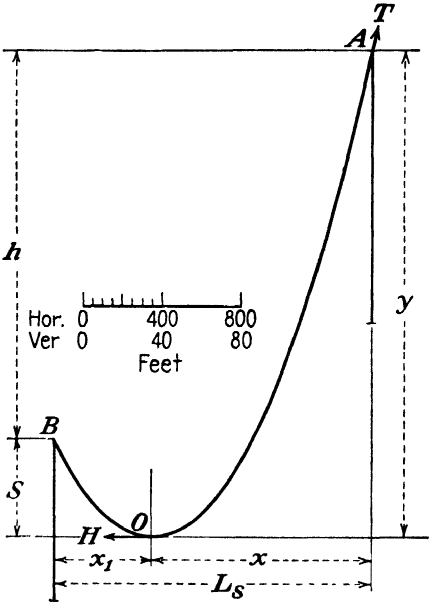


FIG. 46.—Unsymmetrical span.

Unsymmetrical Spans.—It is generally believed that unsymmetrical spans, or spans with supports at unequal elevations, present an elusive problem. An unsymmetrical span may be considered as consisting of two half spans of unequal length, the relative lengths of which are the same for only one condition of conductor loading. When either the temperature or the conductor loading changes, the low point of the cable shifts, thus altering the relative length of the two half spans. For this reason, unsymmetrical spans cannot be treated in the same manner as symmetrical, or level, spans. Insofar as sag alone is concerned, either half span gives the same results as would a level span twice its length.

This should not cause a great deal of concern, since generally the sag and tension will be taken care of automatically. It is not necessary to analyze or calculate the sag in an unsymmetrical span unless the conductors are to be dead-ended at both ends of the span. If the horizontal distance between supports and their difference in elevation are such as to require a horizontal tension different from that applying to the line in general, it may be necessary to calculate the sag as well as to determine other properties of the span.

Except for very special conditions, the sag template used for spotting structures on the main part of the profile is used for both level and hillside spans. The template, in addition to the curve used for determining the position of the conductor at maximum sag, should contain a cold curve, which is used to indicate the position of the conductor at minimum sag.

The distance from the lower support to the lowest point of the cable will be maximum at maximum sag and minimum at minimum sag. Let us assume, for example, that Fig. 46 represents an unsymmetrical span in which the 3/0 copper cable we have discussed is to be installed. The two half spans of unequal length are represented by x and x_1. The horizontal distance between supports is represented by L_s, and the difference in elevation of supports A and B is represented by h. The sag below the upper support is represented by y from which

$$S = y - h.$$

Probably the most satisfactory method of sagging conductors in spans with supports at unequal elevations is to use a tension dynamometer, working from the lower end of the span. When it is not convenient to use a dynamometer, the next most satisfactory method is as indicated by Fig. 47. Targets are installed on the structures at equal distances from both A and B, such that the cable when sagged to a distance S'_c will be tangent to the line of sight DC. It is not practical to attempt to measure the sag below either support when the low point of the cable falls below the base of the upper support. When the half span x exceeds about 600 ft in length, were copper conductor is used, the mean length method should be applied to compensate for the error in y derived from the parabola solution.

For all practical purposes, the apparent sag S'_c in Fig. 47 will be the same as the sag S' at the center of a level span equivalent in length to L_s, the horizontal distance between the two supports A and B. The problem then is to set up values for S'_c, the vertical distance from the slope line AB to the cable at the center of the span, projected horizontally, such that a true value of S_{cx} will be obtained.

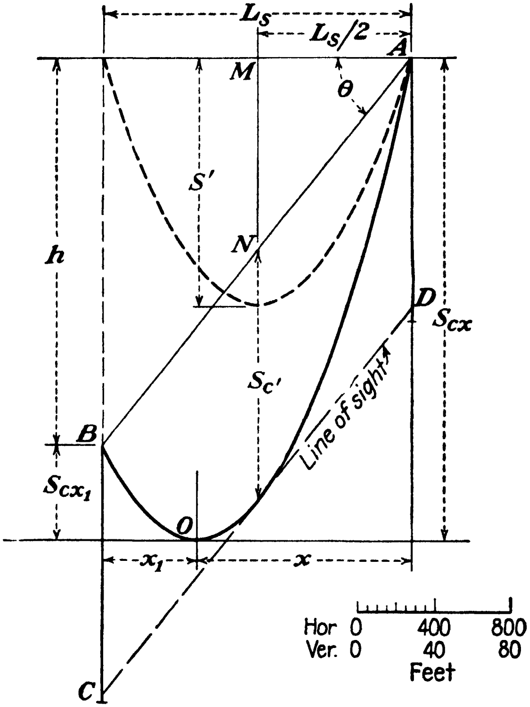


FIG. 47.—Unsymmetrical span indicating method of sagging conductor.

Assume the curve AOB to represent the cable at a temperature of 120° and that H = 1,297 lb (see Chart 1). w_{vo} = 0.5181 lb per ft.

$$x_1 = \frac{L_s^2 - 4ah}{2 \times l_s} \tag{52}$$

When $4ah = L_s^2$, the low point of the cable will coincide with support B. When $4ah$ is greater than L_s^2, the low point of the cable will fall outside the lower support, and there will be uplift at B. This condition rarely occurs for long spans but is quite common with short and medium-length spans.

Values at 120°F.

$$\text{Let } L_s = 1{,}600 \text{ ft} \quad a = \frac{1{,}297}{2 \times 0.5181} = 1{,}251.69$$

$$h = 200 \text{ ft}$$

$$x_1 = \frac{1{,}600^2 - (4 \times 1{,}251.69 \times 200)}{2 \times 1{,}600} = 487 \text{ ft}$$

$$x = 1{,}600 - 487 = 1{,}113 \text{ ft}$$

$$y = \frac{0.5181 \times 1{,}113^2}{2 \times 1{,}297} = 247.42 \text{ ft} \quad \text{(parabola)}$$

$$L = 1{,}113 + \frac{2 \times 247.42^2}{3 \times 1{,}113} = 1{,}149.6675 \text{ ft} \qquad L_m = 1{,}131.3338 \text{ ft}$$

$$S_{cx} = \frac{0.5181 \times 1{,}113 \times 1{,}131.3338}{2 \times 1{,}297} = 251.49 \text{ ft}$$

The catenary solution gives a value of 251.55 ft, or less than ¾ in. difference.

$$S_{cx1} = 251.49 - 200 = 51.49 \text{ ft}$$

For other values of H (Chart 1), compute the sag y. Also, compute the length L and L_m, from which S_{cx} is determined at other temperatures in the same manner as above.

Assume a level span of 1,600 ft, as indicated by the dotted lines in Fig. 48, and compute the sag at 120° as follows:

$$S' = \frac{x^2}{4a} \qquad x = 800 \text{ ft} \qquad 4a = 5{,}006.76 \text{ ft}$$

$$S' = \frac{800^2}{5{,}006.76} = 127.83 \text{ ft} \qquad \text{(parabola)}$$

$$L = 1{,}600 + \frac{8 \times 127.83^2}{3 \times 1600} = 1{,}627.233 \text{ ft} \qquad L_m = 1{,}613.6165 \text{ ft}$$

$$S'_c = \frac{0.5181 \times 1{,}600 \times 1{,}613.6165}{8 \times 1{,}297} = 128.92 \text{ ft}$$

(mean length method)

For other values of H (Chart 1), compute the value of a, which changes for each value of H, and perform the operations in the same manner as above. All essential values are compiled in Table 39.

At 120°, $S'_c - S' = 128.92 - 127.83 = 1.09$ ft in the 1,600-ft span, which will compensate for the difference in sag below A of $S_{cx} - y = 251.49 - 247.42 = 4.07$ ft.

TABLE 39.—PROPERTIES OF UNSYMMETRICAL SPAN

Temp., deg F	H, lb	x_1, ft	x, ft	S_{cx}, ft	S_{cx1}, ft	S'_c, ft
120	1,297	487.00	1,113.00	251.49	51.49	128.92
90	1,303	485 63	1,114.37	250 93	50.93	128.31
60	1,311	483.70	1,116.30	250.22	50.22	127.52
30	1,317	482.25	1,117.75	249.71	49.71	126.93
0	1,325	480.32	1,119.68	249.02	49.02	126.15
−20	1,330	479.12	1,120 88	248.82	48.82	125.66

When tensions differing from those in other parts of the line are required, the sag in an unsymmetrical span is computed for a level span equivalent in length to the horizontal distance between the two supports, using the parabola method only. Then one structure is dropped down to indicate the actual difference in elevation of the two supports. Compute the distance x_1 for each given value of H. The work is then carried out in the same manner as described above for S_{cx} and S'_c.

The sag at maximum horizontal tension of 3,778 lb, 0°F, is approximately the same as for the 120° tension of 1,297 lb, from which $T = 3{,}778 + (1.5091 \times 251.49) = 4{,}157$ lb. This is less than 60 percent of the ultimate tension of the cable and is permissible.

In the solution to these sag problems, no attempt has been made to minimize the effort that must be put forth in arriving at definite and final conclusions. On the other hand, an effort has been made to emphasize the simplicity of the mean length method when applied to the solution of level or unsymmetrical spans.

Another important point is that, when the horizontal component of tension is used as a basic assumption, a constant value is available for any given conductor loading. The tension at the support, which varies with the sag and span length, is always readily determined for any given sag and conductor weight. When the tension at the support at maximum cable loading, which is one of the first things to be determined, exceeds the permissible value, it is only necessary to reduce the assumed maximum horizontal tension and proceed with the solution; since, for other cable loadings, we are not necessarily concerned about the tension at the support.

CHAPTER X

LINE LOCATION AND SURVEY

The shorter a given transmission line is between its terminal points, the lower the over-all cost and the transmission losses will generally be. It is therefore desirable to have a line adhere as closely as feasible to a straight line. In the early stages of power transmission, there was a great deal more open and sparsely settled country than at present; lands were not so valuable, and buildings were more scattered. Lines could therefore generally be located to best advantage with a minimum amount of preliminary survey work.

In transmission-line surveys, some other items to be considered are of as great, if not greater, importance than the actual running of the line. The most important item is that of final location. Any good instrumentman can run a line between two given points when that is his only consideration. This is not the condition, however, with which a transmission-line surveyor is confronted. The surveyor knows where to begin the survey and where to finish it, but running the line properly from the standpoint of location is a different matter. His problem, which requires skill and training, is to determine the best possible location for a given line.

As the initial step in the preliminary survey for an important transmission line, it is advisable to travel roads that are available and relatively close to the route along which the line is to be built, in order to determine major items of possible obstruction, such as small towns, swamps, manufacturing plants, and undesirable terrain.

In fairly open sections of the country where roads are accessible, an aerial survey will rarely be justified. On the other hand, for lines of appreciable length, in either swampy or mountainous country, much of which may be virtually inaccessible, an aerial survey is advisable. Photographs should be taken of a strip at least 1 mile on either side of the proposed transmission line.

After the aerial photographs are finished, they are matched and pieced together to form a map of the desired area, the scale of which should be such that buildings, fields, streams, and woodlands are readily distinguishable.

One of the most satisfactory methods for determining the approximate location of a transmission line may be carried out with a minimum of field work in states where a county agent system exists. It is as follows:

In most counties, the county agent has on file aerial photographs (for agricultural purposes), which are usually in the order of 22 by 24 in. in size, and reproduced to a scale of 660 ft to the inch, or 8 in. to the mile. They are not for general distribution, but the county agent will usually agree to having consecutive sections roughly matched. It is then an easy matter to identify the area for some considerable distance on either side of the proposed line location. The aerial photographs are numbered, serially, for each flight. From these serial numbers and a form that may be secured from the county agent, reproductions may be purchased from the Agricultural Adjustment Administration, Washington, D.C.

When the reproductions are received, they are cut, matched, and pieced together to form an aerial map to be used for determining the tentative line location. Because of the large scale, buildings and other objects are clearly distinguishable. All buildings and other objects indicated on the map which are adjacent to the proposed line are ringed in pencil for easy identification. A string or an ordinary sewing thread, used to simulate the transmission line, is then stretched between the two terminals of the line. The ends of the string are held stationary, and the rest of it is moved laterally and adjusted to clear buildings and other undesirable areas until a tentative and as nearly satisfactory location as possible is obtained. Thumb tacks are inserted at each angle to hold the string in place, and a pencil line is drawn on the map to coincide with the tentative line location.

After the above procedure is completed, the map is trimmed along the sides where necessary. Sufficient width is left on each side of the line to permit further lateral revisions if and when they are found to be necessary. The aerial map is then cut in sections that can be readily handled in the field. Matching numbers or letters are entered on adjacent ends of consecutive sections for ready identification.

It is not always possible to obtain recent aerial maps, and houses and buildings may have been constructed after the aerial photographs were made. The county agent will generally know of any conditions that may be a possible source of obstruction.

Even though there is no apparent reason why the line cannot be built as indicated on the aerial map, right-of-way difficulties may be encountered, lands may have been optioned or purchased for air-base operations, or other obstructions may make it necessary to revise the tentative line location at one or more places.

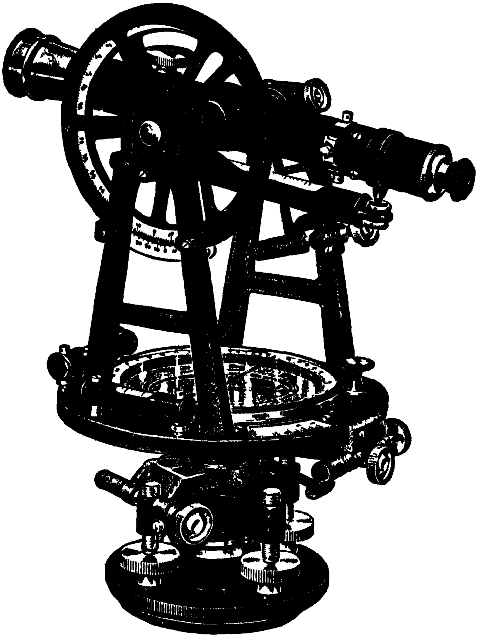


Fig. 48.—Transit.

Field Work.—The survey crew, which should as far as practicable be trained in transmission-line survey work, will usually consist of a chief of party; an instrumentman; two rodmen, or chainmen; and one or more axmen, depending on the amount of trees and brush to be cut.

The major items of equipment required for a transmission-line survey, exclusive of transportation equipment, include a transit (Fig. 48); level (Fig. 49); range rod either wood or steel (Fig. 50*a*); 100-ft engineer's steel tape, commonly called chain, (Fig. 50*b*); stadia rod (Fig. 50*c*); and level rod (Fig. 50*d*).

The field work will consist of a preliminary instrument survey line, a considerable portion of which usually turns out to be the final location. This phase of the work is called "hubbing," since a hub stake is set at each instrument point. Two methods of measuring distances are specifically adaptable to transmission-line surveys, namely, "chaining" and "stadia." The method employed on a given project will depend largely upon the terrain and the method with which the surveyor has the most aptitude and familiarity.

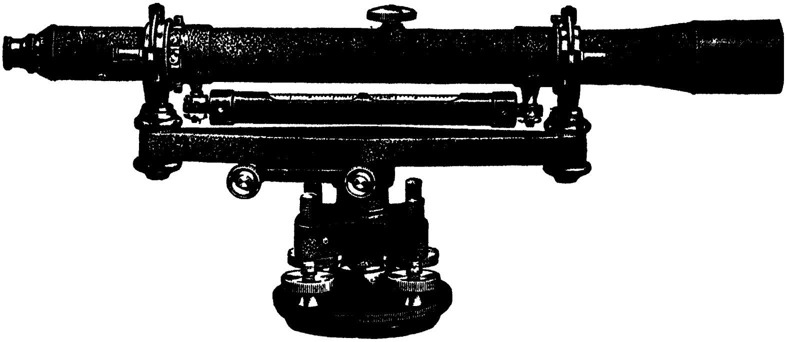


Fig. 49.—Level.

Measuring Distances by Chaining.—Chaining, the direct method, as applied to transmission-line surveys consists in measuring horizontal distances by the use of a 100-ft steel tape (see Fig. 50*b*), the first and ninety-ninth foot of which are graduated in tenths of a foot. The other 98 ft are graduated at intervals of 1 ft. The procedure is as follows:

The transit is set up over the hub stake at the point from which the distances are to be measured and is focused on the line. The head rodman is given all but one of a set of chaining pins, and the back rodman is given the one remaining pin. (Eleven chaining pins make a full set.) The head rodman takes the zero end of the tape, called "little end," and drags it out until the back rodman calls "Chain," which is a signal that the tape is approximately all out. The back rodman holds the 100-ft end of the tape called "big end," even with the plumb line under the transit or with a chaining pin inserted in the ground by the side of the hub stake at the zero point. The head rodman pulls the tape taut, holding it in a horizontal position with the end even with the range rod, which is held in his other hand and lined in with the transit. As soon as the measurement is taken, a

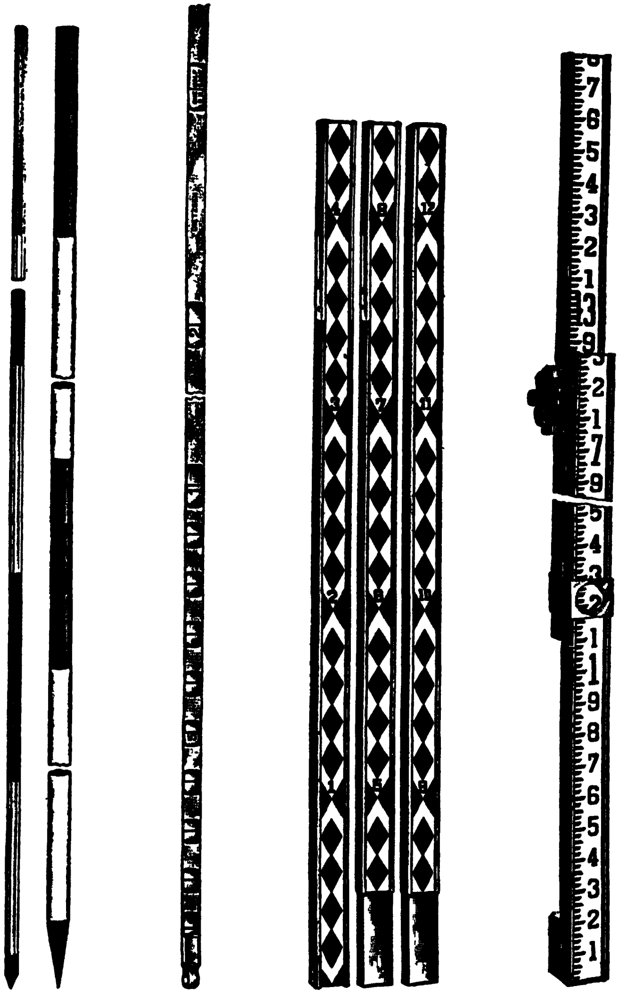

(*a*) (*b*) (*c*) (*d*)

Fig. 50.—(*a*) Range rod—steel and wood; (*b*) engineer's steel tape; (*c*) stadia rod—three piece; (*d*) level rod.

chaining pin is inserted in the hole made by the rod, and the back rodman pulls the chaining pin at his end of the tape. Both rodmen then go ahead and repeat the process.

The measured distances are recorded, and notes are taken, usually by the chief of party or the instrumentman. Although this procedure is the generally accepted practice, there are no definitely established rules to be followed. As a rule, each surveyor will have his own ideas as to the procedure necessary to obtain a precision demanded by the nature of the work on which he is engaged.

When horizontal distances are measured on steep inclines or where the ground is badly broken, the tape cannot be held in a horizontal position for its full length, and several measurements may be required for one length of the tape. Great skill and care are therefore needed to obtain results with a high degree of precision. However, for most transmission-line surveys, only average precision is required, and the results obtained are generally quite satisfactory. For broken tape measurements, it is well to use a plumb bob instead of the range rod. Whether the plumb bob is held by the head or by the back rodman will depend upon whether the measurements are being taken down or up the incline.

Measuring Distances by Stadia.—To facilitate taking long sights, stadia rods should have a length of 12 to 15 ft, but for ease in transportation it is desirable to have the rod made in sections with sockets or clamps provided for quickly and firmly assembling the rod in the field. There are many different kinds of stadia rods, some of them homemade, which serve their purpose adequately. For transmission-line surveys, it is well to provide a rod with graduations representing a clearly defined figure or diagram that may be distinguished for distances up to about 1,500 ft.

Transits equipped for use with stadia rod have two additional hairs mounted horizontally on the cross-hair ring in the telescope. Distances are measured by observing through the telescope the space on a graduated rod included between the stadia hairs. If the rod is held vertical at different distances from the transit, different space intervals on the rod are included between the stadia hairs, the space on the rod being proportional to the distance from the instrument to the rod; and the intercepted space on the rod is therefore a measure of the distance to the rod.

Actual distances are derived from formulas, with a stadia slide rule, or from a stadia reduction table.

The stadia method of measuring distances is particularly adaptable to transmission-line surveys. Intervening spaces do not have to be traveled, and it provides a convenient means of measuring distances across inaccessible places, such as rivers and ravines. The accuracy of the stadia measurements depends upon the atmospheric conditions, the length of sights, and the observer.

When it is necessary to take long sights, the space included between the stadia hairs may be beyond the range of the rod. For example, the height of the instrument *HI* will ordinarily be from 4.5 to 5 ft; and, since the crosshairs must be focused on the rod a distance of *HI* above the ground, with an *HI* of 4.5 ft, the intercepted space on the rod would be limited to 9 ft. When the lower stadia hair falls below the lower end of the rod, the rod interval is taken as twice the interval between the cross hairs and the upper stadia hair, which would be the value used for determining the distance from the transit to the rod.

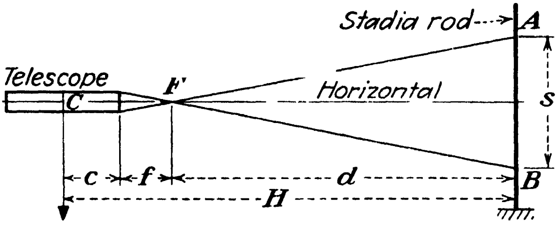


FIG. 51.

Figure 51 represents the principle of the stadia method as described above for measuring distances over comparatively level ground. In the figure, the line of sight is seen to be horizontal, and the stadia rod is in a vertical position. The rod interval is represented by the distance s between the apparent positions of A and B on the rod.

$$H = Ks + (f + c) \tag{53}$$

K is called the "stadia interval factor," the nominal value of which may be taken as equal to 100. $(f + c)$, for all practical purposes, may be considered as a constant and equal to 1 ft.

If, for example, the rod interval is read as 4.13, substituting in (53) and solving for H, we have

$$H = 100 \times 4.13 + 1 = 414 \text{ ft}$$

Other values are determined in the same manner and require only a mental calculation.

Figure 52 illustrates the principle of determining horizontal distances and differences in elevation by use of the transit and stadia rod. It is obvious from the figure that trigonometric equations are required for determination of H, the horizontal distance from the instrument to the rod, and V, the difference in elevation of the instrument and rod. The equations from which H and V are derived are somewhat involved. The work required for their solution is not only laborious but requires a table of trigonometric functions.

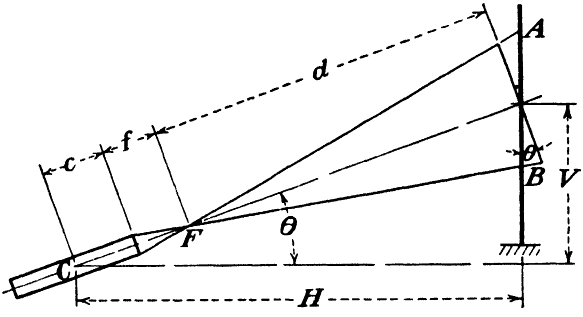


FIG. 52.

$$H = Ks \times \cos^2 \theta + (f + c) \cos \theta \qquad (54)$$
$$V = 0.5 \times Ks \times \sin (2 \times \theta) + (f + c) \sin \theta \qquad (55)$$

As an example of how the horizontal and vertical components of the inclined distances are determined, assume the rod interval to be 4.5 ft, and the vertical angle θ 11°28′. From a table of trigonometric functions, cos 11°28′ is 0.98 and the sine is 0.1988. $\sin 2 \times \theta = \sin 22°56' = 0.38$ nearly.

$$\text{From (54) } H = 100 \times 4.5 \times 0.98^2 + 1 \times 0.98$$
$$= 450 \times 0.9605 + 0.98 = 433.2 \text{ ft}$$
$$\text{From (55) } V = 0.5 \times 100 \times 4.5 \times 0.38 + 1 \times 0.1988$$
$$= 225 \times 0.38 + 0.1988 = 87.87 \text{ ft}$$

To avoid calculating the values of H and V, stadia reduction tables have been compiled by various authors and may be found in practically all publications on surveying. A stadia slide rule is also available for this work. It is preferred by many surveyors, since it may be carried in the pocket and provides a ready means of computing actual distances from stadia-rod readings.

It is apparent that, unless the telescope of the transit is correctly focused on the stadia rod, the vertical angle will be incorrectly indicated, and erroneous values of V, the difference in elevation of the transit and rod, will result. To determine the value of V correctly, the line of sight, or cross hairs, must be focused on the rod at a distance from the bottom of the rod equal to *HI*. Only then will the instrument indicate the true slope of the ground.

The probability of error in reading the rod interval may be reduced by setting the lower stadia hair at a 1 ft graduation on the rod. This facilitates reading the interval s, since fractional parts of a foot will appear only at the upper stadia hair. After the rod interval is read, the telescope is revolved vertically and the cross hairs are set at the correct height on the rod.

Running the Levels.—The importance of the levels of a transmission-line survey is often not given the consideration it justly deserves. Since the structure locations are dependent upon the profile, which is plotted from a given set of field notes, an error in reading the rod or recording the reading can result in improper structure location and short ground clearance. It is therefore important to have the work performed by trained men and carried out with a better than average precision. The level party will consist of a levelman and rodman. A reading is taken on the rod at each station and on the line between stations at each rise or depression in the ground.

When the slope of the ground transverse to the line is such that the elevation under the outside conductor is in the order of 18 in. and more above that on the center line of structures, sidehill elevations should be taken. This is necessary to provide information from which to determine minimum clearance from the conductor to earth at any and all points along the line.

Levels may be taken with transit and stadia rod, but the method is not generally satisfactory for transmission-line work. Since elevations at consecutive rod locations will rarely be the same, excessive manipulation of the transit is required for the focusing of the cross hairs on the rod for each setup. For comparatively small differences in elevation between the instrument and rod, the tendency to disregard the small vertical angle, because of the calculations required, results in low-precision work. This condition is not inherent in the level and level-rod method.

Preliminary Survey.—The survey should adhere closely to the line indicated on the aerial map unless some unforeseen obstacle is encountered that may require a change in the tentative location. It should be explained, however, that only an approximate bearing can be determined from the aerial map. The bearing of the line on the map, however, may be determined with a sufficient degree of accuracy by the following procedure:

Set the transit up at the beginning point of the survey to coincide, as nearly as can be determined from the aerial map, with the origin of the line. Take a long foresight with the rod held vertically at a point on the line, as indicated on the aerial map. Next, move the instrument ahead to the first rod position, and take a backsight on a rod held at the point of beginning. Then take another long foresight. From these two setups, and observable objects on the aerial map, the divergence, if any, from the line on the map may usually be determined. It is then only a matter of returning the instrument to the beginning point and turning off the angle required to make the correction.

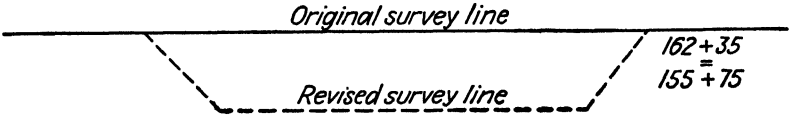
From the preliminary survey, the distances of which may be measured by chaining or stadia, all buildings and other objects that are close to the survey line should be spotted with respect to their distances from the line. The distance between the survey line and the buildings that should be spotted will depend upon the voltage and type of the proposed line. Since it is rarely certain what will be encountered ahead, it is well to spot all buildings within a distance of several hundred feet of the survey line. The locations of yards, gardens, cemeteries, the boundary lines of fields crossed, orchards, and woodlands should be recorded. In addition, all telephone, telegraph, transmission, and distribution lines, farm and county roads, state and United States highways, railroads, streams, and fences should be recorded. The preliminary survey line, including all buildings, etc., should be drawn to scale on durable comparatively heavy, detail paper.

After the map of the completed preliminary survey is drawn, the string described in connection with the aerial map can be used to determine the portions of the preliminary location that may be considered acceptable for final location.

When the line has been run over a route that is acceptable for alignment and on which it appears that there will be no serious difficulty in securing necessary rights of way, the final survey is run. This work is done carefully. All brush and trees are cut from a strip of sufficient width to make for greater precision in instrument work and chaining. All angles are turned (the compass being used only as a check), a hub stake is set, and a tack is driven in the top of the stake at each instrument point. A stake is driven at each "station," or every 100 ft, measured from the beginning point of the survey.

Stakes are 18 in. long sawed material and should be of sufficient rigidity to permit of driving in hard ground. Stakes should be numbered only as they are driven; otherwise there is danger of getting mixed on the numbers and driving a stake improperly numbered at a given station. This will result in a minimum error of 100 ft.

Each stake is plainly marked with the proper "station." The beginning point of the survey is marked 0 + 00, read "naught plus naught-naught." The first station is 100 ft from the beginning point and is marked 1 + 00. The fifteenth station is marked 15 + 00. If an instrument point is required 75 ft ahead of station 15 + 00, the stake is marked 15 + 75 and is driven about 1 ft to the left of the hub stake. A station between the 100-ft stations is called a "station-plus."



It is not always possible to have the station numbers run consecutively throughout the entire length of a given line. For example (see sketch), intersection points of revised and original survey lines call for an "equality." If the station numbers of two intersecting survey lines are 162 + 35 and 155 + 75, respectively, the stake is marked 162 + 35 = 155 + 75. All subsequent records should indicate this equality, since the survey line up to this point is increased in length by

$$(162 + 35) - (155 + 75) = 660 \text{ ft}$$

An intersection point of two survey lines, in the language of the surveyor, is called "*PI*" (abbreviation for point of intersection).

Staking the line in 100-ft stations is not absolutely necessary but greatly facilitates the securing of property information, as well as the staking of structures in the field, since it is never necessary to measure a distance greater than 100 ft to stake the structure as indicated on the profile.

When it is necessary to rely entirely on an instrument survey, it is virtually impossible to obtain a line location, especially for alignment, that will compare favorably with the location obtained when aerial maps are available. It is difficult to determine the direction, or bearing, of a line between two distant points. It is also extremely difficult to locate buildings and other objects in rolling or wooded country until the surveyor is near them.

When the instrument survey is started, a definite course is followed until it becomes necessary to change direction. When an obstruction is in the path of the line, it is necessary to abandon possibly a mile or more of the survey and to back up and turn an angle in order to divert the line around the obstruction. To get back on the same line requires a minimum of three angles. This may happen several times in a comparatively short distance and not only requires an excessive amount of time but makes for an undesirable line location. This condition is typical of practically all instrument surveys, especially where the country is quite thickly settled.

Property Information.—For securing property information, the 100-ft stationing is of invaluable assistance in determining the location of property lines. In connection with this work, a great deal of information can usually be supplied by people who have resided for many years in the immediate vicinity. These "old-timers" are often able to point out the location of corner stakes from which the property line may be traced to its intersection with the survey line. It is then only a matter of locating the station-plus of the property line with respect to the nearest 100-ft station. This can usually be determined with sufficient accuracy by pacing. The station-plus and approximate bearing of the property line should be recorded.

When both boundaries of a given parcel of land are obtained, it is only a matter of subtraction to determine the distance across the property. Many property lines coincide with fences, ditch lines, roads, streams, etc., simplifying to a certain extent the securing of property information.

When the property information has all been secured, a property list is compiled, giving each tract of land crossed, by parcel number, the names of those owning or controlling the property, the station number at entering and leaving boundaries, and the distance traversed across the property. This property list is used in connection with the negotiations and purchase of the rights of way. The property list is preserved and made a part of the permanent records pertaining to that particular project.

It should be kept in mind by all members of the survey party that, once a given line is constructed, the operators of the line must practically live with the people whose lands the line crosses. Every effort should therefore be made to keep on friendly terms with people in the community, whether they are affected by the line or not, to avoid arousing a spirit of antagonism toward the sponsors of the proposed line. Friendly relations can most easily be established and maintained by delegating someone to precede the survey party for the purpose of securing permission to run the survey line over the property. The person sent ahead should preferably be the one who is expected to carry out the negotiations, at a later date, for procurement of the rights of way.

The purpose of the survey should be explained, with the further information that the work is of a preliminary nature to determine a suitable location for a proposed transmission line, and that compensation will be made for any damage to crops, or otherwise, that may be inflicted by the survey party. Occasionally during these discussions, a property owner will remark, "I do not want the line to cross my place at such and such location but would not object to having it cross at some other designated location." This may alter the situation appreciably. At least, it provides information as to what may be anticipated in connection with the final negotiations for purchase of rights of way across the property.

Even though the proposed line is sponsored by a public utility, which by virtue of the law in most states possesses the right of eminent domain, with legal right to run a survey line across property without first securing permission from the property owner it is not advisable to proceed under these legal rights except as a last resort.

CHAPTER XI

TRANSMISSION-LINE DEVICES, STRUCTURES, AND STRUCTURE SPOTTING

Sag Template.—For spotting transmission-line structures on a profile, a sag template is required. It should preferably be made of transparent celluloid and should consist of four curves, as will be explained. For spotting structures on steep inclines, the template should be constructed to include spans exceeding in length the horizontal distance between actual structure locations. For this reason, when a comparatively short span has been used as a basis for calculating the sag in a given line conductor, as in this case, it is not safe to use a template constructed from values based on the assumption that the sag is proportional to the square of the span length (parabola).

On the other hand, the template should be constructed from values derived from the mean length method, using as a starting point the sag from a set of sag-tension curves similar to Fig. 39. For example, the 120° sag for the 3/0 twelve-strand copper in the 500-ft span is given as 12.56 ft. The sag in a 1,000-ft span from the parabola method would be $4 \times 12.56 = 50.24$ ft. By the mean length method, the sag would be 50.41 ft. The sag in a 2,000-ft span by the parabola method would be 200.96 ft, and by the mean length method 203.68 ft.

To construct the sag template, enter on a piece of regular profile paper span lengths to a scale of 400 ft to the inch horizontally, and to a scale of 40 ft to the inch vertically, as indicated in Fig. 53. Plot on this sheet, from calculated values, the required number of curves: one curve to indicate the position of the conductor at minimum temperature, three maximum temperature curves, one curve to indicate the position of the conductor at maximum temperature, one curve to indicate minimum ground clearance at maximum temperature, and one curve to indicate the tentative location for a given height of structure, as indicated in Fig. 54.

When the curves have been plotted, fasten a piece of transparent celluloid securely over the sheet, using for this purpose

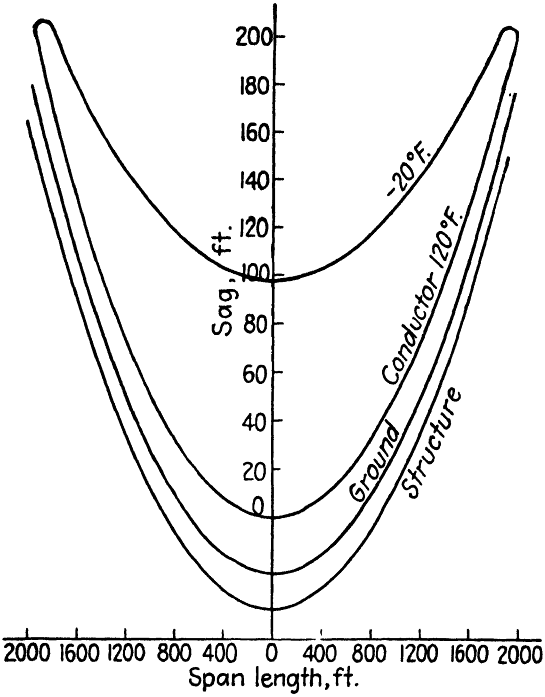


Fig. 53.—Method of constructing sag template.

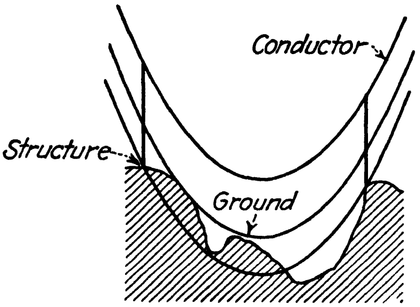


Fig. 54.—Method of using sag template.

scotch tape or thumb tacks. With a sharp-pointed steel scriber, cut the celluloid at the minimum and maximum temperature conductor sag curves. Before taking up the celluloid, inscribe a thin line to coincide with the ground line and structure curves. Inscribe also a vertical line at the center of the template. Construct a graduated gauge in the upper right-hand corner of the template to indicate pole heights, above a base line, from ground line to conductor at the structure for various lengths of poles with the insulators in a suspended position. Construct a similar gauge in the upper left-hand corner to indicate height

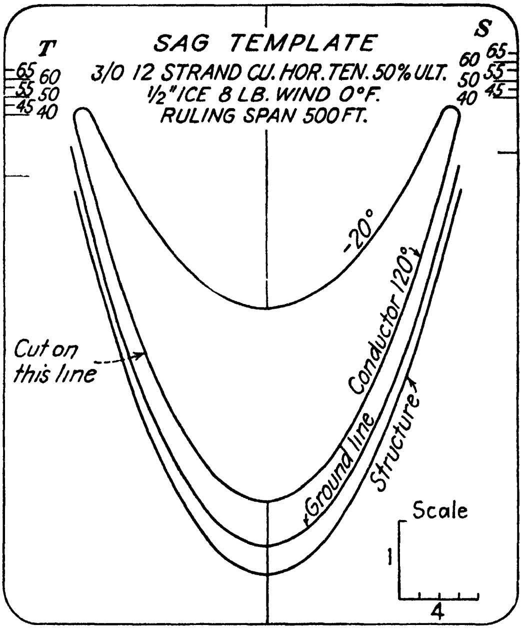


FIG. 55.

from ground line to conductor at the structure with the insulators in a strain position. All essential information, such as conductor size and loading, should be inscribed on the template as shown. The completed template will have the appearance of Fig. 55.

The inscription on the template, "500 ft ruling span," is for the purpose of indicating the basis from which the original calculations were made and also indicates the basis from which the mean length method of calculating sag in the longer spans is to be applied. For example, the sag in a 2,000-ft span (parabola) is equal to $12.56 \times 2{,}000^2/500^2 = 200.96$ ft. The length of conductor in the span at 120°, from Eq. (42), Chap. IX, is computed to be 2,053.85 ft, from which the mean length L_m is equal to 2.026.92 ft.

$$S_c = \frac{0.5181 \times 2{,}000 \times 2{,}026.92}{8 \times 1{,}289} = 203.68 \text{ ft}$$

It is necessary to apply the mean length method only for spans having a length in excess of about 1,000 ft.

Spotting Structures.—Figure 56 depicts a section of plan and profile of a transmission line. To use the template for spotting structures on the profile, proceed as follows:

Hold the template in a vertical position as determined by the vertical line on the template, with the ground-line curve just tangent to the profile of the ground at the point in the span that will be the point of minimum ground clearance at maximum temperature (see Fig. 54). With the template in this position, draw in the conductor. By use of the pole gauge, draw in the desired height of the structure. Slide the template to the right or left, as the case may be, and repeat the procedure. Care should be exercised to avoid locating structures on peaks, for this generally makes for poor line grading. By care in determining structure locations and using variable pole heights, satisfactory grading of the line can usually be obtained. Refer to structures 2 and 3. The difference in elevation at the ground line of the two structures is about 19 ft, but the difference in elevation at the conductor supports is only about 2 ft. Obviously proper line grading provides for minimum crossarm stresses, a matter of utmost importance.

The nine structures on the section of profile in Fig. 56 represent five different types. The severity of loading and kind and size of conductor will of course have considerable bearing upon the type of structure to be installed at any particular location. The approximate stress in unguyed unbraced structures may be determined as described in Chap. VII. No definite value can be set up for the maximum unguyed span that will apply under all conditions. The span length will be governed by the type and class of pole, the kind and size of conductor, the factor of safety required, the loading district, and soil conditions.

From a casual inspection of the structure spotting in Fig. 56, it appears somewhat unblanced. An analysis reveals, however, that it is not. This section of profile was selected for illustration because it represents a condition that requires a good deal of thought and care in order to determine the most feasible structure locations. Considering the irregularity of the profile, the line is seen to be comparatively well graded.

The dotted portions of the profile represent side-hill elevations under the outside conductor on the uphill side of the structure. Where side-hill elevations are shown, this should be the governing factor in the determination of minimum ground clearance.

Spotting structures on a profile embodies both engineering and economics. Two items of considerable importance should be kept in mind: (1) In general, it is more economical to reinforce the line by being quite liberal with X bracing and guying than by installing additional structures. (2) It is too late to correct for mistakes in spotting after the line is constructed.

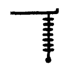


FIG. 57.—Type H, tangent structure.

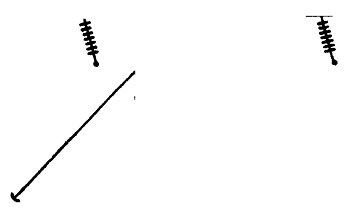


FIG. 58.—Type HLA, light angle structure not over 5 deg.

When the designer is satisfied that the spotting is such as to give the most satisfactory line, consistent with good design, as many prints as are required are made from the plan and profile, usually in one continuous roll. The surveyor is supplied with a print to be used as a guide for staking structures. Each structure stake is marked with the station-plus, type of structure, and height of poles called for at that particular location. The construction forces need this information in order to have delivered to each structure location the proper length and class of poles.

Since it cannot be determined in the office whether the ground at all indicated structure locations is suitable for the installation of a structure, it is often necessary to relocate the structure in one direction or the other. The spotting should be such as to

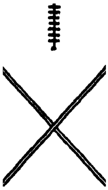


FIG. 59.—Type HX, tangent structure for long spans.

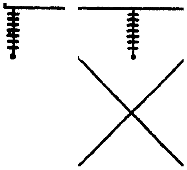


FIG. 60.—Type HG, cross-guyed structure for long spans.

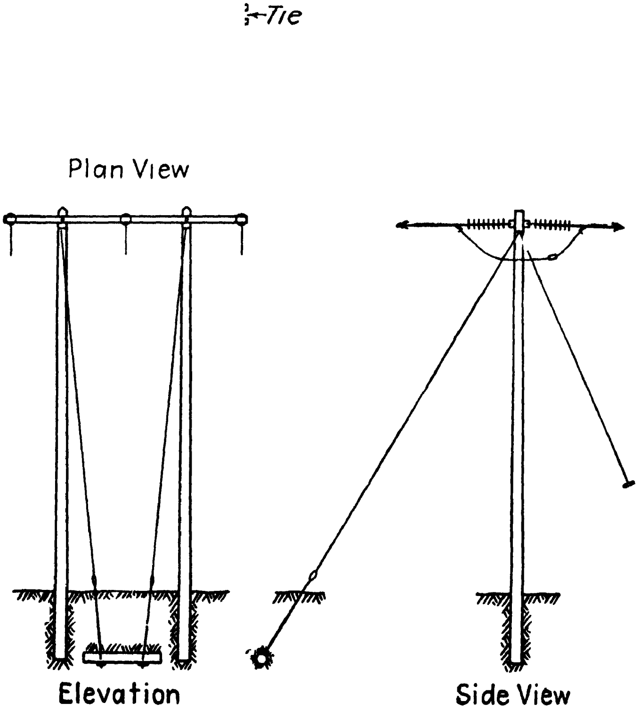


FIG. 61.—Type HT, two-pole tension structure.

permit changing the location from 10 to 15 ft in either direction without having to increase the height of the poles. The surveyor should be given authority to change the location, within the prescribed limits, when an obstruction or unsuitable ground is encountered.

Various type of two- and three-pole structures are illustrated by the sketches in Figs. 57 to 64. Figure 57 is a flexible, or hinge-type, structure and is extensively used. For this type of structure with overhead ground wires, see Fig. 84. Figure 58, as indicated, is used at very light angles. This type of structure is not suitable for use at angles of any appreciable size. The side guy, in order to clear the conductor the required amount, must be attached to the pole at a distance below the crossarm that will produce excessive bending moment at the guy attachment; and the poles therefore have a tendency to bow.

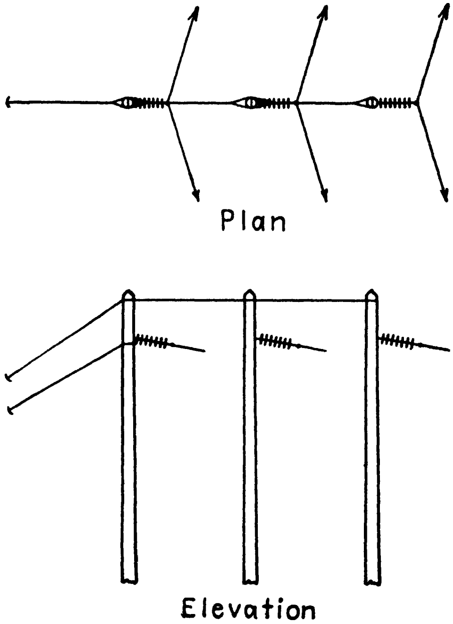


FIG. 62.—Type 3PA, suspension or pull-off angle structure. Limited to about 30-deg. line angle.

Figure 59 illustrates an X-braced structure that is meeting with increasing favor. This type of structure is especially adaptable where in general long, approximately uniform spans are feasible. Figure 60 illustrates a cross-guyed structure, a type that is used with great success where right of way is comparatively narrow. When this method of guying is used, anchors may be installed on the right of way, thus permitting the purchase of a limited width of right of way throughout. Figures 61, 62, and 63 illustrate a two-pole and two three-pole structures of more or less standard design, which are used, of course, at tension and angle points.

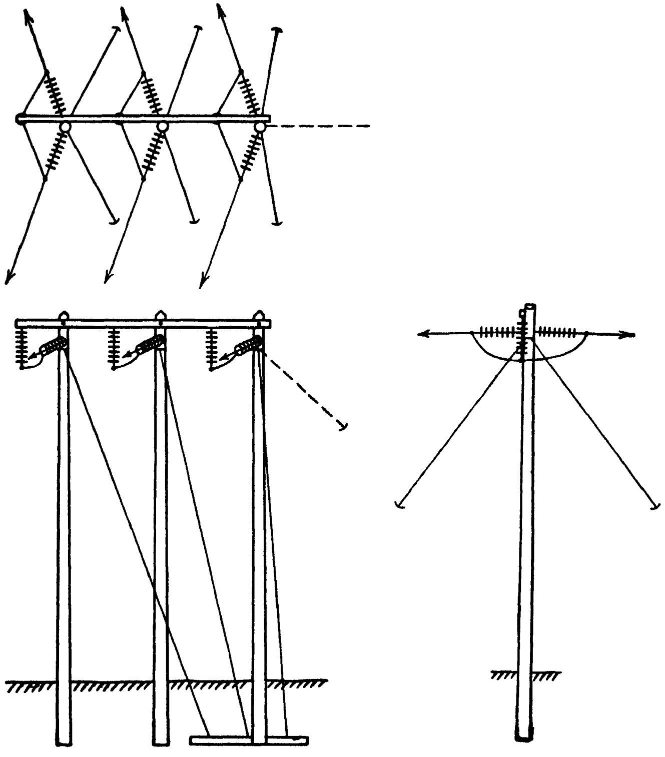


FIG. 63.—Type T3, three-pole angle structure.

Figure 64 shows a transposition structure of extremely simple design. Two transposition structures are required for each third of a complete transposition, as indicated by Fig. 65.

For important transmission lines of any considerable length, it is desirable to have an angle construction chart, similar to Fig. 66, to be used as a guide in determining the type of structure to be installed at angle points. The areas, as indicated on the chart, will differ for different types of line and various loading districts. When the angle falls within a given area on the chart, the design should be such that the type of structure indicated in that particular area may safely be used.

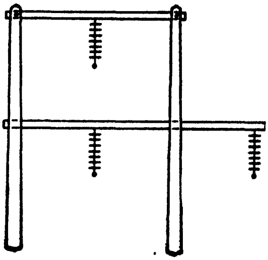


Fig. 64.—Transposition structure.

It should be noted that the 3PA structure is limited to an angle of 30 deg, with large conductors, regardless of the fact that this is a three-pole structure. As explained in Chap. VII, this limitation is made because unequal stresses are set up in the conductor at the clamp attachment. Obviously the sharper the angle is the more unequal the stresses will be, especially in large conductors. Angles in excess of 30 deg are permissible when a double-clamp arrangement or some method is devised that will increase the radius of curvature at the clamp attachment. The span is limited to comparatively short lengths, because otherwise the vertical component of load on the conductor may prevent sufficient clearance at the structure with small angles.

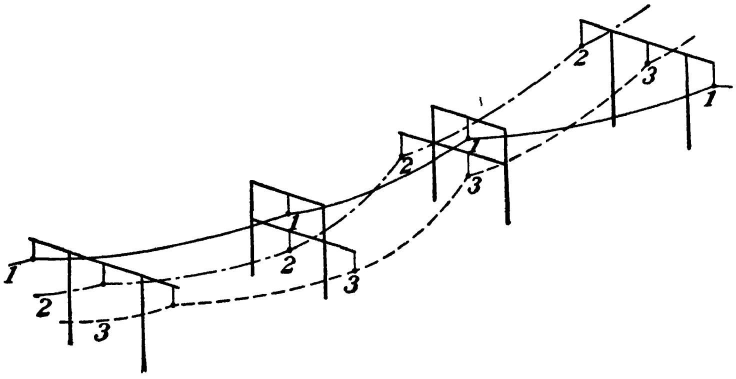


Fig. 65.—Method of transposing conductors.

For angles in excess of 60 deg in the line, the pull due to the angle exceeds the conductor tension. In such cases, special consideration should be given in the area marked "special" to the guying and the type of structure to be installed. The three-pole structure T3 may often be used for angles up to 90 deg.

Insulator Swing.—For approximately level spans, regardless of their inequality in length, the deflection of insulators in a suspended position produced by transverse wind loads is not generally of any consequence. On the other hand, when a structure is installed where the distance from the structure to the low point of the conductor in adjacent spans is small compared with the total length of the two adjacent spans, the effect of the wind load is to produce excessive deflection of the insulator strings transverse with the line. This is called "insulator swing."

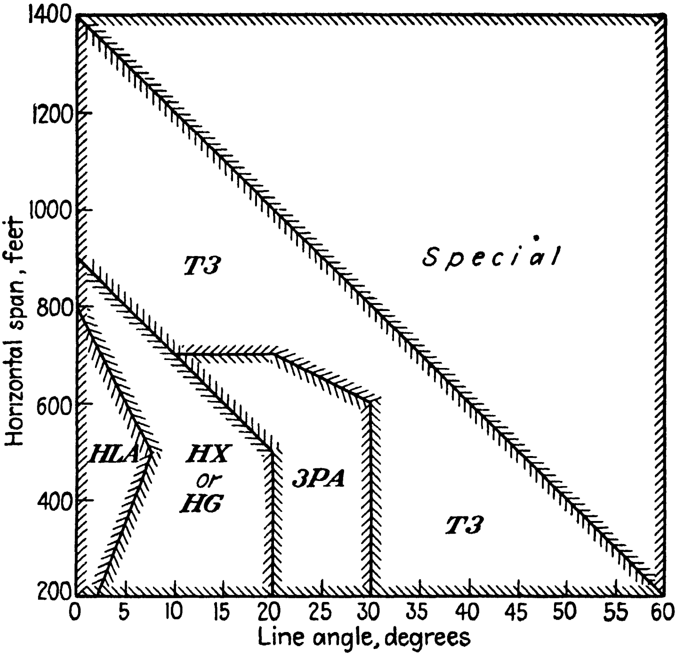


FIG. 66.—Angle construction chart.

As a basis for calculating the deflection angle of a suspension string of insulators, the wind pressure to be used should generally be taken at a minimum of 8 lb and a maximum of 12 lb per sq ft on the bare conductor. For wood crossarms with unbonded hardware, an insulator swing in the order of 55 deg is generally considered permissible. For wood crossarms with bonded and grounded hardware, or steel towers, the maximum permissible insulator swing should not exceed about 45 deg, unless some method is provided whereby safe clearances are maintained between the conductor and the component parts of the supporting structure. Three convenient methods are available, one or the other of which may be resorted to for this purpose.

1. Suspend the insulator string from a bracket under the crossarm that will drop the assembly about 12 in.

2. Install a taller structure for increasing the vertical component of load supported by the insulator string (see Fig. 67*A*).

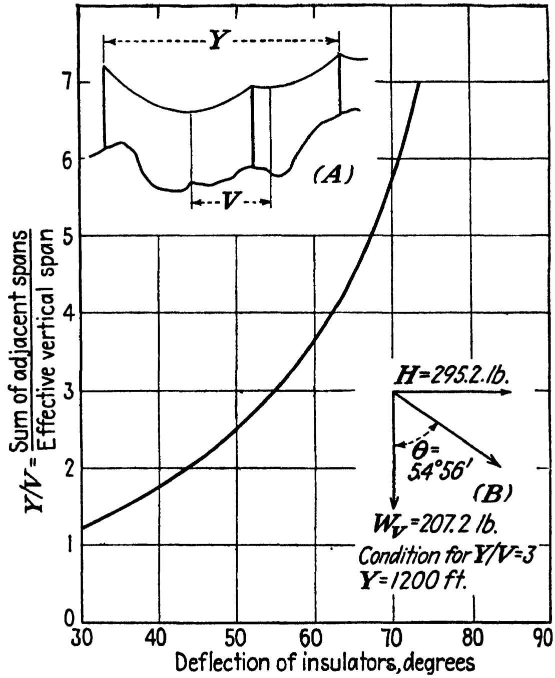


FIG. 67.—Curve for determining the angle of insulator swing 3/0 twelve-strand copper cable.

3. Install a tension structure. This method is comparatively expensive and should be resorted to only when methods 1 or 2 cannot be made to answer the purpose.

In mountainous sections, structure locations are frequently fixed within quite narrow limits. It is therefore obvious that the problem of insulator swing enters quite prominently into the design. Preventive measures should be taken at the time of spotting the structures on the profile.

To facilitate the work, a curve such as that indicated in Fig. 67 should be constructed. For any particular ratio of *Y*, the sum of the lengths of the two adjacent spans, to V, the effective vertical span, the angle of insulator swing may be determined almost immediately from the curve for any given kind of conductor. This particular curve is for use with 3/0 twelve-strand copper and is plotted from values derived from a wind pressure of 12 lb per sq ft on the bare conductor.

The diameter of the 3/0 twelve-strand copper cable is 0.492 in., from which the wind load at 12 lb per sq ft is

$$\frac{12 \times 12 \times 0.492}{144} = 0.492 \text{ lb per lin ft}$$

The weight of the bare cable $w_{vo} = 0.5181$ lb per ft. Let us assume that $Y/V = 3$ and that $Y = 1{,}200$ ft, V is equal to $1{,}200/3 = 400$ ft.

The horizontal span supported by the intermediate structure $= 1{,}200/2 = 600$ ft. The horizontal component of load per conductor $= 600 \times 0.492 = 295.2$ lb. The vertical component of load supported by each insulator string on the intermediate structure $= V \times w_{vo} = 400 \times 0.5181 \doteq 207.2$ lb. The tangent of the angle of insulator swing $\theta = 295.2/207.2 = 1.4247$. $\theta = 54°56'$. (See Fig. 67*B*.)

It is simple to construct a curve similar to this for any particular kind and size of conductor. Several curves, each based on a different wind pressure, may be plotted on the same sheet if desired. The simplest method is to assume different angles of insulator swing and compute the ratio Y/V. For example, assume the deflection to be 45 deg. The tangent of 45 deg equals 1.00, from which $w_{vo} = 295.2/1 = 295.2$ lb. $V = 295.2/0.5181 = 570$ ft. $Y/V = 1{,}200/570 = 2.11$, which corresponds to 45 deg on the curve. Other values of Y/V are derived in the same manner, and the curve may be plotted from them.

The values of Y/V and the angle of insulator swing from which the curve was plotted do not take into consideration the weight of the insulators. The deflection of the insulator string will be less than that given by the curve when the weight of the insulator assembly is included. Because there is considerable uncertainty regarding wind pressures, however, it is well to neglect the effect of the weight of the insulators. The error is on the safe side, thus providing a slightly better factor of safety.

When it is desired to consider the weight of the insulators, the curve may still be used, with little additional calculation. Assume the insulator assembly to have a weight of 90 lb. To use the curve, convert one-half the weight of the insulator assembly into its equivalent length of one conductor, and add this to the previously computed value of V. In this case, $90/2 = 45$ lb and $45/0.5181 = 87$ ft. V now equals the equivalent of $400 + 87 = 487$ ft. Y/V is now equal to

$$1{,}200/487 = 2.46$$

The angle corresponding to this value on the curve is seen to be approximately 49°30′.

The generally accepted equation from which the tangent of the angle of deflection is derived takes into account one-half the weight of the insulator assembly, which is added to the weight of one conductor in the vertical span. For example,

$$\tan\theta = \frac{295.2}{207.2 + 45} = 1.1705 \quad \theta = 49°30'$$

which checks the value derived from the curve.

There is no definite temperature at which insulator swing may be said to be a maximum. However, for a given wind velocity, the lower the temperature the greater will be the deflection of the insulators, assuming, of course, that the condition is as indicated by Fig. 67*A*, where the intermediate structure is lower than either of the adjacent structures.

A study of meteorological data for the entire country, covering a long period of time, reveals that approximately 65 percent of the days on which the wind velocity exceeds 40 mph occur during the six colder months, that is, November through April. Approximately 60 percent of the days with high wind velocity during this 6-month period occur in February, March, and April, with the highest number of days falling in March. It is therefore evident that high wind velocities occur most frequently at comparatively low temperatures.

From a theoretical standpoint, it would no doubt be advisable to determine the vertical span V somewhere in the 30 to 60° temperature range, depending to a certain extent upon the loading district. From a practical point of view, however, this is not convenient, because sag templates are not generally made to include conductor sags except at maximum and minimum temperatures. On the other hand, it is more convenient to locate the low points of the conductor with the cold curve. This

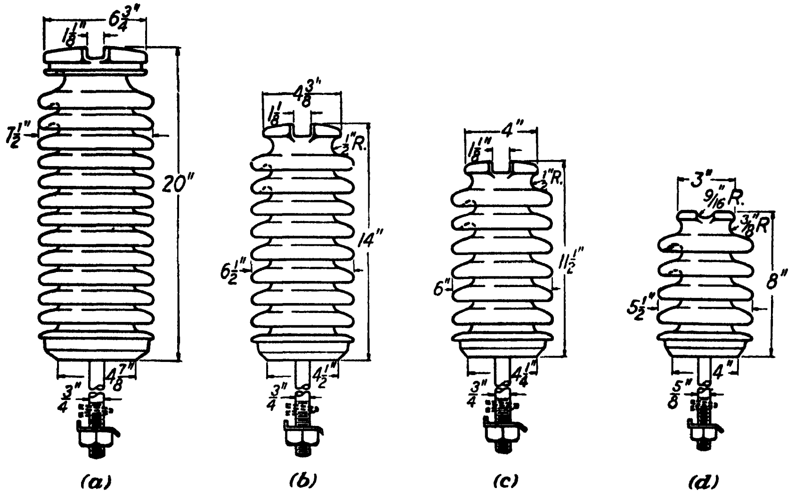


FIG. 68.—Line post insulators. Nominal line voltage: (a) 66 kv, (b) 45 kv; (c) 35 kv; (d) 25 kv. (*Courtesty of Lapp Insulator Company.*)

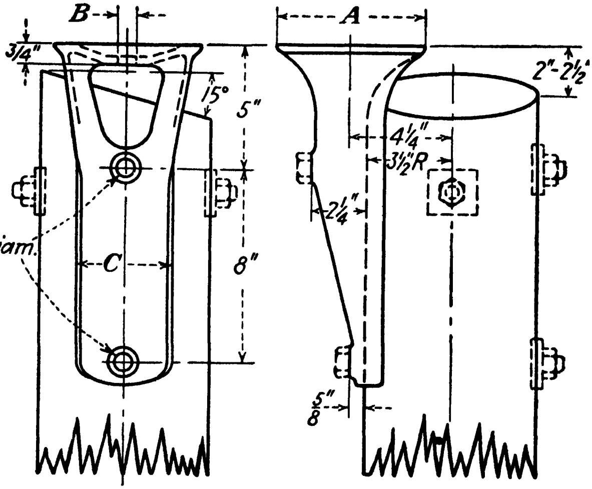


FIG. 69.—Pole-top bracket for mounting line post insulators. This bracket is made in three different sizes to accommodate the full range of line post insulator sizes. (*Courtesy of Lapp Insulator Company.*)

will give a higher value of Y/V, but the insulator swing at $-20°$ will be approximately equal to the swing at 60° with 80 percent of the wind velocity. It may, logically be concluded, therefore, that the cold curve, combined with the lower wind pressure, may be used for the determination of insulator swing, since it is unlikely that high wind velocity will occur at extremely low temperatures.

Line Post Insulators. *Mechanical.*—Line post insulators, illustrated in Fig. 68, have distinct advantages over the conventional pin-type insulator from a standpoint of construction and susceptibility to damage from mechanical attack. For example, the line post, in the same voltage class though longer than the well-known conventional pin-type insulator, is much smaller in diameter. This facilitates double-pole top mounting and double-arming (see Fig. 71*b*). This cannot generally be done with conventional pin insulators of 35-kv rating and above, since the diameter is such that double-arming plates or some method of increasing the distance between the cross-arms is required to prevent the petticoats of the two insulators from fouling.

The additional height of the line post over the conventional type, though making it susceptible to greater cantilever action, serves a useful purpose in overbuild jobs. The extra height often makes it unnecessary to replace a given pole with a pole 5 ft longer to provide the necessary vertical clearance from the conductors of other circuits on the same pole.

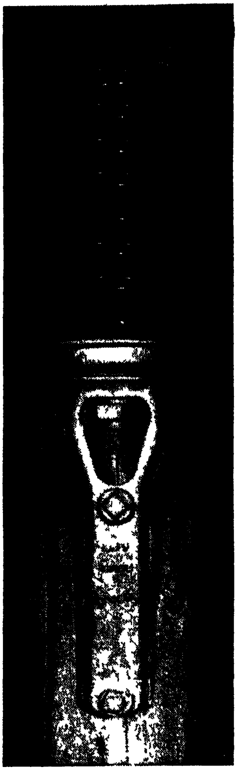


FIG. 70.—Pole-top mounting of line post insulator. (*Courtesy of Lapp Insulator Company*).

The line post is designed and constructed to withstand severe impacts from stones and bullets with a minimum of damage. A direct hit with a rifle bullet may break off considerable of the porcelain but will generally not break away a sufficient amount to cause flashover and a line outage.

A close-up of a single line post pole top mounting is shown in Fig. 70. Detail of the pole-top bracket is shown in Fig. 69.

Radio Interference.—Line post insulators in the 66-kv nominal voltage rating are practically free from radio interference, without special treatment, up to approximately 115 percent of the operating voltage. The spread between the operating voltage and that at which radio interference might be expected to start is appreciably greater for line posts having voltage ratings below 66 kv.

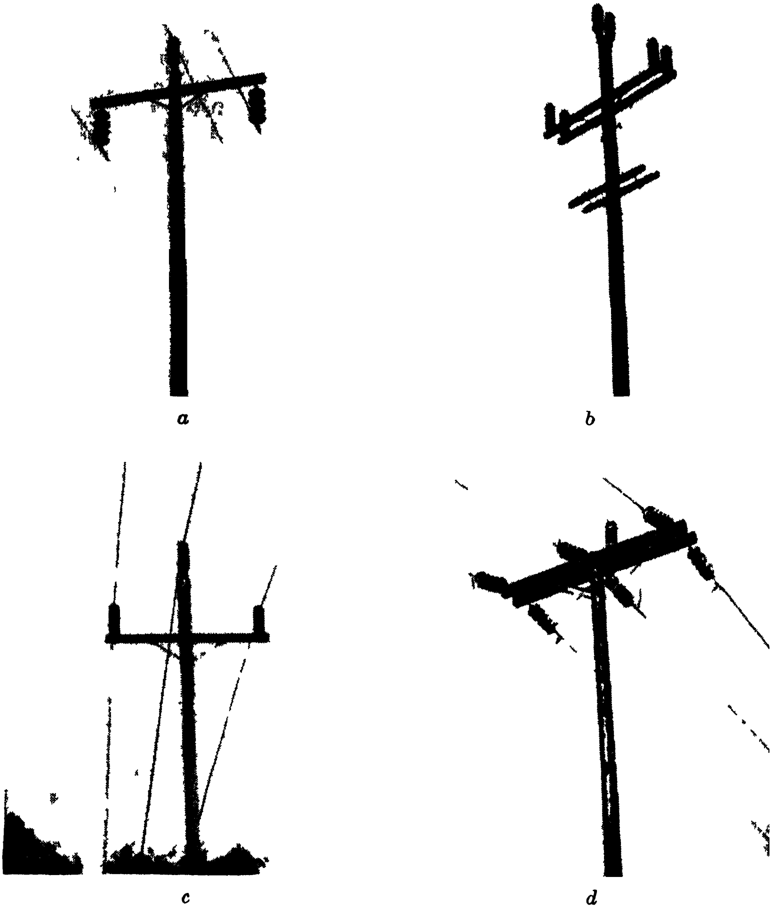


Fig 71 —Modern 66-kv single-pole line construction (*a*). Tangent structure with two suspension strings of insulators and one-line post insulator (*b*) Double-line post-angle construction (*c*) Tangent structure with three-line post insulators. (*d*). Balanced dead-end construction. Middle conductor dead-ended on through bolt with loop carried over on line post insulator.

The clamp-top line post, shown in Fig. 72, embodies practically the same mechanical and electrical characteristics as those of the same voltage rating shown in Fig. 68. This insulator, as will be seen, embodies a specially designed built-in steel cap to provide for the assembly of a pivotal-type suspension clamp. This clamp, in addition to providing protection from radio interference, which might develop from a loose tie wire, provides a smooth streamlined effect for the conductor at the clamp attachment. The rocking effect of the clamp permits the conductor to assume a position such as to produce a minimum inequality in conductor stresses at the clamp. This is important where there is considerable difference in the length of adjacent spans or at the upper support of steep hillside spans where the conductor drops sharply away from the upper support.

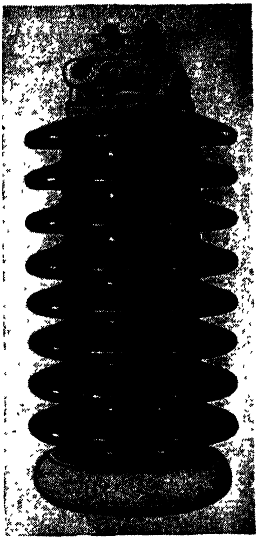


Fig. 72.—Clamp-top line post insulator. *(Courtesy of Ohio Brass Company.)*

In the preceding illustrations, devices and types of insulators that are well known or obsolete have not been included. Only the latest in design and manufacture have been illustrated, and

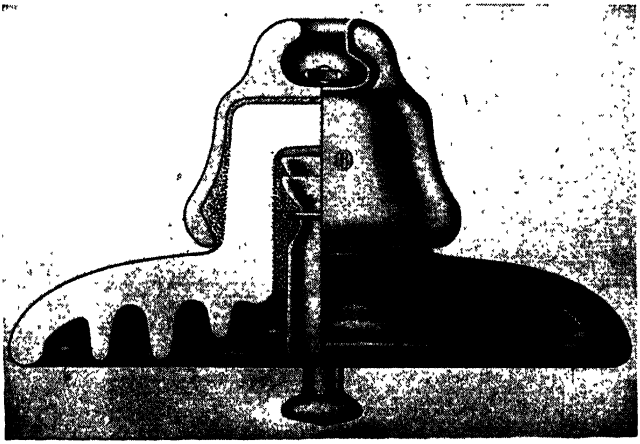


Fig. 73.—Suspension insulator. *(Courtesy of Ohio Brass Company.)*

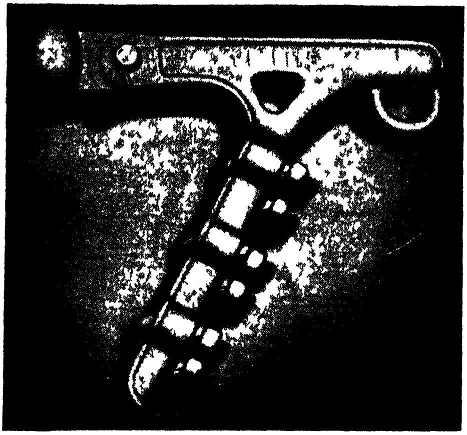


Fig 74 —Strain clamp

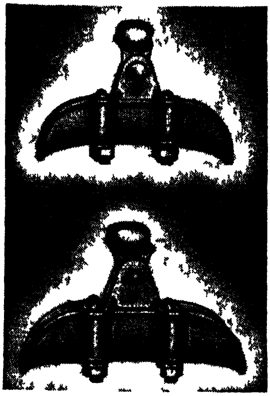


Fig 75 —Suspension clamps (*Courtesy of Ohio Brass Company*)

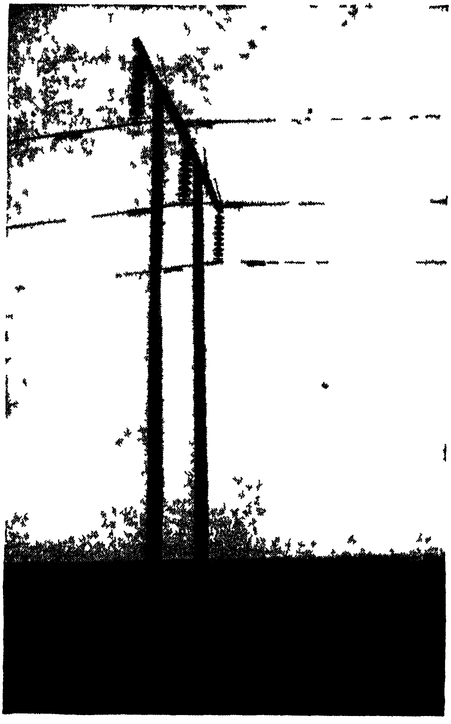


Fig. 76 —Stockbridge vibration-damper installation at 110-kv H-frame (*Figs* 76 *to* 80 *Courtesy of The Aluminum Company of America*)

no conventional pin-type insulators are shown. This type, in the various voltage classes, is so well known that further illustration would serve no useful purpose.

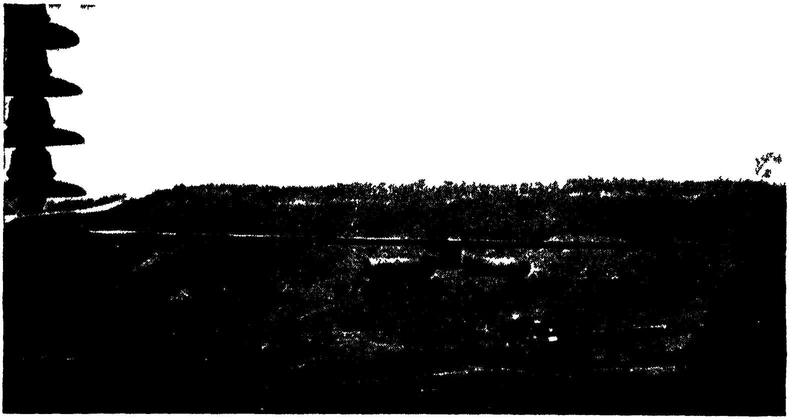


FIG 77 —Close-up of Stockbridge damper installed on a conductor

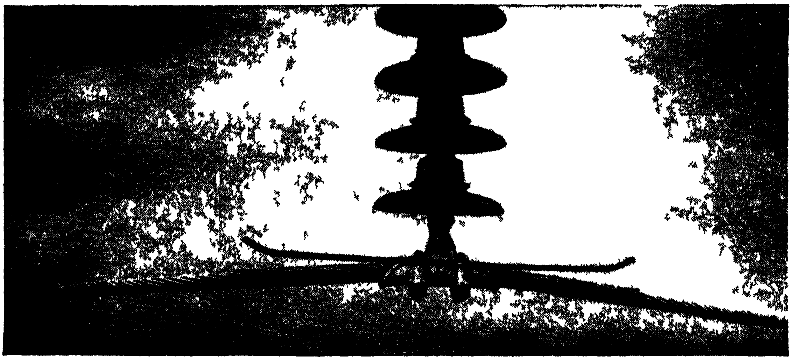


FIG 78 —Close-up of tapered armor rods installed on a conductor at suspension insulator assembly

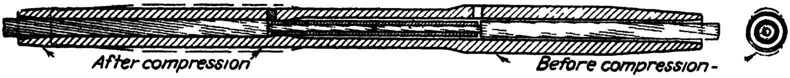


FIG 79 —Compression joint for ACSR

Steel Tower Lines.—In transmission lines with a voltage in excess of 154 kv, steel is almost universally employed for supporting structures. It is perfectly feasible, however, under favorable conditions, to use wood-pole construction for lines up to and including 220 kv, and no doubt this will be done to an increasing extent in certain sections of the country. It is readily apparent that steel has decided structural advantages over wood for double-circuit high-voltage lines. Steel can be fabricated more readily into shapes for supporting a circuit on each side of the tower, for which the over-all width will be approximately the same as for a single-circuit H-frame line. The excess height required for two circuits presents no difficulty with steel-tower construction, whereas it may be a difficult problem with wood poles. In general, spans will approximate the same lengths for a double-circuit steel-tower line as for a single-circuit wood-pole H-frame line of the same conductor and voltage.

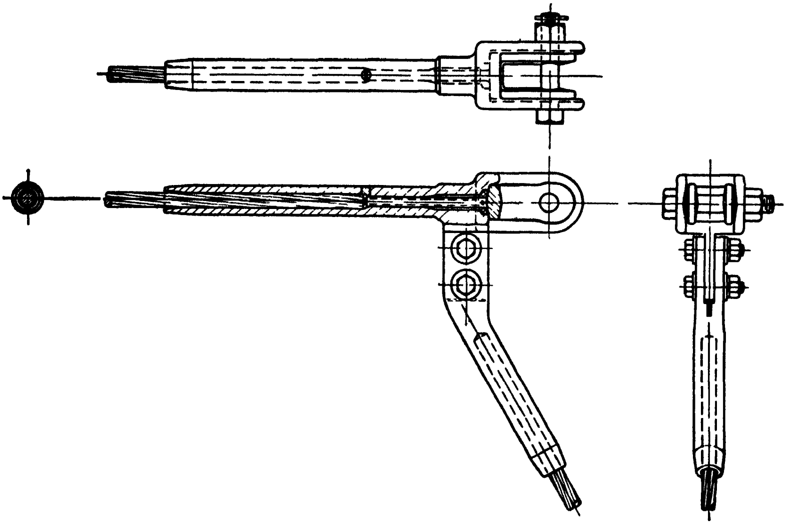


FIG. 80.—Compression dead-end body for ACSR.

As a result of operating experience, tower design has undergone considerable change in recent years in regard to the relative positions of overhead ground wires and line conductors. Modern towers, as will be seen from Figs. 86 to 88, make provision for greater vertical clearance between the overhead ground wires and the top conductors than did most earlier designs. It will also be seen that the horizontal separation of the two overhead ground wires is approximately the same as that between the line conductors on the middle, or longer, crossarm of double-circuit towers. This is an important feature, since it provides more effective shielding of the conductors. It should be particularly noted that the maximum vertical angle between each of the overhead ground wires and either of the three line conductors on the same side of the tower is very small.

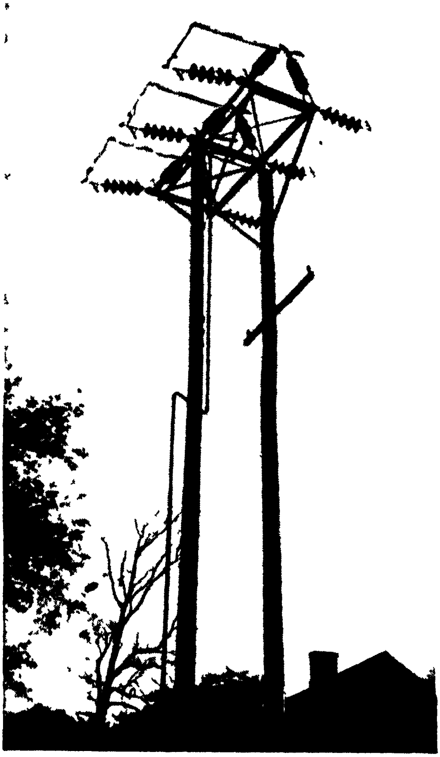


FIG. 81.- A 66-kv sectionalizing-switch installation.

Transmission-line design engineers are usually not structural engineers and therefore are not required actually to design the towers themselves. It is the responsibility of the transmission designer, however, from information furnished by the tower manufacturers as to test loads, etc., to determine the type of tower to be installed at any given location to give the required strength.

To facilitate installation on side hills and uneven ground, towers are provided with variable leg arrangements. Single leg extensions of 5, 10, 15, and 20 ft may be supplied when required. For turning large angles (see Fig. 88), square-end crossarms are provided. The conductor is anchored to the corners of the square end of the arm on the outside of the angle. This provides the necessary clearance from the corners of the tower and conductors.

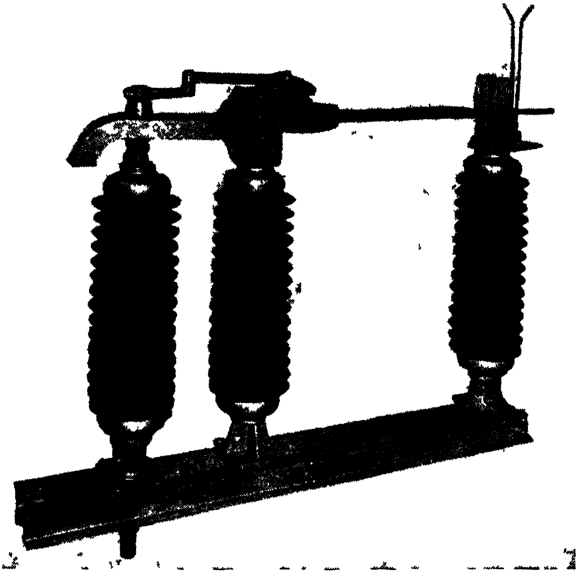


FIG. 82.—One phase of 138-kv line-sectionalizing switch. (*Courtesy of James R. Kearney Corporation*)

Several types of tower footings, or anchors, are available. Tower manufacturers, in general, have their own footing designs to meet the requirements for any type of soil that may be encountered. The footings may be depended upon to develop the required holding power when properly installed in the kind of earth for which they are intended. The type of footing to be used for any particular line will depend largely upon the manufacturer who supplies the towers.

When the required tower height cannot be obtained by the use of single leg extensions, a square extension may be installed below the standard section. With some types of towers, however, neither the standard section nor the square extension is designed to be attached directly to the footing legs. Single leg extensions are then required. Square extensions are generally supplied in 10-, 15-, 20-, and 25-ft lengths. Obviously, since the tower leg batter is of constant slope, the higher the tower, the greater will be the spread of the legs at the ground line.

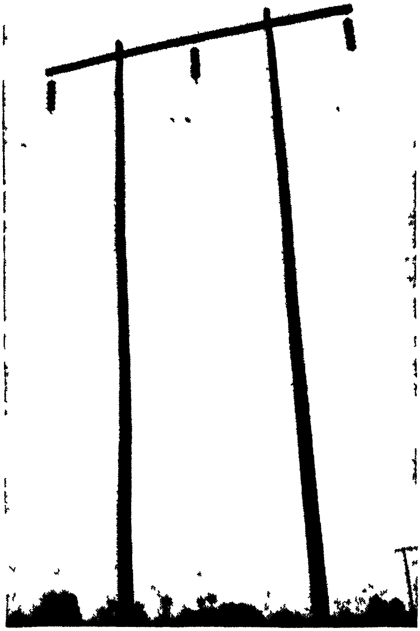


FIG. 83.—A 110-kv H-frame suspension structure.

For the benefit of the construction forces, it is necessary to follow a definite procedure in numbering the tower legs. The method indicated on the section of profile in Fig. 89 is recommended. The legs are numbered from left to right around the tower from one direction throughout the length of the line, as indicated by the arrow on the profile. The type of tower; the length to the lower crossarm, called "tower height"; the square extension, if any; and the type of footing are given. The single leg extension required at each corner of the tower is also given. Thus $\overset{+}{5}$ indicates that a 5-ft leg extension is required at tower leg No. 1. The letter *G* indicates that a grillage footing is required.

Since it is impractical to attempt to determine the amount of rock and various kinds of soil that may be encountered, especially in mountainous sections of the country, it is well to place an order for a few extra anchors of various kinds to preclude the probability of delay in completing the project on account of a shortage of certain kinds of material.

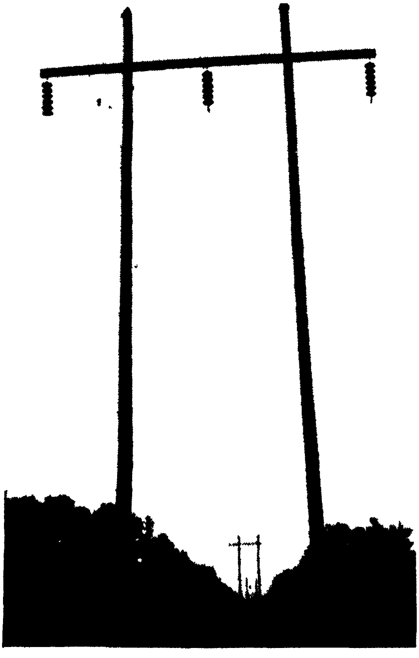


FIG. 84.—A 110-kv H-frame with overhead ground wires.

Because materials must be ordered in time for deliveries to be made before the approximate date set for beginning construction, some method must be devised for determining the tower heights and various leg lengths from the profile. To determine the length of leg extensions with a fair degree of accuracy, the following method, which is comparatively simple to use, may be applied:

Take, for example, the type K suspension tower in Fig. 86.

The distances from the center of the tower to the leg, across the face of the tower, for the standard section, and various leg lengths, are approximately as follows:

Standard section	11.6 ft
5-ft leg	12.3 ft
10-ft leg	13.1 ft
15-ft leg	13.8 ft
20-ft leg	14.6 ft

Plotting and Tower Templates.—Refer to Fig. 90, which represents a plotting template. The graduated scales at the sides of the template represent elevations on the 45 deg diagonals above and below the horizon taken on the survey line at the tower station. The sides of the template also represent the vertical scale of the profile.

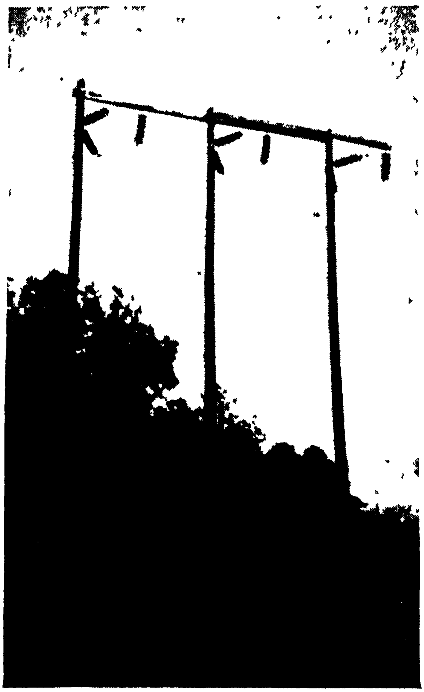


Fig. 85.—A 110-kv three-pole angle structure.

Plot ground-line slopes a, b, a', and b' at proper elevations as referred to the horizon. aCb represents the profile of the ground line through the center of the tower on the survey line. a' and b' represent elevations above the horizon 13.1 ft out and at right angles to the survey line. When working from the profile with side-hill elevations taken on only one side of the survey line, it is necessary to assume uniform slope of the side hill and to plot a'' so that $a' - a = a - a''$. Also plot b'' so that

$$b' - b = b - b''.$$

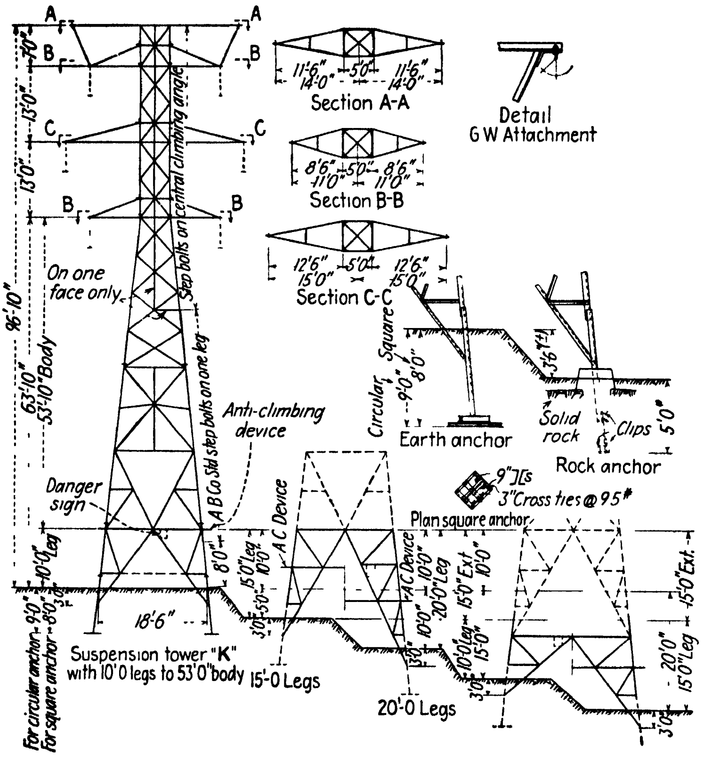


FIG. 86.—Type K 132-kv suspension tower. (*Figs.* 86 *to* 88, *Courtesy of American Bridge Company.*)

This will generally give the approximate elevations on the 45-deg diagonals.

In order to determine the length of leg required at each corner of the tower, it is necessary to use a tower template in conjunction with the plotting template. The tower template is shown by Fig. 91. The sloping sides of the tower template represent the hip batter of the tower legs, *i.e.*, looking diagonally through the tower. The scale is the same as that of the plotting template.

By means of the sag template (Fig. 55), determine the location of the tower and height to the lower crossarm, making allowance for the length of the insulator string, to provide the necessary ground clearance of the conductor. The tower height and length of the insulator string are known and will fix the location of the point *O* with respect to the top of the tower template, to which when added the tower height, will give the total height from *O* to the lower crossarm.

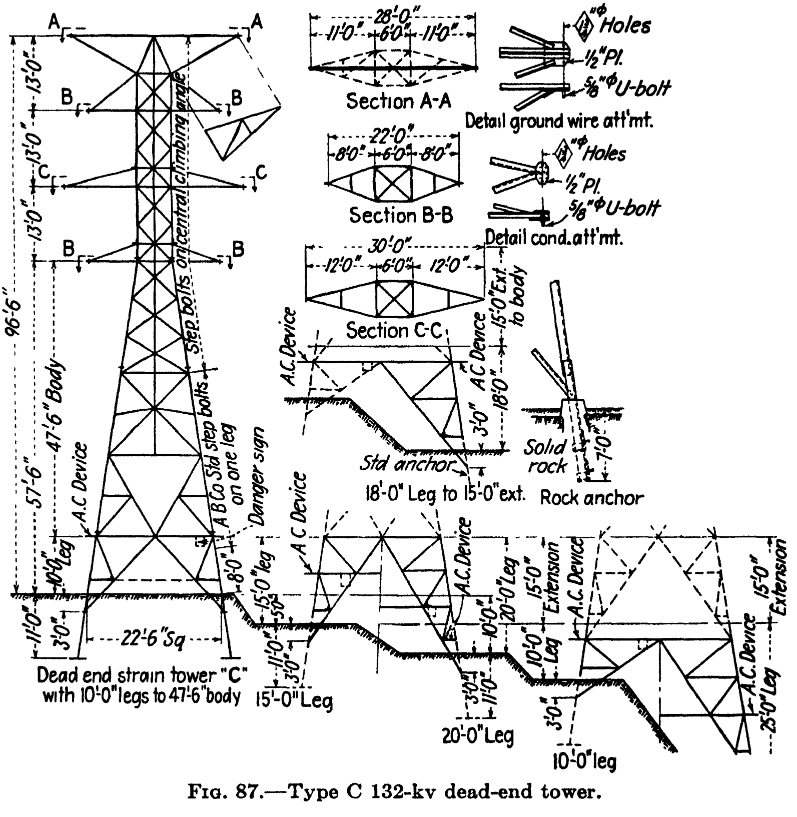


FIG. 87.—Type C 132-kv dead-end tower.

The next step is to superimpose *O* of the tower template over *C* of the plotting template (see Fig. 92), from which the length of legs to compensate for the side-hill elevations will be determined by the points where a', b', a'', and b'' fall on the tower template. The leg extensions may also be determined by transferring lines Ca', Cb', etc., to the tower template, using point O as origin.

It will be necessary in some cases to shift the tower template vertically to obtain the best average footing depth. This vertical shifting is limited, however, since a footing leg should not extend more than 1 ft above the ground. The tower should not be staked until it is ready for construction, and levels should be run to establish the slopes accurately.

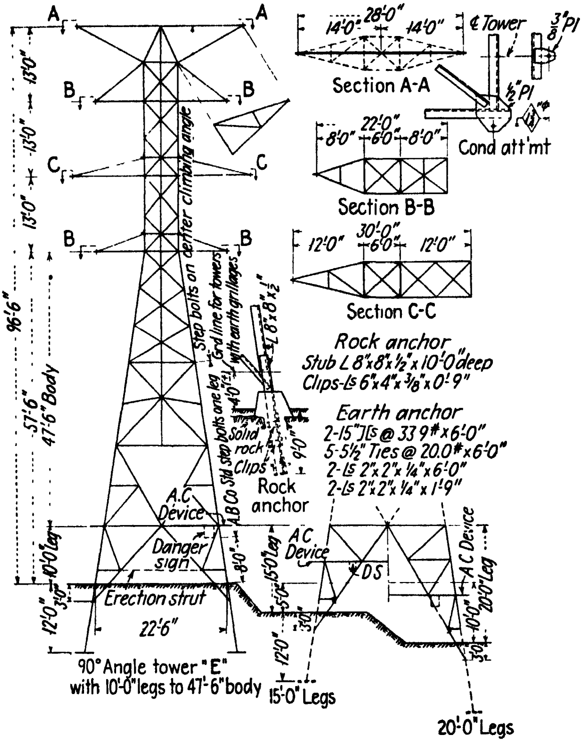


FIG. 88.—Type E 132-kv 90-deg angle tower.

With white background for the plotting template and a transparent material, such as tracing cloth, for the tower template, one set of templates may be used for several towers. The type of tower, length of legs, etc., should be recorded and

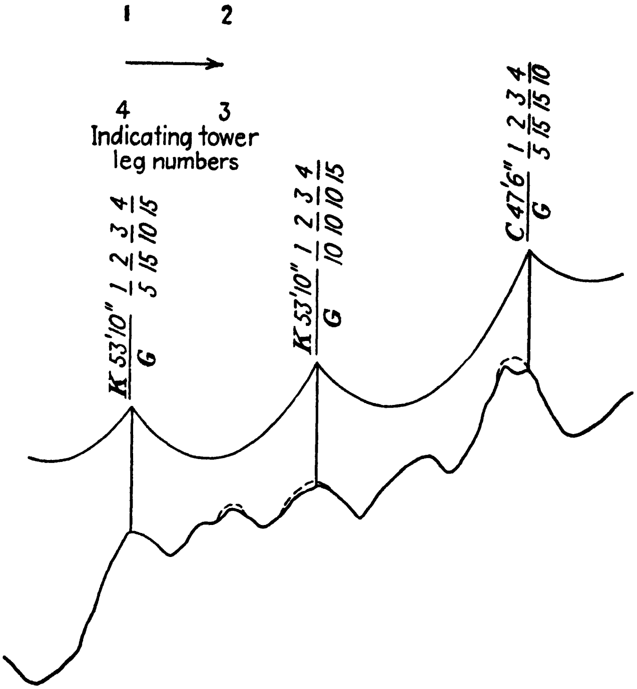


FIG. 89.—Indicating type of tower and footing, length of standard section, and leg arrangement.

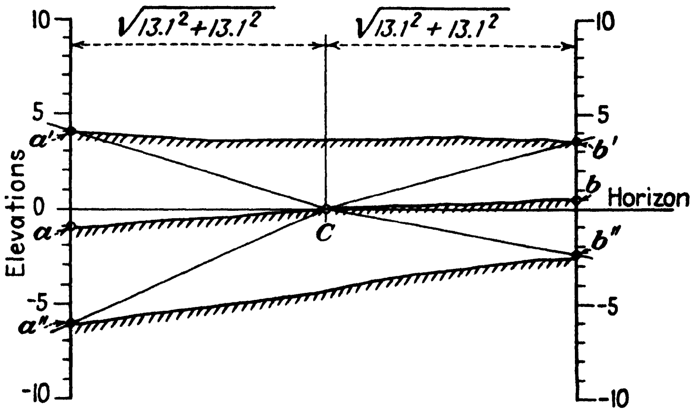


FIG. 90.—Plotting template.

transferred to a form especially prepared for the purpose. There will then be one of these forms for each tower in the line.

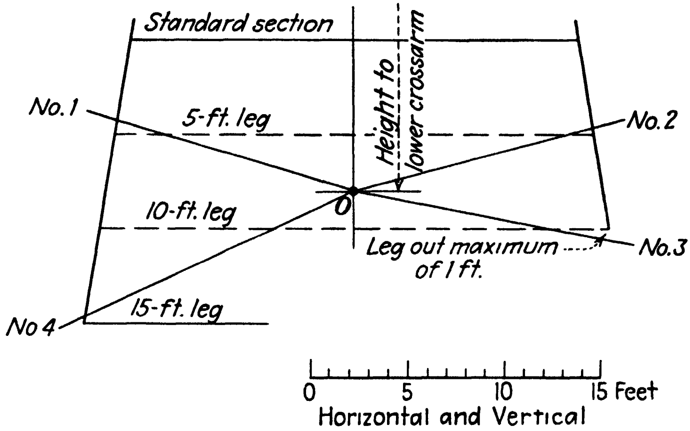


FIG. 91.—Tower template.

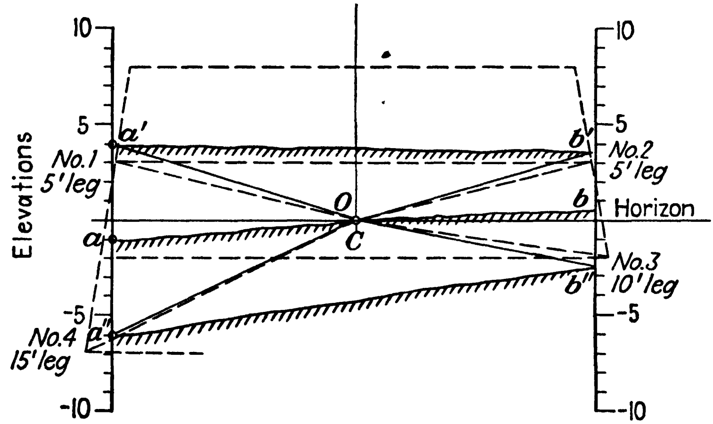


FIG. 92.—Plotting template (full lines) having point O of tower template (dotted lines) superimposed on point C demonstrating method of determining tower leg lengths.

The above discussion gives a general outline of procedure that can readily be followed. However, a considerable amount of plotting to a workable scale will be required where extensive side-hill elevations are encountered.

CHAPTER XII

CONSTRUCTION OF WOOD-POLE TRANSMISSION LINES

In the early stages of transmission-line construction, lines were generally of comparatively low voltage, short span, and small conductor; and their erection required a minimum of effort. Teams were used for wire stringing, transportation, and deliveries of materials, and only a small capital investment in tools and equipment was needed. During periods when little or no line work was in progress, the teams could generally be used for maintenance and other work. Construction of transmission lines was almost always performed by company forces.

As time went on and the expansion of utility systems began in earnest, more and heavier lines were necessary; and more and heavier equipment was required for their construction. Fortunately, about this time, automobiles and trucks became available, and the equipment problem ceased to be a matter for serious concern. It was difficult in those days to build lines fast enough to keep pace with the demand for extensions; consequently, there were no extended periods of time between construction projects. Equipment could therefore be used to its fullest advantage.

About fifteen years ago, contractors specializing in transmission-line construction began to replace company forces, and now practically all jobs of major importance are let to contractors. This works to the advantage of the owners, even though the construction cost may be somewhat increased. The tools and equipment required to carry on system expansion require a substantial investment, and much of the equipment may remain idle between construction projects. On the other hand, a contractor's only source of revenue is from the use of construction equipment, which he must be able to utilize to its fullest advantage.

Most transmission-line construction contracts are negotiated on a unit cost basis rather than for a stipulated amount for an integral part of, or the entire project. This method protects the sponsors of a given project, since the contract provides that payment will be made only for work actually performed, at a stipulated amount per installed unit. This usually applies to installation cost only, for materials are generally furnished by the sponsors.

There is considerable rivalry between the various contractors; and, since work is done on a unit cost basis, the time element is important. A contractor's cost is virtually fixed per unit of time, regardless of the number of units installed. It is therefore obvious that, the more work done in a given time, the greater will be the profit per installed unit. A successful contractor provides himself with good equipment and maintains that equipment in good working order at all times. He can then reduce lost time to a minimum.

Clearing Right of Way.—It is desirable to have all right of way for a given line secured before releasing the job for construction. Right of way can usually be acquired for comparatively short lines before starting construction; but, for lines of any considerable length, it is often not possible to acquire all right of way in advance without unduly delaying the beginning of construction. The first actual work to be done on a transmission-line project is clearing the right of way. The width to be cleared will depend on the type and location, the voltage, and the continuity of service required of the line. This may range from 25 to 100 ft or more on each side of the line. Right-of-way clearing must generally be done under quite rigid specifications as regards height of stumps, disposition of logs and brush, etc.

The clearing crew will usually consist of not less than 10 and not more than 16 men. The size of the crew required will depend largely upon the width of right of way to be cleared, amount and size of timber to be cut and disposed of, the time allotted for this phase of the work, and the number of men available. In general, the maximum number of men that can be used to advantage in a right-of-way clearing crew, where brush must be burned, is about 16 men, consisting of the following classifications:

1	foreman
1	saw filer and tool sharpener
12	men cutting, trimming, and piling
2	men burning brush

Where roads are in close proximity to the line and cities or towns where room and board may be obtained are within reasonable distances, it usually will not be necessary to set up camp for housing and boarding the men. In some cases, especially between crop seasons, farmers residing in the community are glad to have the opportunity of making a little extra money by working on such projects. They will travel to and from their homes, and the subsistence problem will then not generally be important. This class of labor, more often than otherwise, is is found to be quite satisfactory. Men who have had logging-camp experience make the best class of labor for right-of-way clearing projects, however.

Clearing transmission-line right of way is a hazardous occupation. Only a comparatively small number of men should therefore be assigned to one foreman or sub-foreman. Where the growth of underbrush is heavy, there is danger of the workmen unknowingly getting too close together, with the liability of one workman's being injured by the tools of another. It is therefore imperative that the men have close supervision. Men cutting underbrush should be spaced out well across the right of way and well in advance of the men cutting trees, and the men trimming and piling should keep well to the rear of the men cutting trees, so that in neither case will there be danger from falling trees. Obviously the number of men that can work efficiently will be governed by the width to be cleared. To facilitate trimming, sawing to length, handling logs, and disposition of brush, trees of a particular group should be felled in one direction and parallel with the right of way.

Equipment Required.—The equipment required on a right-of-way clearing job will generally consist of the following:

1	truck for transportation of men and tools
4	crosscut saws
4	sledge hammers
6	wedges
12	axes
6	bush axes

Suitable size of rope or cable, and tools for sharpening saws and axes

For heavy logs, a tractor-mounted revolving crane may be used to advantage. This type of crane operating from the center of the right of way virtually throws the logs to the edge of the right of way. For example, when it is desired to pile the logs along the edge of the right of way, the crane picks up the log and revolves the boom in one direction, giving the log a swing. It then reverses quite rapidly, starting the swing in the opposite direction. At the proper instant, the cable is released, throwing the log well out. This method of handling logs requires an experienced and alert operator.

For medium-sized logs, or when a crane is not available, the next best method is to use a two-wheel logging cart and teams that have been broken in to logging.

FIG 93

Trees within the construction and patrolling limits must be cut so that the stumps will not interfere with tractors and trucks. Whether it is necessary to cut all trees and underbrush for the entire width of right of way will depend on the width of right of way acquired, the type of line, and the height of trees. Danger trees, whether inside or outside the right-of-way limits, should be cut down.

No fixed rule has been established as to just what constitutes a "danger tree." Some engineers define it as a tree whose height is equal to the horizontal distance from the nearest conductor. Others define it as a tree whose height plus 10 ft is equal to the horizontal distance from the nearest conductor. In any event, all dead trees and trees that lean toward the line and would strike the nearest conductor when falling should be cut down. The importance of the service required of the line and provisions contained in the right-of-way easements may have considerable bearing on the subject. Near coast lines where high winds are prevalent, and in storm areas, it will usually be necessary to clear a greater width than in less severe storm areas. Also, the width to be cleared will be more on the uphill side of a line than on the downhill side.

Figure 93 shows a transmission-line right of way after the trees and underbrush have been cut but before the logs have been removed. Figure 94 shows a section of the same right of way after the logs have been removed. Figure 95 indicates a heavy swamp-clearing job. It is obvious that an enormous amount of work is required to clear and dispose of the brush and logs. It is well to remember that clearing right of way often requires as many man-hours as all other operations in building a transmission line. The time required depends on the size and type of trees, their density, and the type and density of underbrush. Terrain is another controlling factor. Swamps, particularly those with few or no access roads, add materially to the time required. Inaccessible mountainous country also adds to the time. The disposition to be made of brush and logs may also have considerable bearing on the time element.

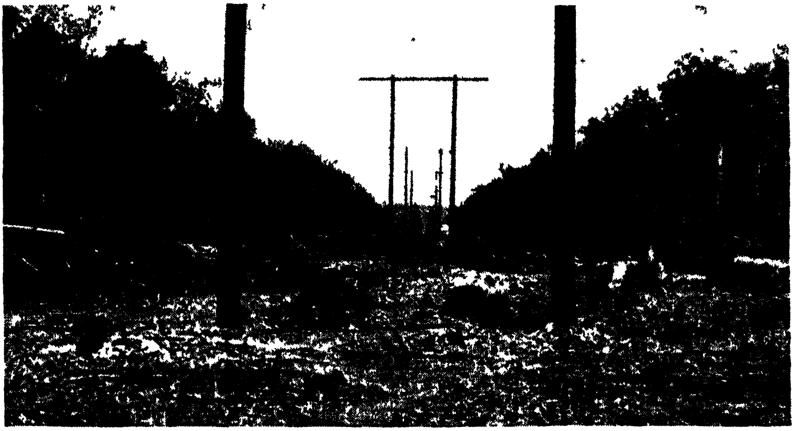


FIG. 94

With well organized and managed right-of-way clearing jobs, the time required will vary over a wide range. In level or rolling country, with good access roads, comparatively light clearing can be done for less than 100 man-hours per acre. On the other hand, in swamps or mountainous country, inaccessible to roads, with heavy timber and underbrush, 600 man-hours per acre is not uncommon.

Clearing right of way is often the operation that determines the rate of progress of the construction work. Clearing should start well in advance of the actual line construction, if the construction is to be carried on economically and at a rate of progress usually maintained by competent contractors or company forces.

Fig. 95.

Line Construction.—The construction of a transmission line is essentially a transportation and rigging job, until the actual wire stringing starts, when transportation again becomes a major item.

To expedite the construction and to maintain an economical cost level, equipment and methods must be selected with care.

There are two general methods that may be used in the construction of a transmission line.

1. A minimum amount of equipment may be used, and the same gang may be used for all the different operations on the job. A crawler tractor, with winch; a truck and trailer; and line trucks in sufficient quantity to serve the gangs are the minimum amounts of equipment that should be used.

It would not be possible to set up the average job on a fast production schedule with the amount of equipment listed above. The general method of procedure would be about as follows: The pole truck and trailer, with sufficient men to handle the poles, would be used to haul the poles from the point of storage to the road point nearest the point of erection. The tractor would be used to take the poles from this point to the stake. The crossarms and other material would be hauled and carried into the line in the same manner.

It would be necessary to dig holes by hand, and the setting would probably be done with a boom on pole-raising derrick using the same tractor that was used for bringing poles and other material from the road to the structure locations. After all or a part of the structures are set, the wire stringing would be started. Unless teams or other methods for pulling out the wire were used, it would be necessary to use for this operation the same tractor that was used for moving poles and other materials from the road to the stake, and for setting the structures. The individual operations to be followed for this type of construction will be given later in this chapter.

If this general plan of building is followed, the rate of construction must necessarily be slow, since sufficient equipment is not available to carry on all operations at one time, with one operation following closely behind the other.

2. Many transmission lines are being built by contractors who have sufficient equipment to carry on work at a much faster rate. Individual gangs are placed on each operation and furnished with separate pieces of equipment necessary to carry on their work. With enough small gangs, properly equipped, it is possible to carry one operation just behind the other. If the work is scheduled properly, the conductor stringing will be completed shortly after the last structure is set. It should be remembered, however, that the rate of construction with this method is a great deal faster than the rate at which right of way can be cleared. As previously pointed out, right-of-way clearing should therefore be started well in advance of the actual beginning of line construction.

The detailed descriptions here of the various operations and the various methods of handling these operations are the results of actual experience. In some cases, more than one method of carrying on an operation is described. It is necessary to determine in advance the methods to be followed on the line to be constructed. They will be determined largely by the length of line, the difficulties of the terrain, the amount of equipment, and the number of gangs available. In general, it is usually not desirable to attempt to build short lines on a production basis with separate gangs and equipment for each operation. It will usually be found more economical to build longer lines by this method.

Too much stress cannot be laid upon the necessity for always organizing and scheduling the work properly, regardless of the method to be employed.

Hauling and Handling Poles.—When an ordinary truck and trailer are used for hauling poles, a sufficient number of men to load and unload by hand will be required. The equipment needed will be axes, bars, sledge hammers, cant hooks, carrying hooks, bolt cutters, extra winch line, timber, skids, log binders, etc. Poles are usually unloaded from flat cars, or they may be hauled from an existing pole pile. The number of men required will be determined by the size of poles, the accessibility of the pole pile to the truck, and whether or not they are to be unloaded from cars.

In mountainous country with sharp curves in the roads, a side-loading truck equipped with a power-driven winch and loading bar is used by some contractors (see Fig. 96). With this type of truck, poles up to 60 and 65 ft can be handled from the ground and loaded and unloaded by a truck driver and two men.

Poles can be trucked to the road points nearest to the location on the line and from there are usually dragged into or down the right of way with teams or a caterpillar tractor. A tractor driver and two additional men can handle the poles.

In moving poles along the right of way to the structure locations, a two-wheel dolly or sleds may be needed to protect the poles in rocky country or, if the distance is great, in other types of country.

Distributing Other Materials.—Crossarms, hardware, insulators, and guying materials must be trucked out to the nearest road points and then moved into their locations. It is customary to sack, or box up, all the small material required for a structure and to mark it accordingly. Some of this material may be moved in by the same crew handling the poles, but it may be necessary to provide other crews for part of the distribution. The material is moved into the locations by teams, or tractor, on carts or sleds.

The importance of having every piece of material at the structure location in advance of the erection crews should not be overlooked.

For a transmission line, all or a considerable part of which is located in a mountainous country, the cost of transporting and delivering materials to many of the structure locations may be greatly in excess of the cost of erection. It is therefore apparent that the moving and handling of poles and other materials should be considered an operation of major importance.

FIG. 96.—Side-loading truck with loading bar.

Digging Pole Holes.—There are so many different conditions with which a hole-digging crew may be confronted, any one of which may require a change in method, that no definite procedure can be set up with the assurance that it may be adhered to throughout the length of a given line. A construction force must therefore be equipped for applying various methods to the digging of pole holes.

The method to be applied to the digging of pole holes will depend largely upon the composition of the soil and the length of line. Length of line may be the deciding factor as to whether hand or machine digging should be employed. A comparatively long line may make it advisable to use a hole-digging machine. For comparatively short lines, however, the use of a machine may not be justified. In some kinds of soil, holes cannot be left open for any length of time without caving. A hole-digging machine, equipped for pole setting, can usually be used to good advantage in such soil.

Fig. 97.—Machine for boring pole holes.

There are different types and makes of hole-digging machines. The type that is probably most commonly used is illustrated by Fig. 97. This machine is seen to be tractor-mounted and equipped with a derrick, or gin, for pole setting. It is important that holes be bored at the proper location. Tractor-mounted machines are readily maneuvered into position. It should be explained, however, that hole-boring machines equipped with a gin for setting poles are used primarily for rural-line work where comparatively short and light poles are installed. They are not generally adaptable for setting poles and structures on lines of transmission voltage. Hole-boring machines may be adapted for mounting on either a tractor or a four-wheel-drive truck.

By manipulation of a system of levers, the auger is raised and lowered as desired. It is run up or down along a vertical gear rack, indicated quite clearly in Fig. 97. The auger is withdrawn from the hole at frequent intervals and given a spin. It's thus cleaned by centrifugal action, of the earth that has been withdrawn from the hole. As shown, this piles the earth around the hole, and it is used later for tamping around the pole.

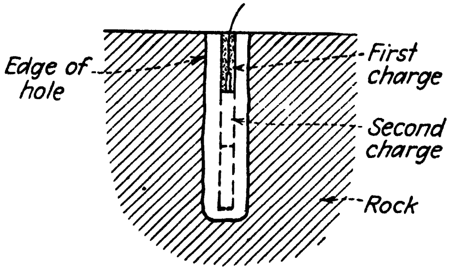


Fig. 98.

Where the soil consists of sand or of comparatively loose earth, the only tools required for hand digging are a shovel and spoon. With more firm and comparatively hard soil, a digging bar will also be required. One experienced workman should dig from one to about four holes per day, depending upon the nature of the soil and the size and depth of the holes.

Where the soil is a mixture of sand and water, it will be necessary to use sand barrels, or similar apparatus, to prevent the earth from caving. The barrel is worked down into the hole as the earth on the inside is removed.

Rock Holes.—When blasting pole holes in solid rock, it is not generally advisable to attempt to blast the hole to its final depth at one shot. On the other hand, it is recommended that the bore hole be put down about 18 in. and loaded with a charge of dynamite, designated "first charge," as indicated in Fig. 98.

All fragments are cleaned out, and another hole about 18 in. deep is drilled. A second charge is put in. This procedure is continued until the desired depth is reached.

Erecting Poles.—Little change has taken place in the last several years in methods of erecting poles insofar as the actual raising of the poles is concerned. The major difference is that, under present conditions, the pole, crossarm, and insulators are completely assembled and raised intact. The assembly work was originally done after the pole was in place.

The modern method is to hoist the pole by means of a boom mounted on the rear end of a truck body. A steel cable with a hook in the end is placed around the pole above its center of gravity; and the pole, with crossarm and insulators attached, is raised clear of the ground in approximately a vertical position. The pole is then swung into position directly over the hole and is lowered into the hole with the winch cable.

By means of pikes, the pole is straightened up, canted if necessary, until the crossarm is at right angles to the line. The pole is then lined up, usually by the crew foreman. The top is piked in accordance with hand signals; and the butt is shifted, when necessary, with a digging bar, to bring it into line. The hole should be sufficiently large to permit of shifting the pole for lining up and still to leave room all around for the tamping bars. When the pole is lined up ready for tamping, the ends of all the pike poles are pressed into the ground, to hold the pole in place while the tamping is being done. This is called "grounding the pikes." At least three tamps should be used for each shovel; otherwise the earth may be filled around the pole faster than it can be firmly tamped.

For jobs of any considerable size, it may be more economical to use compressed-air tamping. The air tamp can be handled by one man and is driven from an air compressor with a capacity of not less than 60 cu ft per min.

The earth should be piled up around the pole for several inches. The tamped earth will then be less firm than the undisturbed earth around the pole and will recede appreciably after a heavy rain. When the settling has taken place, the line should be gone over and back-filled, so that water will have a tendency to drain away from the pole.

When the insulators are of the suspension type, an effective and convenient method that may be employed to prevent insulator breakage due to violent swinging of the top of the pole, cable slippage, etc., is to leave the wood crate in which the insulator assembly was shipped in place until the pole is set. The crate is then detached and lowered to the ground.

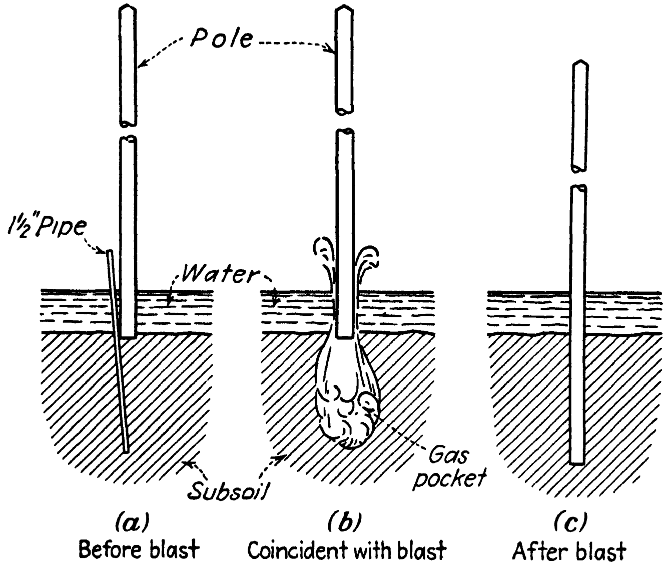
Men Required.—In a pole-erection job, the crew is necessarily divided into two parts, the assembly crew and the pole-setting crew. The assembly crew must work in advance of the pole-setting crew. Two men are required for assembling the pole, crossarm, and insulators, neither of which must necessarily be a lineman. For the actual setting of the pole, six men, in addition to the foreman, can work to advantage. These men will include a truck driver, one lineman, and four groundmen. The groundmen are used for making the hitches, manipulating the pole into place, and handling the pikes.

Setting Poles with Dynamite.—Where the soil is comparatively soft and more or less plastic in formation, poles may be set successfully by blasting the hole and setting the pole at one operation. Where water is standing at or near the surface and is not too deep, an excavation, the depth of which will depend upon the soil conditions, can be made at the pole location. Rope guy lines should be used for holding the pole erect in the excavation, since pikes are too short to be handled safely by the men while blasting and cannot be grounded and permit the pole to drop freely into the cavity made by the blast. For poles longer than 40 ft, as a matter of safety, to prevent the pole from getting out of control and falling, ½-in. blocks should be used on the rope guy lines.

When the pole has been raised, a 1½-in. pipe may be driven in the ground as indicated by Fig. 99*a*. The pipe will of course be partly filled with earth as it is driven down, but a ram may be used for forcing the earth out through the pipe at the bottom end. Then a charge of dynamite, primed with an electric blasting cap, is pushed into the pipe, to which should be attached about 75 ft of ordinary drop cord. The pipe should be withdrawn and all hands should be in the clear before detonation. Any of the well-known methods of detonation, or firing the charge, may be used. A convenient method, however, is to fire the charge from the spark plug of a car or truck, which should be placed not less than about 50 ft from the pole.

The explosion should create a pocket, or cavity, which will resemble somewhat that indicated by Fig. 99*b*. As the gases rush upward around the pole, they will carry a considerable amount of water and loosened earth with them, and the pole will settle into the cavity made by the blast. The pole will generally settle deeper when it is cradled, or rocked back and forth with the guy lines, just as the charge is fired.

When the subsoil contains a considerable amount of sand, the water rushing in from the surface and mixing with the sand will have a tendency to settle around the pole, and no further backfilling and tamping may be required. The pole may be to approximately specified depth, as indicated by Fig. 99*c*. It may, however, require additional backfilling and tamping.



(a) Before blast (b) Coincident with blast (c) After blast

FIG. 99.

The size of the charge will depend upon the nature and toughness of the soil. An experienced workman will generally know about the size of charge required to settle a given pole a given amount in any particular kind of soil. Forty percent dynamite is generally recommended for this class of work. If the explosion is too fast, there is danger of splitting the bottom end of the pole. If the charge is too large, there is danger of both splitting the bottom end and breaking off the top of the pole.

Another method of setting poles with dynamite is used quite successfully. Instead of resting on the ground where the hole is to be blasted, the pole is swung clear of the ground and held in this position with a quick releasing winch until the charge is ready to be fired. Just as the charge is fired, the winch cable is released, dropping the pole in the cavity made by the blast. The additional momentum obtained in this manner will generally settle the pole deeper than will the first described method. The pole should be held just clear of the ground and released very quickly at the proper time; otherwise, the effect of the added momentum may be lost.

In setting H-frames with dynamite, either one or both poles may be shot down at the same time. Since it is practically impossible to shoot both poles to the same depth, it will usually be necessary to jack up one of the poles in order to level the crossarm.

Erecting Wood-pole H-frame Structures.—Almost all H-frame structures are erected by raising the structure intact with some form of boom, hoisting derrick, or gin pole. Several methods are employed. Contractors and other construction forces are always on the alert for new and improved methods that will enable them to speed up the work and thus reduce the cost per unit of installation.

One successful method of erecting H-frame structures is known as the "falling gin method." It requires a tractor equipped with drum and cable, and a gin pole about 25 ft in length with a top diameter in the order of 5 in. The gin pole is rigged at the top with three cables of the required size and length. Preparatory to raising the structure, the tractor is maneuvered into position anywhere from 150 to 200 ft from the butt of the poles and approximately in line with them. Enough cable is payed out from the drum to connect with one of the gin cables.

The gin pole is raised and held in place with rope side lines. Two of the cables are equipped with a hook in one end and are attached to the poles just below the crossarm, with the gin leaning slightly toward the crossarm end of the poles. The other cable should be equipped with a loop in the free end for connecting directly to the cable from the tractor drum. Two butting boards are used, one in each hole, to act as a stop for the poles.

After the lines, which are used for holding the structure in position while it is being raised, are in place and properly manned, a signal is given to take up on the cable with sufficient tension to slide the poles against the butting boards. As further tension is taken in the cable, it begins to wind on the drum and pulls the top of the gin toward the tractor while at the same time elevating the top of the structure. When the top ends of the poles are approximately in line with the top of the gin and cable drum, the gin will have performed its duty. The top of the poles will be in such position that further elevation can be accomplished by the cable leading directly from the structure to the drum on the tractor. When this point is reached, the gin will swing free and

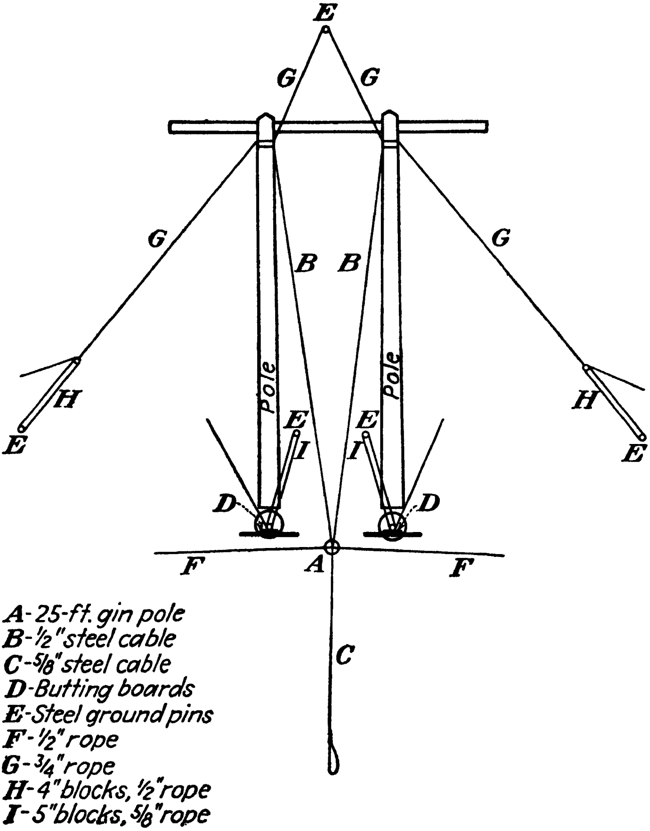


FIG. 100.— Hook-up preparatory to raising structure.

hang on the cable until the poles drop in their respective holes. When the cable tension is released, the gin will fall to the ground.

Equipment Required.—The equipment required for raising H-frame structures by the falling gin method is as follows:

1	tractor equipped with drum and cable
1	gin pole about 25 ft long and having a top diameter in the order of 5 in. The butt of the pole should be relatively large, cut from the woods, and should be of comparatively light weight. Black gum makes an excellent pole for this purpose.
5	1½-in. by 5 ft steel ground pins
2	pair 4-in. two-sheave blocks
2	pair 5-in. two-sheave blocks
100	ft ½-in. steel cable
60	ft ⅝-in. steel cable
300	ft ½-in. rope
100	ft ⅝-in. rope
400	ft ¾-in. rope

In addition to the above, other tools necessary for performing work of this nature, such as sledge hammers for driving the ground pins, steel bars for skidding the poles into place, shovels, and tamping bars, will of course be required.

Figure 100 indicates a schematic hook-up preparatory to raising the structure and indicates the position of the various items of equipment and the purpose for which they are used. The butting boards are 3″ × 12″—9′ and equipped with a piece of 1″ × 6″ board about 3 ft long fastened about 3 ft from the top to prevent it from dropping to the bottom of the hole, and an eye bolt for attaching the blocks.

Figure 101 shows the structure when it has been raised to a point at which the gin pole is just about to swing clear of the ground. The gin pole, rigging, and butting boards may be seen quite clearly, but the tractor is out of view. Figure 102 shows the same structure just as the poles are dropping into the holes. The gin is seen to be swinging clear of the ground. The tractor may be seen in the background.

A falling gin of the A-frame type makes the use of transverse side lines unnecessary; but the advantage to be gained in this manner will generally be more than offset by the extra weight and structural shape, which make it much less adaptable for handling and transporting from one structure location to the next. Since the gin must be unloaded and loaded again at each structure, ease of handling is an important feature.

A tractor used for raising H-frame structures should have ample power and be equipped for producing slow drum speed. It is often necessary to stop and start one or more times while raising the structure to make adjustments, etc., before the top

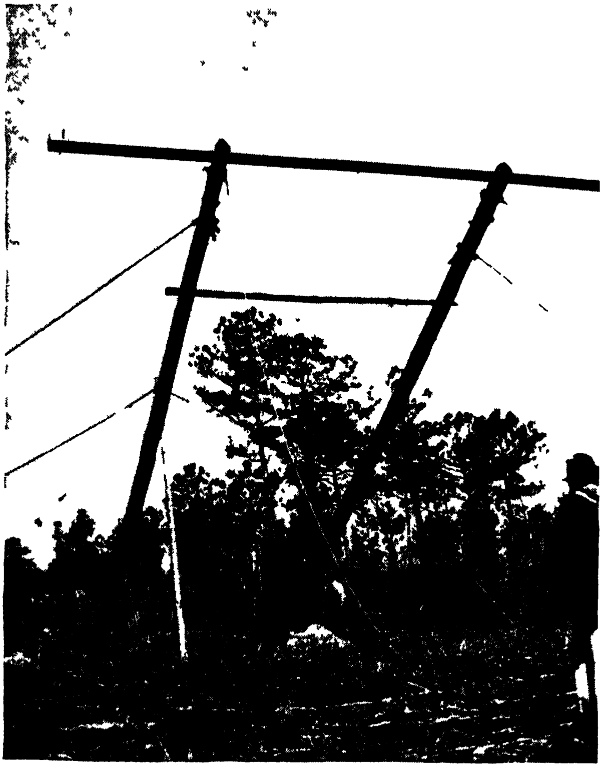


FIG 101.

ends of the poles have reached an appreciable distance from the ground. Excess power is then required for starting the load. For satisfactory performance and minimum strain on cables and equipment, the peripheral drum speed should not exceed about 30 ft per minute.

Some method of transporting the gin pole, butting boards, and other tools from one structure location to the next must be provided. A convenient and satisfactory method of doing this is by team and wagon. Speed is not essential, and a team can travel over places that are inaccessible to a motor vehicle. Also, less damage will be done to crops where the line crosses over lands that are in cultivation.

A great deal of time can be saved throughout the day if pikes instead of rope guy lines are used for holding the poles erect. This makes it possible to release the rigging, load it on the wagon or truck, and proceed, along with the tractor, to the next structure location. By the time the members of the crew who have been left behind have finished tamping and have joined the other members of the crew, preparations should be well under way for raising the structure. If this procedure is followed wherever possible, lost time is reduced to a minimum.

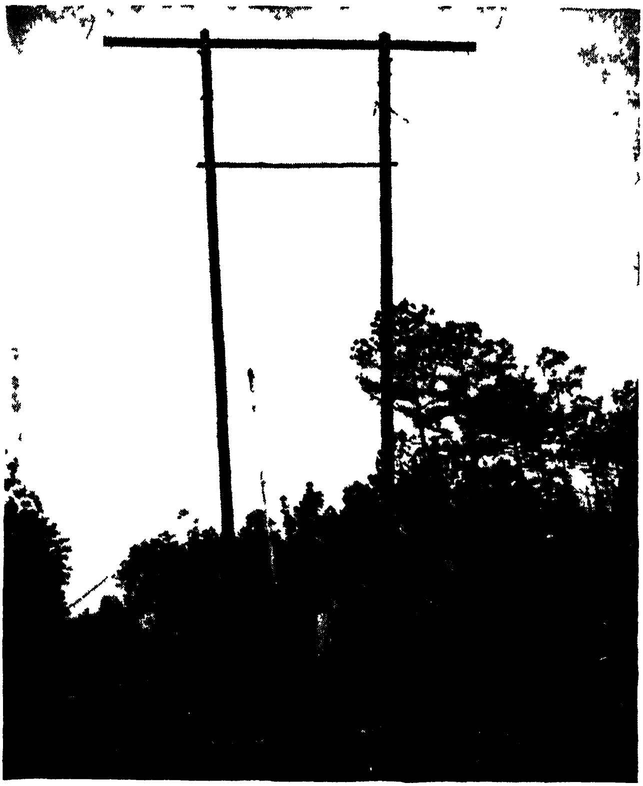


Fig 102

In addition to a truck for transportation purposes, the falling gin method requires only one piece of major equipment, the tractor. It also requires a minimum amount of preparation prior to the actual raising of the structure. With a well-trained crew of 10 to 12 men and fairly level right of way, a good average should be from 12 to 20 structures per day raised and tamped in ready for the guying and wire-stringing crews. The number of structures erected in a given length of time will depend largely upon the length and the class of poles.

The framing and assembling crew, which for H-frame work will consist of three to five men, depending on the size of poles and crossarms, precedes the erecting crew. When they arrive, they have only to erect and tamp in the structure.

Pole Derrick Method.—The method of erecting H-frame structures with a derrick has been in successful use for many years. The method consists essentially of the following:

The work of digging the holes and framing and erecting the structure is performed virtually at one operation by one crew. Framing the structure and attaching the guy lines and tackle are done by the crew and then the structure is raised intact by means of a tractor-mounted or tractor-drawn, tractor-operated derrick.

The major difference in this and the falling gin method lies in the fact that, instead of using butting boards and up-ending the poles (previously described), the structure is raised clear of the ground, maneuvered into position, and lowered into the holes.

Crew and Equipment Required. The crew and equipment needed for this operation will consist of about the following:

1 foreman
2 linemen
1 tractor and winch operator
1 digging machine operator
6 helpers

1 truck for transportation
1 tractor
1 hole-digging machine
1 derrick, which may or may not be tractor-mounted
Usual number and variety of small tools

The number of structures erected by this method in a given time will vary widely, depending on the length and class of poles. It will probably vary from a minimum of 4 per day with 60- or 65-ft class 2 poles, to a maximum in the order of 18 with 45-ft class 5 poles, varying between these values for other lengths and classes of poles.

Stringing Conductor.—Wire or cable used for transmission-line conductor is received from the factory or wire mills in coils or on reels, depending on the size of wire or cable and the specifications of the purchaser. Stranded conductor will usually come in lengths of ½ to 1 mile per reel, unless otherwise specified.

Two methods of stringing conductor are employed. The method used will generally depend on whether pin-type or suspension insulators are used in the line. One method is the reverse of the other, as will be seen later. The method of stringing with pin-type lines is to mount the coils or reels on a truck or trailer or a tractor- or horse-drawn vehicle. One end of each conductor is made fast to some stationary object, and the vehicle is driven along the right of way, unreeling the conductors and letting them fall to the ground. This is called "paying out." With single-pole construction, usually used for pin-type lines, three conductors are paid out at one time and all will be on the same side of the poles.

As the paying-out vehicle is driven along the right of way, one lineman and generally two helpers, depending on the size of the conductors, lay the conductors up on the crossarms.

This method of stringing is more adaptable to small than to large conductors. If lines are of any considerable length, roads, telephone lines, rural lines, etc., will generally have to be crossed, and conductors must be cut and respliced where they cross over other lines. This is not too difficult with a small conductor, but it is both difficult and extremely undesirable with large conductors. When a crossing occurs at or near the end of the conductor on a reel, the cutting and resplicing will of course not be necessary.

The method has the further disadvantage that, for reels containing ½ to 1 mile of conductor, it is difficult to mount three reels on one vehicle. When this cannot be done, it will be necessary to pay out one or two conductors at a time, obviously increasing the cost of stringing.

It is not advisable to drag the conductors on the ground or over cross arms for any considerable distance, since to do so may result in abrasion and damage to the outer strands. Regardless of the care exercised in stringing, however, it is practically impossible to avoid a small amount of dragging, both on the ground and over crossarms.

It is important, however, to exercise extreme care to avoid sharp bends or kinks in the conductor, since some of the strands may be overstressed and permanently weakened, resulting in failure under severe loading. A kink may often be straightened or smoothed out so that it is practically impossible to determine from an external inspection where it occurred. Nevertheless, the damage may have been done.

Where the pay-out method of stringing conductor is required, it will facilitate matters greatly if the conductor is put up in coils rather than on reels. Coils may be mounted on regular pay-out reels, which not only are lighter and more convenient to handle but require less space on a truck or trailer. For most transmission lines, however, the conductor will be too large to be conveniently put up in coils.

When heavy conductor is required and other lines are to be crossed, it will generally be found necessary to employ stringing blocks. A comparatively small block may be mounted on top of the crossarm by means of a special wire loop attached to the pin-type insulator. When this cannot be done, it will sometimes be necessary to suspend the stringing blocks from the crossarm and transfer the conductors up and over the arm.

Suspension-type Lines.—In this type of transmission line, the reels must be spaced along the right of way in accordance with the reel length of the conductor. This operation will generally require one truck driver, one tractor driver, and four helpers, one truck, one tractor, one sled, and miscellaneous small tools. The truck is used for hauling the reels to road crossings. The tractor and sled are used for further hauling of the reels to and along the right of way to reel stations.

Figure 103 illustrates reel mounting on jacks with an improvised type of reel brake. This brake works on the principle of a counterbalance. For effective braking, it requires the use of a board in the order of 2″ × 12″—12′, pivoted about 3 ft from one end. The short end of the board is inserted under the flange at one end of the reel, as shown. As the reel revolves, the overbalancing weight of the long end of the board creates considerable friction between the reel flange and the board. This acts as a brake to prevent the reel from overshooting and permitting the conductor to run out, with the resulting danger of its becoming damaged when the slack is again taken up by the pulling-out vehicle.

As far as the writer knows, no completely safe and satisfactory method of reel braking has been devised. Often the pulling-out vehicle is ½ to 1 mile or more from the reel station, possibly out of sight, and rapid signaling is therefore difficult. Often, because of inadequate reel braking, the conductor runs out from one or more of the reels Before a signal can be relayed to the driver, the slack is taken up with such sudden force as to dislodge the reels from their mounting jacks, possibly dragging them several feet

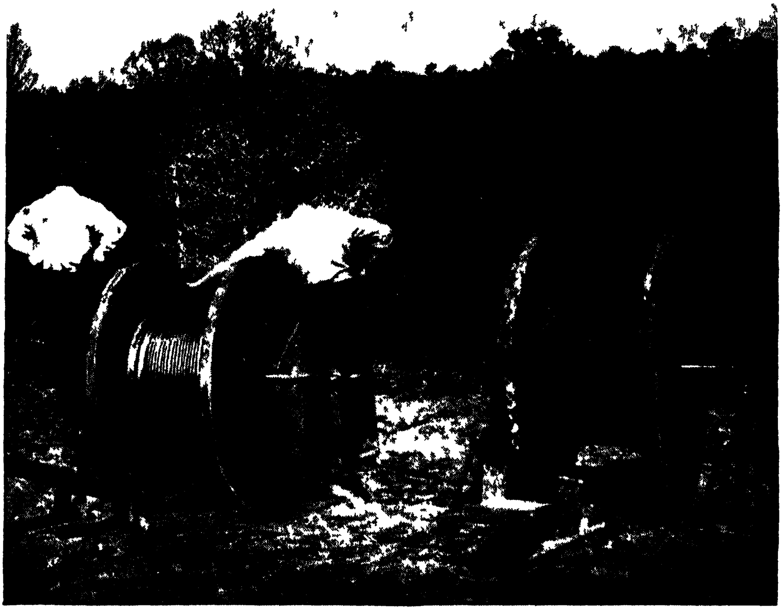


FIG 103—Reel mounting and brake

The conductor is usually badly damaged, and several feet must be cut out. Although the actual loss of material may not be of any particular importance, the stringing crew must return to the reel station, cut out the damaged part, resplice the conductor, and remount the reels. They must then return to their respective positions before stringing can proceed. This will usually require an hour or more. It is therefore obvious that considerable importance attaches to proper reel tending and braking. For comparatively long pulls, it will be necessary for one or more linemen to be in position on poles to relay signals.

Figure 104 illustrates a well-designed type of stringing block that is adaptable for use with large conductors. The blocks are suspended from the crossarm by means of slings, preferably made of 1/4- or 5/16-in. common guy strand. This block is constructed of a wood housing with a metal reinforcing band. The sheave is usually made of aluminum and equipped with roller bearings. The slings should be made with a served loop in each end and have a length such that, when they are passed over the crossarm with the stringing block hook inserted through the two loops, the conductor will lie in the top of the sheave at approximately the same elevation as the seat of the suspension clamp at the bottom of the insulator string. This is necessary because the conductor is sagged while hanging in the blocks, and the elevation and ground clearance will not be altered when it is transferred from the blocks to the suspension clamp.

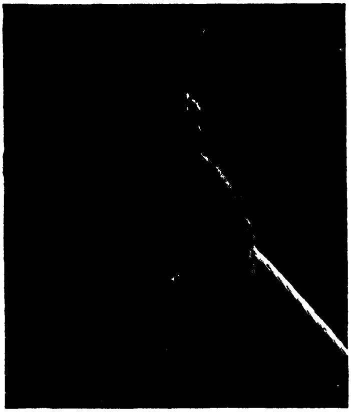


FIG. 104.

Good wire-stringing blocks are quite expensive and should be given care and attention in order to prolong their useful life. They should be inspected periodically for broken sheaves, defective bearings, and loose housing rivets. Stringing will be facilitated if the sheaves are kept in a free-running condition.

Sagging and Clipping in.—Transmission-line conductors, regardless of the kind of material of which they are composed, will elongate as tension is applied, the amount of stretch depending upon the nature of the material and the unit stress to which the conductor is subjected. When stress is initially applied, the elongation will consist of both elastic and non-elastic stretch. For repeated loadings, the elongation will be elastic only, provided that the unit stress is not greater than at initial loading. For example, should stress be initially applied and then entirely removed, the conductor will not return to its original length.

The length will be reduced an amount corresponding to the elastic stretch. The ratio of unit stress to unit elastic stretch is what is known as the "modulus of elasticity" of the material.

The permanent stretch of a conductor may be thought of as an adjustment taking place under initial tension which tends to remove the slight internal strains and stresses that may be attributed to manufacture and handling. If, upon repeated application, the unit stress is increased beyond that at initial application, the permanent stretch will also be increased.

The magnitude of permanent stretch can be determined only by stress-strain tests on a given length of wire or cable. Stress-strain tests consist in applying an increasing tension to a wire or cable. The length of a section is carefully measured prior to the test. At maximum applied tension, the length of the given section is again carefully measured, and the total stretch is determined. The tension is then slowly reduced until it is entirely removed. The difference in length at maximum and subsequent to the removal of tension is a measure of the elastic stretch. The difference in initial length and that subsequent to the removal of tension is a measure of the non-elastic, or permanent, stretch.

The elastic and non-elastic stretch applies to both solid and stranded conductors. A stranded conductor as a whole, however, contains a non-elastic property that is not characteristic of a solid conductor or of the individual wires of a stranded conductor. A stranded conductor will have a greater non-elastic stretch than will a solid conductor of similar material and equal total cross section when subjected to an equal tension. This additional permanent stretch may be attributed to a tendency of the strands to straighten out and become more closely embedded under tension. Obviously, the shorter the lay is for a given cable, the greater will be the permanent stretch. Manufacturers have also found that the greater the number of strands is, the greater will be the permanent stretch.

The non-elastic stretch is called the "permanent set" of the conductor. It is the property that makes it necessary either to give the conductor its initial sag and tension at the time of stringing or to prestretch and give the conductor its final sag and tension at the time of stringing.

Although this more or less technical discussion of the elastic properties of transmission-line conductors may seem somewhat out of place in connection with a construction problem, it should be of value in explaining the application of certain methods to the sagging of conductors.

When the cable is given its initial sag, as is the usual practice, two sets of sag-tension charts, initial and final, are required. For initial sag and tension, engineers compute a sag and tension to which the cable is to be strung in a given span at a given temperature such that, when it becomes loaded with ice and wind to its maximum design load, it will take its permanent set and final sag at the corresponding temperature. The final sag-tension chart enables the designer to construct a sag template (see page 159), from which the height and location of structures required to provide safety-code clearances are determined.

It is quite common practice to prestretch a.c.s.r. conductor at the time of stringing, because the slightly higher tension in the aluminum strands of an unprestretched cable makes them more susceptible to fatigue and ultimate failure from vibration. In some sections of the country, the cable may not reach its maximum design load for many years after its erection, during which time the normal stress will be slightly higher than for a similar prestretched cable. In any case, either vibration dampers or armor rods should be installed in conformity with specifications and instructions from the cable manufacturer.

The effect of prestretching may be more clearly understood from a study of the stress-strain curves of Fig. 105. Although these curves are plotted from tests on a piece of high-tensile-strength steel wire, they will serve the purpose adequately.

When tension is initially applied, the increasing unit stress and elongation are as shown by the initial curve until the elastic limit of 130,000 lb per sq in. is reached at *D*. As the tension is reduced, the decreasing stress and elongation are as shown by the dash line from *D* to *B* on the zero stress line. It is to be noted that the non-elastic stretch, or permanent set *AB*, is slightly in excess of 0.0006 in. per in. length of wire. The elastic stretch *B* to *C*, is about 0.0045 in. per in. length of wire.

Repeated stress will be as indicated by the full line curve *BDE*. If repeated tension is applied until the unit stress at *E* is reached and again reduced to zero, the decreasing stress will follow a line from *E* approximately parallel to the dash line *BD*. From this it is evident that the permanent set, after repeated stress, to *E*, will be about double that after initial stress.

The author believes that prestretching should not be a set and fast rule. Prestretching is an operation that should be given careful supervision in order to make certain that a systematic procedure is adhered to. In certain sections of the country, it is more logical to eliminate prestretching by specifying a slightly lower percentage of the ultimate strength of the cable at maximum design conductor tension. This will also give a slightly lower stress under normal operating conditions. Prestretching is expensive and must be properly performed to accomplish the desired result. It also requires extra equipment and considerable temporary guying of certain structures, especially for the larger sizes of cable.

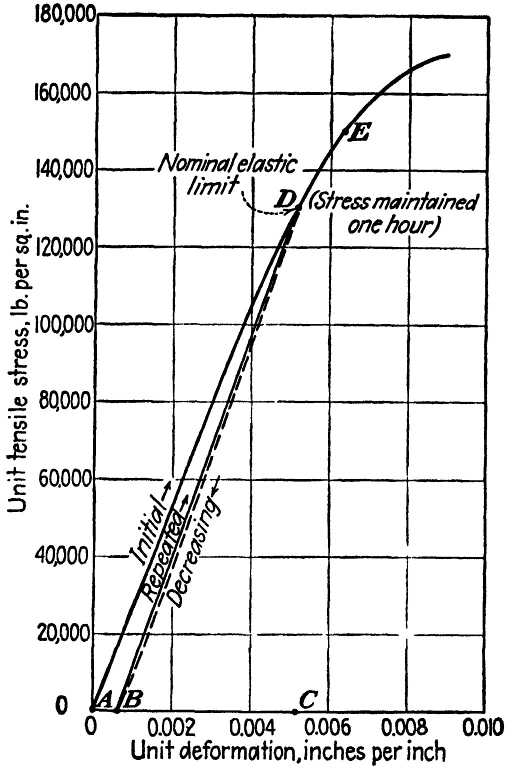


FIG. 105.—Reproduction of stress-strain curve for sample of steel wire from core of 795,000 cir mils, A.C.S.R.

The Copper Wire Engineering Association has prepared and made available to engineers a complete series of initial and final sag and tension charts, covering various sizes and various strandings for hard-drawn copper. These charts cover the range of span lengths most commonly met with in practice and may be used as a basis for preparing stringing curves for field use.

Regardless of the method used in sagging, it is advisable to use a dynamometer for indicating the tension in the conductors corresponding to that required at stringing temperature. The dynamometer used for this purpose should be of such capacity that the pointer will be well up on the scale at stringing tension, for this will generally give a more accurate and reliable reading. For lines of any considerable length, the dynamometer should be checked periodically, by either a known weight or another dynamometer of known calibration. The sag should be checked by means of targets placed on the poles in at least one approximately level span for each pull.

More satisfactory results will generally be obtained if the conductor is pulled slightly higher than normal stringing tension and then let off very slowly until the desired sag and tension are obtained. The conductors are then snubbed off to the butt of a pole or some firm stationary object. A mark should be put on each conductor at the last structure in the pull, a measured distance from its position in the block. When the conductor is released after the splicing has been done preparatory to making the next pull, some tension will be lost, and the conductors will run back to some extent. The mark is necessary, when making the next pull, to indicate when the conductors have been pulled back to proper sag. There should therefore always be one pull length between the stringing and clipping in crews. If the conductors are allowed to hang in the blocks several hours, say overnight, the tension will become fully equalized in all spans.

Men and Equipment Required. The men and equipment required for stringing and sagging should be about as follows:

1 foreman
4 linemen
6 helpers
1 tractor driver

1 Truck for transportation
1 Tractor and winch
Stringing blocks and slings for about 3 miles
Running lines, sleeve wrenches or compressors, bolt cutters, grips, chains, cable, rope, dynamometer, and miscellaneous small tools

The clipping in operation will require the following men and equipment:

2 linemen
2 helpers

Clipping-in, or hook ladder
Small chain hoist
Rope, blocks, hand lines, and small tools. Where a.c.s.r. is used and armor rods installed, two armor rod wrenches will also be required

With single-pole construction using crossarms in the order of 10 ft or less in length, the linemen, with safety belts around the pole, will have no difficulty in reaching out and making the tie on pin insulators, or laying the conductor in the suspension clamp and taking up the clamp bolts when suspension insulators are used. For pin-type lines, this is called "tying in." For suspension-type lines, it is called "clipping in."

With H-frame lines and their necessarily long crossarms, linemen cannot reach out to the conductor with their safety belts around the pole and be in position to work with any degree of comfort; and a clipping-in ladder is therefore required. This ladder is made about 7 ft in length with two strap-iron hooks securely bolted to one end to provide for hanging over the crossarm. Two men can work from the swinging ladder. Figure 106 shows two men working from a clipping-in ladder while installing a set of armor rods on the conductor.

The foregoing discussion of methods, men, and equipment required on a transmission-line construction project considers methods and procedure that are known to give satisfactory results from a standpoint of both cost and production. There is no hard and fast procedure, for different jobs can seldom be handled in the same manner.

For an "all out" production job on a long high-voltage line, for which the necessary men, tools, and equipment would have to be available, clearing right of way, digging holes, framing, erecting, guying, and stringing wire would generally be carried on simultaneously. The right-of-way-clearing crew, or crews, should be kept intact as far as practicable. When they have finished one job they should be moved on to another job or other projects instead of being broken up and assigned to other kinds of work. For good results, men engaged in right-of-way clearing require training different from that for men engaged in other kinds of work.

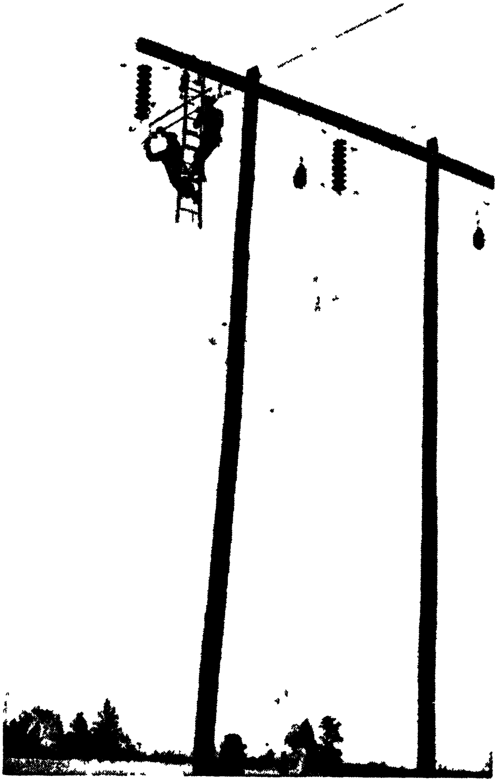


FIG. 106.—Men working on clipping-in ladder.

For construction jobs on which tools and equipment are limited, satisfactory and efficient performance may be obtained by using a smaller number of men for the various operations and also by using the same crew to perform two or more of the different operations. By making the best of the available men and equipment, progress will generally be quite satisfactory.

CHAPTER XIII

CONSTRUCTION OF STEEL-TOWER LINES

Great strides have been made in the development of facilities for electric-power transmission since 1890, when the first long-distance transmission of a.c. electric energy used in the United States for commercial purposes took place. Power was then transmitted from the hydro plant of the Willamette Falls Electric Company, on the Willamette River, to Portland, Ore., the unprecedented distance of 14 miles. The major pioneering of these developments took place in the west because distances from power-consuming areas to sources of production were greater than in other sections of the country, and longer lines and higher voltages were required.

From the inception of long-distance power transmission until about the end of the first quarter of the century, the heaviest towers required were light compared with those now in use. It was therefore the general practice to assemble towers on the ground and then erect the complete assembly and anchor to the previously installed footings. Because of the ever-increasing size and weight of transmission-line conductors and higher voltages that require wider spacings and clearances, heavier and higher towers became necessary; and erecting assembled towers became impractical. The method now in use is to erect steel towers by what is known as "building straight up."

The procedure in steel-tower-line construction is similar to that for wood-pole lines. The work involved in some of the operations differs, however. When the job is released for construction, the right of way is cleared; holes for tower footings are excavated; footings are installed; towers are built straight up; insulators are assembled and installed; and conductors and overhead ground wires are strung, sagged, and clipped in. In recent years, it has also become quite general practice to install counterpoise wires or to drive ground rods to reduce the tower-footing resistance as a means of protection from lightning; but this work is usually done subsequent to the construction proper.

Probably the most important and difficult phase of tower-line construction is the installation of footings, especially where either rock or water in any great quantity is encountered. The work is made more difficult where towers must be installed on steep hillsides. Where the ground at tower locations is practically level, ordinary routine methods may be employed. Extreme care must be exercised, however, to ensure that the tops of all footing legs are at the same elevation after installation.

It would be useless to attempt to go into any great detail as to methods of tower-line construction, since the methods to be employed will be governed largely by the type and voltage of the line and the character of the terrain. Methods that may be utilized to advantage in practically all cases will be discussed, however.

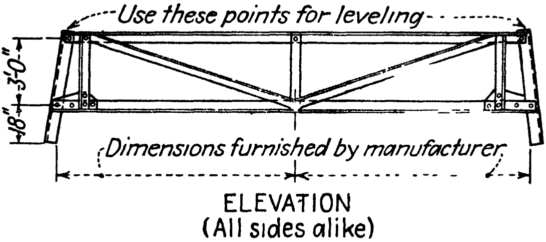

Use these points for leveling
3'-0"
18"
Dimensions furnished by manufacturer
ELEVATION
(All sides alike)

Fig. 107.—Anchor setting template.

Regardless of whether the footings are set in earth or in concrete, the dimensions between footing legs, and their alignment and batter must be such as to permit assembling the tower section and footings without undue strain or distortion in any part of the structure. There are two methods by which this can generally be accomplished: (1) Use an anchor setting template, similar to Fig. 107, fabricated from angles, plates, etc. (2) Use as a template the lower tower section, or extension, as the case may be.

If anchor setting templates are used for all tower footings, several will be required, one for each type of tower and possibly two or more for certain types, depending on the number of different lengths of extensions. For a line of any considerable length, this will be an item of considerable importance from a cost and weight standpoint. The use of anchor setting templates, except for special types of tower and unusual conditions, is not recommended. This will be made clear in the following discussion.

Let us first discuss method (2). When the holes at the four corners of the tower have been excavated to their proper size and depth, a method that may be used conveniently for tangent towers using earth anchors is to assemble on the ground two opposite sides of the lower section, including the footings. Each assembled side is then lifted clear of the ground with a gin or derrick and is lowered into the holes while in a practically vertical position. This is necessary to avoid damage to the footings. One side is held in place with props while the other side is being erected. The two opposite sides are laced together with crossmembers and diagonals; and the assembled section is lined up, made square with the line, and leveled. The proper

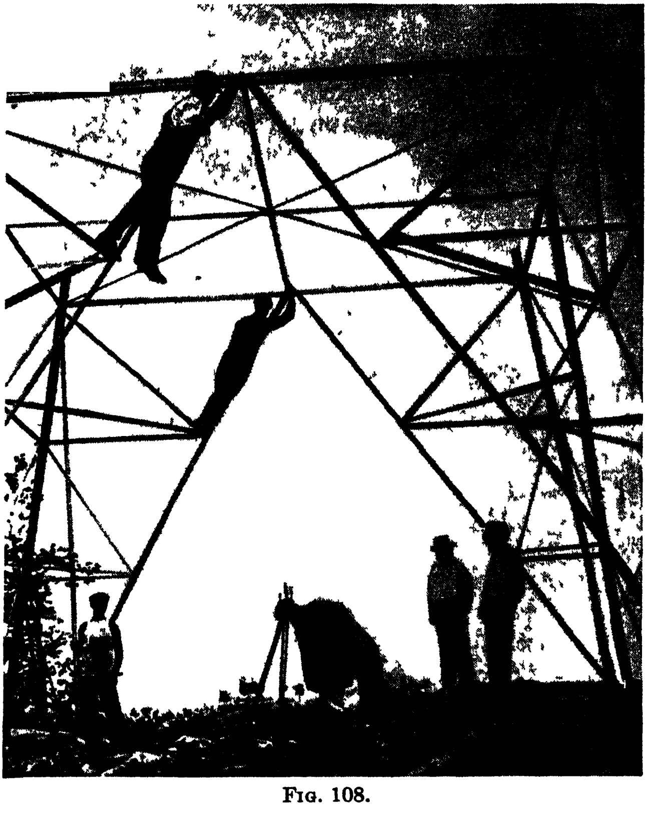


FIG. 108.

elevation and leveling are determined with a transit (see Fig. 108). It will usually be necessary to tamp a small amount of earth or to excavate under one or more of the footings in order to level the section at its proper elevation. When the lining and leveling have been done, the bolts are taken up, and the assembly is blocked and made as rigid as is reasonably possible. If it is an earth footing, it is then ready for backfilling. The tamping is done as the earth is slowly shoveled into the hole; or a few inches of earth may be shoveled into the hole, spread out, and then thoroughly tamped (see Fig. 109). If it is a concrete footing, the forms are built and the concrete poured when the assembled section is leveled (see Fig. 110). Either hand or compressed-air tamping may be used In any case, a check should be made occasionally as the work progresses to ascertain whether the elevations have been disturbed.

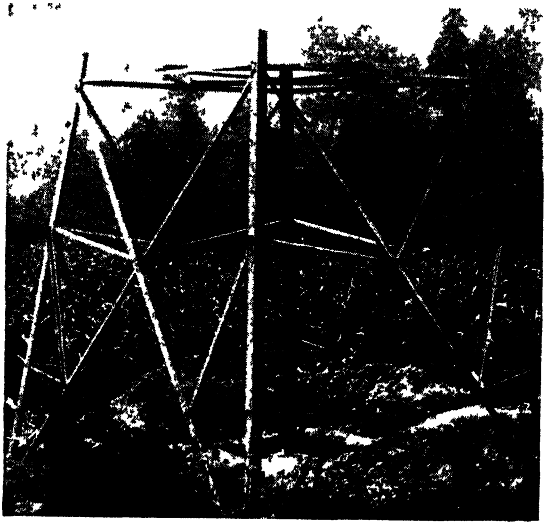


FIG. 109.—Assembled section ready for backfilling

Figures 111 and 112, illustrate difficult situations. It would be practically impossible to use a setting template at such locations.

For heavy towers, it will usually be found necessary to erect one leg at a time and then lace them together to make the completed lower section, as shown in Fig. 109. Also, where rock- or malone-type anchors are used or, for that matter, any type of anchor requiring a drilled hole through which grout must be

Fig. 110.—Concrete footing.

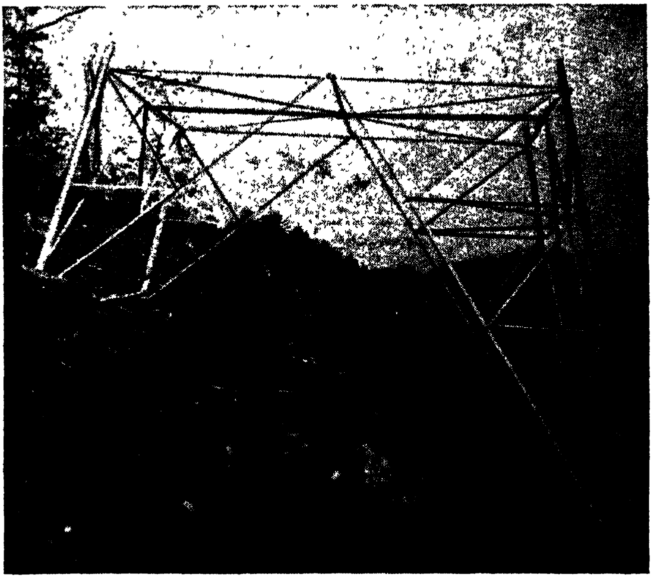


Fig. 111.

poured, the lower section must be assembled to the anchor legs subsequent to their insertion in their respective holes.

The advantage to be gained from the use of method (2) (tower section) is readily understood, since it requires little, if any, more work and time to assemble the tower section than to assemble the anchor setting template. In method (1), (anchor setting template), three operations are required as against one for method (2). The template must be assembled and then dismantled. The dismantling cannot be done until after the backfilling or concreting. Where concrete footings are used, considerable time must elapse, and the crew must return to dismantle the template. The transportation of anchor setting templates between tower locations is in itself an item of considerable importance. Their cost is necessarily chargeable to the project, and they may be of only scrap value when they are no longer needed.

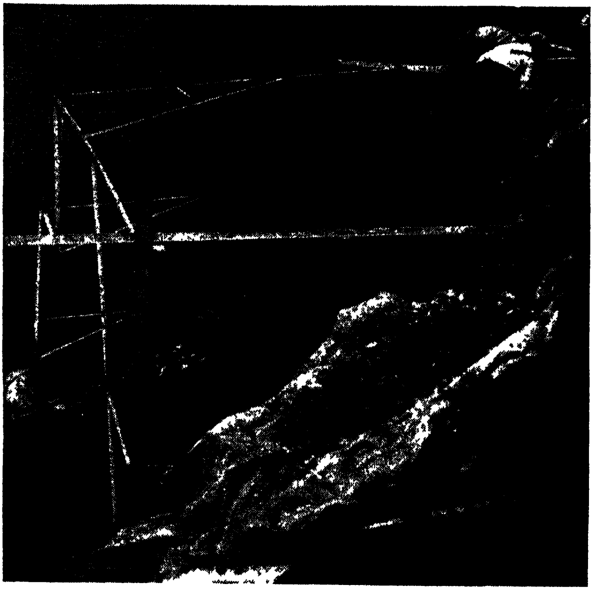


FIG. 112.

For water or soft-bottom holes, it will sometimes be necessary to use an anchor setting template. Where water is standing in the hole and only a concrete mat is required, separation of the agregate may be prevented by pouring it through a chute, as shown in Fig. 113. If only a few cases of this kind are encountered, a template may usually be made from material available in the field.

Building Straight Up.—Towers are usually not built up for several days after the footings have been installed. The elapsed time is not relatively important where earth-encased footings are used; but, where concrete-encased footings are used, employing regular cement, the concrete should be allowed to set for 12 to 14 days before the tower is erected.

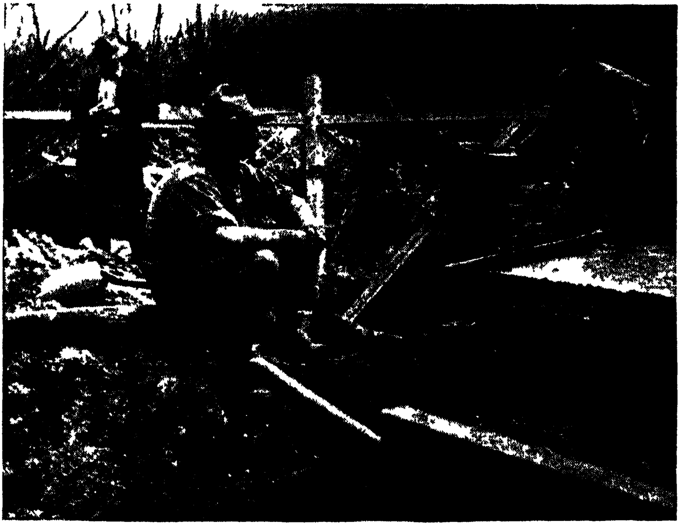


FIG. 113.

Tower erection diagrams are furnished by the manufacturer for each type of tower, including test loads under various loadings, and a complete bill of material required for each tower, extension, footing, etc. To facilitate erection, each member is marked to correspond with that shown on the erection diagram, with the number of pieces of each member required for one tower.

For tower erection, a minimum of eight men are required, four linemen and four helpers. A pole is lashed to one corner of the tower to be used as a gin for hoisting the long members. This is clearly shown in Fig. 114. When the steel is delivered to the tower location, it should be placed in such manner as to be accessible as required. Care should be exercised in handling the steel to avoid such damage as bending and injuring the galvanizing. The bolts should be taken up reasonably tight but should be given their final set after the wire is strung.

To prevent nuts from working loose and setting up a possible source of radio interference, it has become quite general practice to use lock washers at all bolted connections. The writer believes this practice should be adhered to

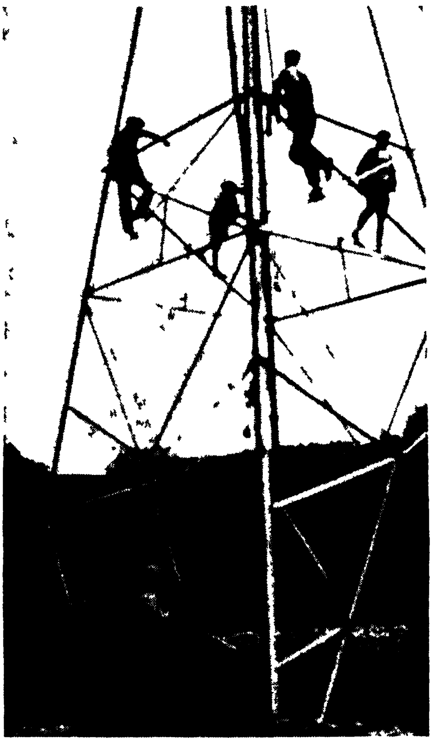


FIG 114.—Building tower straight up

Sagging and Clipping in.—Pulling out, sagging, and clipping in conductor on suspension-type lines is performed in the same manner for tower lines as for wood-pole lines and is fully described in Chap. XII. Figure 115 shows men working from a clipping-in ladder while making up a dead end on a steel tower. Figure 116 shows a dynamometer hookup with the conductor under tension

The character of the terrain over which a given line is to be built will generally be a fairly reliable guide to the logical methods to be employed in tower-line construction.

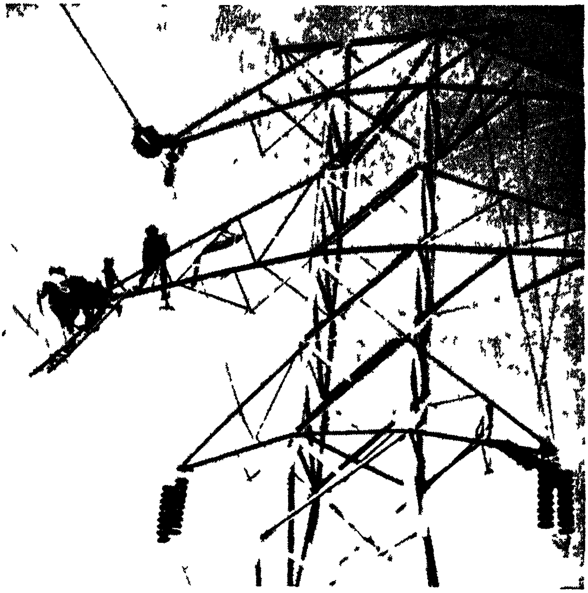


Fig 115 —Making up a dead end

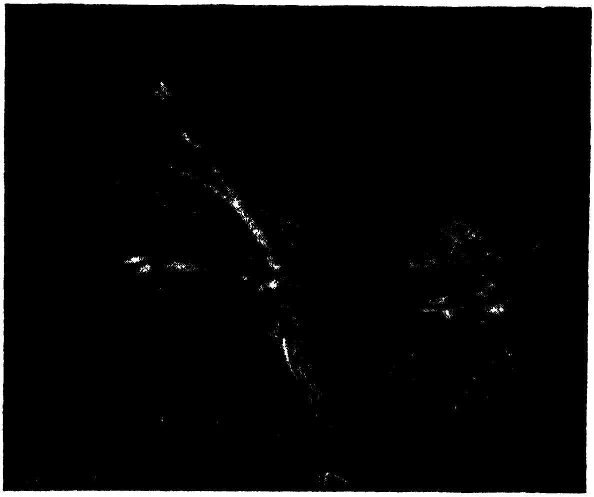


Fig 116 —Dynamometer hookup.

CHAPTER XIV

PERFORMANCE OF HIGH-VOLTAGE TRANSMISSION LINES WITH PHASE CONTROL

A transmission line in some respects may be compared with a mechanically operated machine, in that the mechanism becomes more involved as the operation is made more automatic. As equipment is provided to make a transmission line automatic in its operation, such as maintaining constant supply and receiver voltage under any load condition, the entire installation and the calculations required for determining line performance become more involved. As will be seen from the following analysis, the solution may nevertheless be comparatively simple.

In Chap. V, a rigorous solution is applied to a problem involving the performance of a given transmission line when operating without phase control. No attempt is made to control the supply-end voltage, it being free to vary with the load. The performance of the 110-kv 125-mile line, given in Example 4, is graphically represented in Fig. 24.

Methods for determining performance of short lines will not give results to the same degree of accuracy when applied to the solution of long lines. Short lines have negligible capacitance, and the line charging current is therefore negligible. For lines of any considerable length, however the effect of capacitance must be taken into account.

When a transmission line is operated with phase control, voltage and power-factor corrective equipment, in the form of leading and lagging synchronous condenser capacity, is installed at such locations as may be determined to be most logical. With only one condenser, the installation is usually made at the receiving end of the line. The voltage at both ends of the line may be maintained at a constant and predetermined value over the entire load range, provided that the required leading and lagging condenser capacity is available. It is not generally considered advisable, however, to attempt to maintain the same voltage at both ends of the line, because excess leading condenser capacity will be required at maximum load. The leading condenser capacity should generally be in the order of 60 percent of the normally maximum kva to be transmitted over the line, considered as 100 percent load. For standard equipment, synchronous condensers generally have a lagging capacity in the order of 50 percent of their leading capacity. As will be seen later, 50 percent lagging capacity will not generally be sufficient to maintain the supply voltage constant at the lower load values.

For lines operated with phase control, constant receiver, and constant supply voltage, the receiver power factor will vary with the load. With decreasing load, the receiver power factor becomes more and more lagging.

In order for the problem to fall within the field of practical application, we have chosen to discuss, both analytically and graphically, the performance of a transmission line with characteristics as given in Example 4, Chap. V.

> *Example* 5. 125-mile transmission line, receiving-end voltage 110 kv, three-phase, 60 cycles, 336,400 cir mils a.c.s.r. stranding 26al/7st. Equivalent spacing 15.75 ft, 100 percent load, 30,000 kw at 0.85 lagging receiver power factor.
>
> *a*. Determine the leading synchronous condenser capacity in kva required to correct the power factor from 0.85 lagging to unity at 100 percent load.
>
> *b*. Determine the voltage and current at the sending end of the line at 100, 75, 50, 25, and 0 percent loads.
>
> *c*. Construct the complete performance diagram.

Before a discussion of the above problem, it should be pointed out that the load is assumed to be taken on the high-voltage side of the receiving-end transformers and that the load includes transformer and condenser losses. This is believed to be justifiable, since it would seem inadvisable to complicate the problem by pretending to take into account these small losses, distinct from the other load. It will be shown that the results are affected only at minimum load.

The linear and auxiliary constants were derived in the solution to Example 4, Chap. V, and will be repeated here only for convenience.

Linear Constants

R = 34.75 ohms total resistance

X = 98.125 ohms total inductive reactance

$Z = R + jX = 34.75 + j98.125 = 104.1\underline{/70^\circ 30'}$ ohms total impedance

$B = 5.38255 \times 125 \times 10^{-6} = 0.0006728$ mho total susceptance

G = (taken as zero mho conductance)

$Y = G + jB = 0 + j0.0006728 = 0.0006728$ mho total admittance

Auxiliary Constants

$a_1 + ja_2 = 0.96715 + j0.01156$

Constant $A = 0.96722\underline{/0^\circ 41'}$

$b_1 + jb_2 = 33.99 + j97.18$

Constant $B = 102.95\underline{/70^\circ 43'}$

$c_1 + jc_2 = -0.0000026 + j0.0006654$

Constant $C = 0.0006654\underline{/90^\circ 14'}$

The exact equations of the transmission circuit, with the numerical quantities expressed in vector form, are as follows:

$$\begin{aligned} E_{sn} &= E_{rn} + I_r B \\ &= E_1 + E_2 \qquad (56) \\ I_s &= I_r A + E_{rn} C \\ &= I_1 + I_2 \qquad (57) \end{aligned}$$

where E_{sn} = voltage to neutral at sending end, load conditions.

E_1 = voltage to neutral at sending end, zero load.

E_2 = impedance volts drop, modified by the distribution effect of the circuit.

I_r = current at receiving end, amp.

I_s = current at sending end, amp.

I_1 = current at receiving end, modified by the distribution effect of the circuit, amp.

I_2 = line charging current, amp.

Solution for a. Since the load is assumed to be 30,000 kw at 0.85 lagging power factor, we have 30,000/0.85 = 35,294 kva, the quadrature components of which are equal to

$$35{,}294(0.85 - j0.5267) = 30{,}000 - j18{,}589.$$

The leading condenser capacity required to correct the power factor to unity from 0.85 lagging will be that required to neutralize the lagging reactive kva and is equal to 18,589 kva. At unity power factor, we have

$$30{,}000 - j18{,}589 + j18{,}589 = 30{,}000 + j0 \text{ kva.}$$

Solution for b, 100 *Percent Load*

$$\frac{30{,}000 \text{ kw}}{3} = 10{,}000 \text{ kw per phase at unity power factor}$$

$$E_{rn} = \frac{110}{\sqrt{3}} = 63.510 \text{ kv to neutral}$$

$$I_r = \frac{10{,}000}{63.51} = 157.46 \text{ amp}$$

From (56)

$$\begin{aligned} E_1 &= E_{rn}(a_1 + ja_2) \\ &= 63{,}510(0.96715 + j0.01156) = 61{,}424 + j\ \ 734 \\ E_2 &= I_r(b_1 + jb_2) \\ &= 157.46(33.99 + j97.18) = \underline{5{,}352 + j15{,}302} \\ E_{sn} &= E_1 + E_2 = 66{,}776 + j16{,}036 \\ &= 68{,}675\underline{/13°30'} \text{ volts} \end{aligned}$$

From (57)

$$\begin{aligned} I_1 &= I_r(a_1 + ja_2) \\ &= 157.46(0.96715 + j0.01156) = 152.29 + j\ 1.82 \\ I_2 &= E_{rn}(c_1 + jc_2) \\ &= 63{,}510(-0.0000026 + j0.0006654) = -0.16 + j42.26 \\ I_s &= I_1 + I_2 = 152.13 + j44.08 \\ &= 158.38\underline{/16°10'} \text{ amp} \end{aligned}$$

I_s leads E_{sn} by $16°10' - 13°30' = 2°40'$

The power factor at the sending end of the line, $(Pf)_s$, is therefore equal to 99.89 percent leading.

$$\begin{aligned} (Kw)_{sn} &= 66.776 \times 152.13 + 16.036 \times 44.08 \\ &= 10{,}159 + 707 = 10{,}866 \text{ kw per phase} \\ (Kva)_{sn} &= 68.675 \times 158.38 = 10{,}877 \text{ kva per phase.} \end{aligned}$$

Zero Load (*Approximate Solution*). In the solution for *a,* the leading condenser capacity to correct the power from 0.85 lagging to unity was computed to be 18,589 kva, in which case the minimum installed capacity would be 20,000 kva. With the lagging capacity equivalent to 50 percent of the leading capacity, this would be 10,000 kva, or

$$3{,}333 \text{ kva per phase} = 3{,}333(0 - j1) = 0 - j3{,}333.$$

The no-load current I_{ro} is equal to

$$\frac{(0 - j3{,}333)}{63.51} = 0 - j52.49 = 52.49\underline{/-90^\circ} \text{ amp.}$$

$$\begin{aligned}
E_1 \text{ (previously calculated)} &= 61{,}424 + j\ 734 \\
E_2 = (0 - j52.49)(33.99 + j97.18) &= \underline{5{,}101 - j1{,}784} \\
E_{sno} = E_1 + E_2 &= 66{,}525 - j1{,}050 \\
&= 66{,}533\underline{/-0^\circ55'} \text{ volts} \\
I_1 = (0 - j52.49)(0.96715 + j0.01156) &= 0.61 - j50.76 \\
I_2 \text{ (previously calculated)} &= \underline{-0.16 + j42.26} \\
I_{so} = I_1 + I_2 &= 0.45 - j\ 8.50 \\
&= 8.51\underline{/-86^\circ58'} \text{ amp}
\end{aligned}$$

$$\begin{aligned}
&I_{so} \text{ lags } E_{sno} \text{ by } 86^\circ58' - 0^\circ55' = 86^\circ03' \\
&(If)_{so} = 6.88 \text{ percent lagging} \\
&(Kw)_{sno} = 66.525 \times 0.45 + (-1.05 \times -8.5) \\
&\qquad = 29.94 + 8.93 = 38.87 \text{ kw per phase} \\
&(Kva)_{sno} = 66.533 \times 8.51 = 566 \text{ kva per phase}
\end{aligned}$$

Values at intermediate loads are calculated in the same manner. The essential values are compiled in Tables 40, 41, and 42, from which they are transferred to the performance diagram (Fig. 117).

TABLE 40.—VOLTAGE AND IMPEDANCE DROP

Percent load	$E_{rn}A$ (E_1)	I_rB (E_2)	E_{sn} ($E_1 + E_2$)
100	$61{,}428\underline{/0^\circ41'}$	$16{,}211\underline{/70^\circ43'}$	$68{,}675\underline{/13^\circ30'}$
75	$61{,}428\underline{/0^\circ41'}$	$12{,}396\underline{/59^\circ28'}$	$68{,}675\underline{/9^\circ34'}$
50	$61{,}428\underline{/0^\circ41'}$	$9{,}260\underline{/41^\circ48'}$	$68{,}675\underline{/5^\circ47'}$
25	$61{,}428\underline{/0^\circ41'}$	$6{,}755\underline{/17^\circ36'}$	$67{,}911\underline{/2^\circ21'}$
0	$61{,}428\underline{/0^\circ41'}$	$5{,}404\underline{/-19^\circ17'}$	$66{,}533\underline{/-0^\circ55'}$

TABLE 41.—CURRENT, AMP

Percent load	I_rA (I_1)	$E_{rn}C$ (I_2)	I_s ($I_1 + I_2$)
100	152 30/0°41′	42 26/90°14′	158 38/16°10′
75	116 45/−10°34′	42 26/90°14′	116 22/10°22′
50	86 99/−28°14′	42 26/90°14′	76 49/0°50′
25	63 45/−52°26′	42 26/90°14′	39 34/−11°48′
0	50 76/−89°19′	42 26/90°14′	8 51/−86°58′

TABLE 42—POWER AND POWER FACTOR

Percent load	$(Kw)_r$	$(Kw)_s$	$(Kva)_s$	$(Pf)_r$	$(Pf)_s$
100	30,000	32,598	32,631	100 00	99 89
75	22,500	23,943	23,943	−98 68	100 00
50	15,000	15,699	15,759	−87 53	−99 63
25	7,500	7,773	8,016	−60 00	−96 96
0	0	117	1,698	−0	− 0 688

Minus sign indicates lagging power factor

c. Construction of the Diagram. A study of the performance diagram (Fig. 117) indicates clearly how the line will perform over the entire load range when operated with phase control. The voltage, current, phase angles, and synchronous condenser capacity are indicated at the given loads. Construction of the diagram is best illustrated by performing certain of the calculations as the work progresses.

Lay off vector *OR*, to a suitable scale, to represent that constant receiver voltage to neutral equals 63,510 volts. Let this line be the vector of reference. Locate *B* on vector *OR* a distance from *O* equal to $E_{rn}a_1 = 63{,}510 \times 0.96715 = 61{,}424$ volts. From *B* draw *BC* at right angles to *OR* in a leading direction and equal to $E_{rn}ja_2 = 63{,}510 \times j0.01156 = j734$ volts. Then a line drawn from *O* to $C = E_1 = 61{,}424 + j734 = 61{,}428$ volts. Vector *OC* is of fixed length and constant slope and determines the length and position of the sending-end voltage vector at zero load.

Next, lay off from *C* the 100 percent power factor line making an angle with vector *OR* whose tangent is equal to

$$\frac{b_2}{b_1} \quad \frac{97.18}{33.99} = 70°43' \text{ as shown.}$$

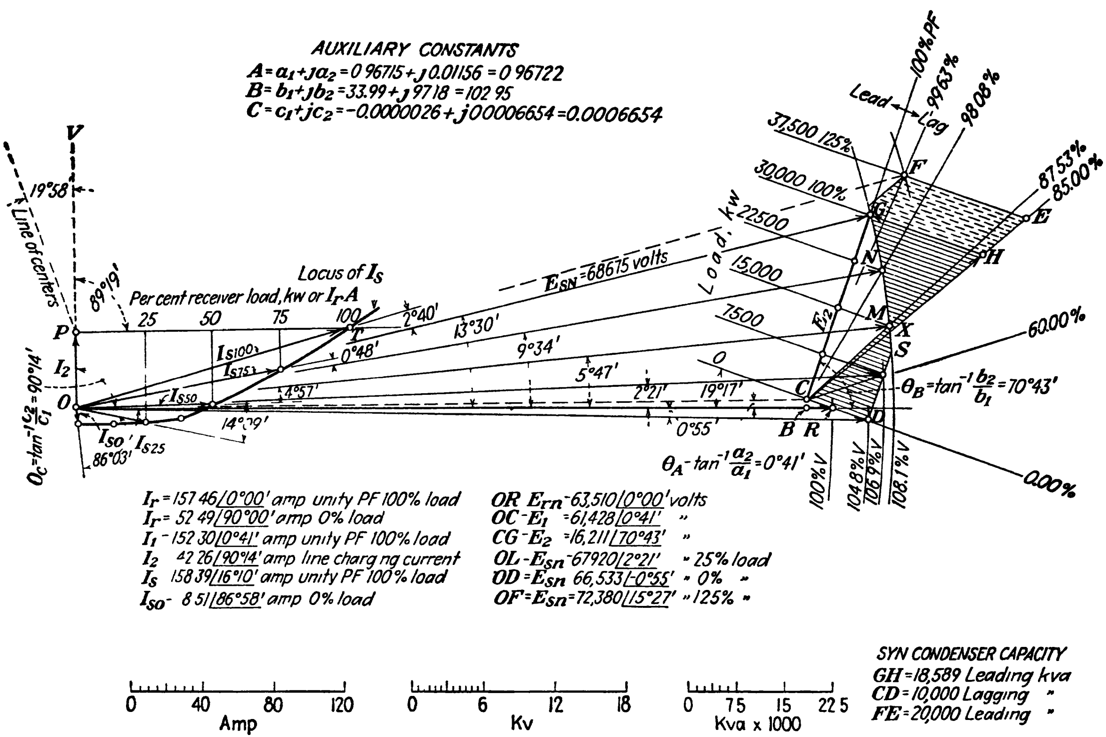

AUXILIARY CONSTANTS
A = a1 + ja2 = 0 96715 + j 0.01156 = 0 96722
B = b1 + jb2 = 33.99 + j 97 18 = 102 95
C = c1 + jc2 = -0.0000026 + j 0.0006654 = 0.0006654
V
19°58'
Line of centers
89°19'
Percent receiver load, kw or IrA
25
50
75
100
Locus of Is
P
I2
O
T
Is100
Is75
Is50
Iso
Is25
86°03'
14°09'
4°57'
0°48'
2°40'
13°30'
9°34'
5°47'
2°21'
19°17'
0°55'
θc = tan⁻¹ c2/c1 = 90°14'
ESN = 68675 volts
Load, kW
37,500 125%
30,000 100%
22500
15,000
7500
0
Lead
Lag
100% PF
99.63%
98.08%
87.53%
85.00%
60.00%
0.00%
F
G
E
H
N
M
X
S
E2
C
B
R
D
θB = tan⁻¹ b2/b1 = 70°43'
θA = tan⁻¹ a2/a1 = 0°41'
100% V
104.8% V
106.9% V
108.1% V
Ir = 157 46 /0°00' amp unity PF 100% load
Ir = 52 49 /90°00' amp 0% load
I1 = 152 30 /0°41' amp unity PF 100% load
I2 = 42 26 /90°14' amp line charging current
Is = 158 39 /16°10' amp unity PF 100% load
Iso = 8 51 /86°58' amp 0% load
OR = Ern = 63,510 /0°00' volts
OC = E1 = 61,428 /0°41' "
CG = E2 = 16,211 /70°43' "
OL = Esn = 67920 /2°21' " 25% load
OD = Esn = 66,533 /-0°55' " 0% "
OF = Esn = 72,380 /15°27' " 125% "
SYN CONDENSER CAPACITY
GH = 18,589 Leading kva
CD = 10,000 Lagging "
FE = 20,000 Leading "
0
40
80
120
Amp
0
6
12
18
Kv
0
7 5
15
22 5
Kva x 1000

FIG. 117.—Performance diagram for 110-kv, 125-mile line with phase control.

Draw from C a line lagging the 100 percent power factor line by 31°47′ (0.85 lagging power factor angle).

Locate point G on the 100 percent power factor line a distance from C equal to $I_rB = E_2 = 16{,}211$ volts to the same scale as vector OR (see Table 40). With center at O and radius OG, draw the voltage arc. This represents the sending-end voltage E_{sn} and is equal to 68,675 volts. Draw lines at right angles to line CG to represent kw loads at 100, 75, 50, 25, and 0 percent loads. Extend these lines to the voltage arc at N, M, etc. Construction of this part of the diagram should be done with extreme care, since the line intersections with the voltage arc determine the kva and power factor at intermediate loads. Lines CN, CM, CL, and CD represent receiver kva and current but, of course, to a different scale.

Line GH represents the leading condenser capacity required to correct the power factor to unity at 100 percent load and, as previously determined, is equal to 18,589 leading kva. The lagging condenser capacity is assumed to be 50 percent of the leading capacity, equal to 10,000 kva, and is represented by line CD to the same scale as GH. At X, where the load is about 12,600 kw, no leading or lagging reactive kva is required to maintain E_{sn} at 68,675 volts, and the condenser is just floating on the line.

It will be seen from the diagram that E_{sn} remains constant from 100 percent load, 30,000 kw at G, to about 11,250 kw at S. Since the 10,000 lagging reactive kva of the condenser is not sufficient to hold E_{sn} constant at 108.1 percent of E_{rn} at loads below S, the locus of E_{sn} will be a straight line from S to D, dropping from 108.1 percent at S to 104.8 percent at D.

As previously noted, since the load is assumed to include transformer and condenser losses, the receiver load cannot drop entirely to zero. Although the line CD represents the maximum lagging condenser capacity (10,000 kva), it does not give an entirely true picture of the situation at minimum load. There will always be a small energy load along the line CG representing the transformer and condenser losses. This is, however, so small as to be negligible. Its only effect is to increase the length of the voltage vector OD slightly and to reduce the angle between vectors OR and CD by a few seconds. It also increases slightly the length of vector I_{so} and reduces its displacement angle.

Emergency Load.—Let us assume that in an emergency the load may reach 125 percent, or 37,500 kw, or 37,500/0.85 = 44,118 kva at 85 percent lagging power factor, the quadrature components of which are $44{,}118(0.85 - j0.5267) = 37{,}500 - j23{,}237$. With the available 20,000 leading kva, represented by line *EF*, we have $37{,}500 - j23{,}237 + j20{,}000 = 37{,}500 - j3{,}237 = 37{,}639\underline{/-4^\circ 56'}$ kva. Receiver power factor is now equal to 99.63 percent lagging. 37,639/3 = 12,546 kva per phase, from which $I_r = 12{,}546/63.51 = 197.54$ amp. $197.54(0.9963 - j0.086) = 196.81 - j16.99$ vector amp. The sending-end values under emergency load conditions are derived as follows:

$$
\begin{aligned}
E_1 \text{ (previously calculated)} &= 61{,}424 + j\ \ \ 734 \\
E_2 = (196.81 - j16.99)(33.99 + j97.18) &= \underline{\ 8{,}341 + j18{,}549} \\
E_{sn} = E_1 + E_2 &= 69{,}765 + j19{,}283 \\
&= 72{,}380\underline{/15^\circ 27'} \text{ volts} \\
I_1 = (196.81 - j16.99)(0.96715 + j0.01156) &= 190.54 - j14.15 \\
I_2 \text{ (previously calculated)} &= \underline{-0.16 + j42.26} \\
I_s = I_1 + I_2 &= 190.38 + j28.11 \\
&= 192.44\underline{/8^\circ 24'} \text{ amp}
\end{aligned}
$$

I_s lags E_{sn} by $15^\circ 27' - 8^\circ 24' = 7^\circ 03'$

$(Pf)_s = 99.24$ percent lagging

$$
\begin{aligned}
(Kw)_{sn} &= 69.765 \times 190.38 + 19.283 \times 28.11 \\
&= 13{,}282 + 542 = 13{,}824 \text{ kw per phase} \\
(Kva)_{sn} &= 72.380 \times 192.44 = 13{,}929 \text{ kva per phase}
\end{aligned}
$$

The 20,000-kva condenser, as will be seen, will not correct the power factor to unity at loads in excess of about 32,000 kw. Emergency loads, or loads above 100 percent, are represented by the area *EHGF*.

The value of E_{sn} at emergency load of 125 percent is represented by the dash line *OF*. The right-hand end of this diagram presents a clear picture of the line performance, insofar as voltage and loads are concerned, within the range of operation.

Sending-end Current Diagram.—With the voltage at each end of the line maintained at constant though not necessarily equal values, determination of the current at given loads and construction of the current diagram becomes somewhat more involved than when receiver power factor is constant and sending-end voltage is variable. The solution to the problem is therefore enhanced and more clearly defined when a sending-end current diagram distinct from the main performance diagram is constructed. Accordingly, before attempting to complete the current diagram at the left-hand end of Fig. 117, let us refer to Fig. 118. This diagram includes certain vectors that are not essential to a performance diagram but are essential to the complete solution.

The calculations are first made and the values compiled as in Table 41, from which values the diagram of Fig. 118 is constructed. To construct the current diagram in Fig. 118, lay off horizontally a line to represent the receiver voltage E_{rn}.

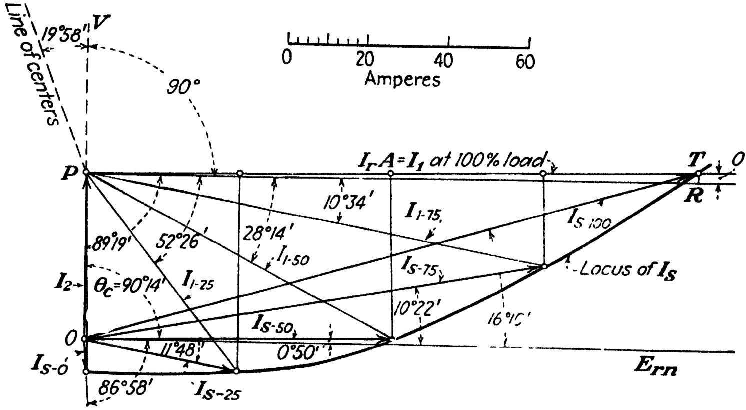


FIG. 118.—Construction of sending-end current vectors.

Then draw vector OP to represent the line charging current, designated I_2, and equal to 42.26 amp displaced from vector E_{rn} by 90°14′ as shown. With origin at P, draw line PR parallel with vector E_{rn}. Then draw line PT displaced from PR by 0°41′ and to a length of 152.3 amp, which is equal to $I_r A = I_1$ at 100 percent load. I_s is equal to $I_1 + I_2$ (vectorially) = 158.38 amp at 100 percent load, represented by vector OT at an angle of 16°10′ referred to E_{rn} and designated I_{s-100}. In this same manner, I_{s-75}, I_{s-50}, I_{s-25}, and I_{s-0} are drawn in on the diagram (the numeral subscripts indicate percentage load).

The coordinates of the center of the current circle may be derived as follows:

$$-l + jm = \frac{a_1 - ja_2}{b_1 - jb_2} = \frac{0.96715 - j0.01156}{33.99 - j97.18}$$
$$= -0.003207 + j0.00883$$

from which the coordinates of the center of the circle are equal to

$$-lE_{rn} = -0.003207 \times 63{,}510 = -203.69$$
$$mE_{rn} = 0.00883 \times 63{,}510 = 560.8$$
$$\text{Radius} = \frac{E_{sn}}{B} = \frac{68{,}675}{102.95} = 667.07$$

These values are measured in terms of amperes.

Line *PV* is displaced from line *PR* by 90 deg. Then the line of centers will be displaced from line *PV* by the angle whose tangent is $-203.69/560.8 = 19°58'$. More simply,

$$0°41' + 19°17' = 19°58'$$

(see Table 40 zero load). It is not essential to locate the center of the current circle. However, as will be seen, it is simple to do so. To avoid confusion, the current vectors for emergency load are not shown on the diagram.

Now refer again to the left-hand end of the performance diagram in Fig. 117, where it will be seen that values essential to determination of the line performance have been transferred from Fig. 118, but to a different scale. The sending-end current vectors at 100, 75, 50, 25, and 0 percent loads are drawn with the same angular displacement from vector *OR* in Fig. 117, as from vector E_{rn} in Fig. 118. The angles given in Fig. 117, however, represent the relation of the sending-end current I_s to the sending-end voltage E_{sn} and therefore determine the power factor at the sending end of the line at the given loads.

If the sending-end current vectors are drawn to the same scale as the receiving-end current vectors *CG*, *CN*, etc. (see right-hand end of Fig. 117), the radius of the sending-end current circle arc will be exactly the same as the radius of the voltage arc *GNMS*. A smooth curve drawn through the points of 50, 75, and 100 percent loads will be the arc of a circle. As previously explained, E_{sn} is a variable below about 11,250 kw, or 37.5 percent load. The locus of I_s in this load range is seen to approach a straight line, as would be expected. To maintain E_{sn} at a constant value of 108.1 percent of E_{rn} would require a lagging condenser capacity at zero load in the order of 75 percent of the leading kva at 100 percent load.

Considerable data have been given in connection with the solution to this problem that would not generally be required in practice. Since the problem represents a condition that may be considered as quite typical, an attempt has been made to include all items of information that will in any way facilitate application of the method to problems of a similar character.

Most Efficient Receiver Power Factor.—The most efficient transmission of power will be obtained when the receiver power factor is slightly lagging. The receiver power factor at which the line will operate most efficiently, or the line of minimum loss, may be determined as follows:

Let P_r represent the component of real power at 100 percent load = 10,000 kw per phase. Let Q_r represent the reactive component of the load. $P_r - jQ_r = 10{,}000 - jQ_r$.

$$Q_r = -\left(\frac{w}{2v}\right) \times E_{rn}^2$$

$$w = b_1c_2 + b_2c_1 \qquad v = a_1b_1 + a_2c_2 \qquad E_{rn}^2 = 4{,}033{,}520{,}100$$

$$w = 33.99 \times 0.0006654 + 97{,}18 \times -0.0000026 = 0.022364$$

$$v = 0.96715 \times 33.99 + 0.01156 \times 0.0006654 = 32.87$$

$$Q_r = -\frac{0.022364}{2 \times 32.87} \times 4{,}033{,}520 = -1{,}372 \text{ kva per phase}$$

This will be the lagging reactive component of the load at minimum transmission losses in terms of receiver load and will remain constant regardless of any variation in the value of P_r. For $P_r = 10{,}000$ kw per phase, we have

$$10{,}000 - j1{,}372 = 10{,}094\underline{/-7°49'} \text{ kva.}$$

The receiver power factor at maximum transmission efficiency is therefore 99.07 percent lagging.

The total load = $3(10{,}000 - j1{,}372) = 30{,}000 - j4{,}116$, from which the required leading condenser capacity at 100 percent load = $-4{,}116 + 18{,}589 = 14{,}473$ kva.

CHAPTER XV

TRANSMISSION OF POWER AT CRITICAL LOAD[1]

Transmission of power for long distances has now become so necessary that it is not only desirable but imperative to work lines at their maximum safe load. It would be convenient if transmission lines could be worked practically continuously at their most economical load. Unfortunately, however, it is rarely possibly to work them most economically for more than comparatively short periods during a particular load cycle. It is therefore essential that engineers and operators familiarize themselves with the properties and behavior of transmission lines when they are operating at or near their critical load.

To the writer's knowledge, only two papers dealing with electric-power transmission at critical load have been published in this country. It is therefore not surprising that there is little material available on the subject. Probably no other information provides such a clear conception of what actually takes place in a transmission circuit as does a knowledge of its properties at critical load.

For the benefit of those who may not be familiar with the expression "critical load," as applied to a transmission circuit, it means simply a state of what may be defined as "constant ratio," *i.e.*, the ratio of voltage and current are constant throughout the entire length of the line at critical load. Any variation from the critical load or critical-load power factor destroys this constant ratio. The leading and lagging components of kva, here referred to are properties of the line itself and not of the load.

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[1] In the preparation of this chapter, the following publications have been consulted quite freely:

NESBIT, W.: "Electrical Characteristics of Transmission Circuits," Westinghouse Technical Night School, 1926.

Trans. AIEE, 1923 and 1909.

WOODRUFF, L. F.: "Principles of Electric Power Transmission and Distribution," John Wiley & Sons, Inc., New York, 1925.

LOEW, E. A.: "Electric Power Transmission," McGraw-Hill Book Company, Inc., New York, 1928.

Transmission lines of medium voltage will normally operate above their critical load. High-voltage lines will normally operate near or below their critical load.

In ordinary transmission-line problems, we either know or assume the approximate maximum kva or kw load and use this as a basis for determining the size of conductor required for a given voltage and transmission distance. The solutions of line problems involving the critical load are worked out in the reverse order. Here we take either the receiving- or the sending-end voltage and the size of conductor and find the load and power factor. The hyperbolic theory is the basis of the solution. The calculations should be carried out in considerable detail to demonstrate the procedure adequately.

To begin with, we have no conception of the kva, line current, or power factor. It should be further explained that, in transmission-line problems, two distinct properties, the properties of the line itself and the properties of the load, are considered. In this particular case, we are primarily concerned with the properties of the line.

Critical Load.—

1. For every transmission circuit, there is a given load, called the "critical load," the electrical characteristics and power factor of which are determined by the action of the line itself. Under critical load, the power factor is uniform throughout its entire length.
2. The current at critical load will vary throughout the length of the circuit, being directly proportional to the voltage.
3. The critical load is proportional to the square of the voltage and inversely to the surge impedance, being independent of the line length.
4. The critical load has an impedance equivalent to the surge impedance of the line.

Percy H. Thomas, who has long been prominent in the field of electric-power transmission and is nationally recognized as an authority, discovered the properties of the critical load and presented his findings in a paper read at the annual convention of the American Institute of Electrical Engineers, Frontenac,

N. Y., 1909. That was long before the time of high-voltage transmission of power as we know it today.

The discussion by Mr. Thomas, although recognizing that ohmic resistance would be present in the line conductors, was based for the purpose of explanation on the assumption of zero ohmic resistance, in which case the critical load would be imposed on the line at unity power factor. An attempt is here made, it is believed for the first time, to present the problem from a more practical point of view. The ohmic resistance of the line conductors, a property of considerable significance, must therefore be taken into account.

It was pointed out in the Thomas paper that, with the leading and lagging components of energy of the line equal, balanced, and out of phase, at critical load, a condition of unity power factor, $E/I = \sqrt{L/C}$, and the power factor and current will be uniform.

The author wishes to point out that, when the ratio of leading and lagging components of energy of the line are equal to the ratio of z to y, which is not a condition of equal, balanced, out-of-phase energy, $E/I = \sqrt{z/y}$ at critical load. The power factor will be uniform and the current will be out of phase with the voltage in a leading direction. The current will increase as the distance from the receiving end of the line increases, being proportional to the voltage. In this case, the expression $\sqrt{L/C}$ is replaced by the expression $\sqrt{z/y}$, where L and C are the unit inductance and capacitance, and z and y are the unit impedance and admittance.

At the midwinter convention of the American Institute of Electrical Engineers in 1923, H. Goodwin, Jr., presented a paper entitled "Qualitative Analysis of Transmission Lines," in which the properties of the critical load were treated quite extensively.[1]

If line conductors were entirely devoid of ohmic resistance, the critical load of a given line would be determined by application of the following equation:

$$\text{Critical kva} = \frac{10^3\,(kv)^2}{\sqrt{L/C}} \tag{58}$$

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[1] See *Trans. AIEE*, Vol. XLII, p. 24, 1923.

where Kv may be taken to neutral or between phases, depending on whether per phase or total kva is required.

L = unit inductance.

C = unit capacitance.

Since the denominator of (58) involves inductance and capacitance only, the leading and lagging components at critical load are balanced and out of phase, from which the power factor is unity.

Since the linear constants of any transmission-line conductor consist of both ohmic resistance and inductive reactance, which combine vectorially, Eq. (58) is of little practical value. Actually, the critical load is determined as follows:

$$\text{Critical kva} = \frac{10^3(Kv)^2}{\sqrt{z/y}} \tag{59}$$

where z = unit impedance, vector ohms.

y = unit admittance, mhos.

$\sqrt{z/y}$ = surge impedance = z_o.

The surge impedance of a transmission circuit (natural impedance) at critical load is a measure of the voltage current ratio E/I. The meaning of surge impedance of a circuit is not generally so well understood as are some of the more frequently used constants. However, this constant is relatively important, since it limits the current that can be produced by a transient voltage.

One important item that the reader should fix firmly in mind is that a load having the properties of the critical load can be imposed on a given circuit at only one power factor, which is determined by the action of the line itself. The power factor is a function of the line impedance and capacitance and will therefore depend on the material, size, and spacing of line conductors. For the load to be critical, *i.e.*, to have the properties of the critical load, the equivalent impedance of the load must equal the surge impedance of the line. Otherwise, as previously pointed out, the constant ratio properties no longer hold, even though the kva may be equal to the critical kva.

The load impedance = $(E_{rn})^2/(Va)_n$, where E_{rn} is the receiver voltage to neutral and $(Va)_n$ the volt-amperes to neutral. By computing the impedance of the load and the surge impedance of the line, it is immediately determined whether the actual load is greater or less than the critical load. When the impedance of the load is of a higher value than the surge impedance of the line, the load is less than the critical load, and conversely.

For the purpose of demonstrating the method and procedure to be applied in the analysis in order to determine the properties of the circuit at critical load, the following conditions are assumed to apply: 250-mile line, 132 kv, three-phase, 60 cycles, consisting of 4/0 seven-strand copper, having an equivalent conductor spacing of 15 ft.

The calculations are carried out to give a high degree of accuracy for the assumed conditions. This is necessary, as will be seen, in order to verify certain statements with respect to the properties of the line at critical load and also to give values that will provide a reliable check as to the reasonableness of the basic theories involved.

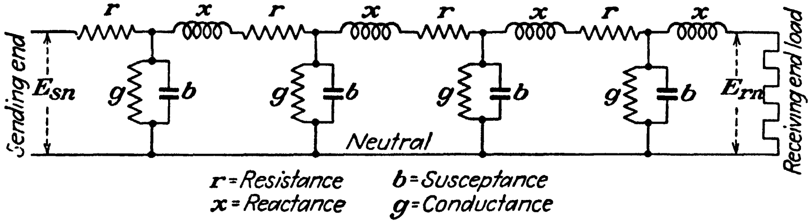

FIG. 119.—Equivalent circuit to neutral.

Figure 119 represents, symbolically, a physical conception of what takes place in a transmission circuit. This diagram is the same as Fig. 23 but is reproduced here for convenience. The calculations are based on the hyperbolic theory and are carried out as follows:

UNIT CONSTANTS AND COMPUTED VALUES

$r = 0.2776$ ohm effective 60 cycle resistance

$x = 0.832$ ohm inductive reactance

$z = r + jx = 0.2776 + j0.832 = 0.87709\underline{/71^\circ 32' 55''}$ ohm impedance

$$C = \frac{0.038829}{\log_{10}(2D/d)} = \text{capacitance, microfarads per mile}$$

$D = 15$ ft $= 180$ in. equivalent spacing of conductors

$d = 0.522$ in. diameter of conductor

$$\frac{2D}{d} = \frac{(2 \times 180)}{0.522} = 689.655, \ \log_{10} \text{ of which is } 2.83863$$

$C = \dfrac{0.038829}{2.83863} = 0.01368$ microfarads per mile of single conductor

$g =$ leakage (taken as zero)

$b = 2\pi fC \times 10^{-6} = 377 \times 0.01368 \times 10^{-6} = 5.1569 \times 10^{-6}$ mho susceptance per mile of single conductor

$y = g + jb = 0 + j5.1569 \times 10^{-6}$

$= 5.1569 \times 10^{-6}\underline{|90^\circ}$ mho admittance

$$z_0 = \sqrt{\frac{z}{y}} = \sqrt{\frac{0.87709/\underline{71^\circ32'55''}}{5.1569 \times 10^{-6}\,\underline{|90^\circ}}} = 412.41\overline{\diagdown 9^\circ13'33''}$$

$=$ surge impedance in ohms at an angle

$\alpha = \sqrt{zy} =$ the hyperbolic angle

$$= \sqrt{0.87709/\underline{71^\circ32'55''} \times 5.1569 \times 10^{-6}\,\underline{|90^\circ}}$$

$$= 0.00212675/\underline{80^\circ46'28''}$$

$\alpha + j\alpha_2 = 0.000341 + j0.00209924$

Length of the 250-mile line in terms of its electric propagation

$$= \frac{(x_1\alpha_2)}{2\pi} = \frac{250 \times 0.00209924}{6.2831835} = 0.0835 \text{ wave length}$$

Full wave length $= \dfrac{250}{0.0835} = 2{,}993$ miles $= 360$ deg

$E_{rn} = \dfrac{132{,}000}{\sqrt{3}} = 76{,}212/\underline{0^\circ}$ volts to neutral

$$I_r = \frac{E_{rn}}{z_0} = \frac{76{,}212/\underline{0^\circ}}{412.41\overline{\diagdown 9^\circ13'33''}} = 184.8/9^\circ13'33'' \text{ amp}$$

Power factor $= \cos 9^\circ13'33'' = 0.987065$ leading. Say 98.71 percent

From (59) Critical kva $= \dfrac{(E_{rn})^2}{z_0} = \dfrac{(76{,}212)^2}{412.41}$

$= 14{,}084$ kva per phase

$= 0.9871 \times 14{,}084 = 13{,}902$ kw per phase

Let E_p and I_p represent volts to neutral and current in amperes at any point p on the line. $\alpha_1 =$ the real component of the hyperbolic angle $= 0.000341$; and $x_1 =$ distance in miles from the receiving end of the line. The voltage at any point p on the line at critical load is derived as follows:

$$\log_e E_p = \log_e E_{rn} + \alpha_1 x_1,$$

where e is the base of the natural logarithm.

This may be expressed in terms of common logarithms as follows:

$$\log_{10} E_p = \log_{10} E_{rn} + 0.43429(\alpha_1 x_1) \tag{60}$$

By the use of Eq. (60) and a table of common logarithms, the calculations are reduced to their simplest form. The essential values have been computed and tabulated under appropiate column headings in Table 43. Angles are referred to the receiving-end voltage vector, minutes and seconds being expressed in decimal form in order to reduce the space required. The table values may be explained as follows:

TABLE 43.—CONSTANTS VOLTAGE AND CURRENT AT VARIOUS POINTS ON THE LINE

x_1 miles	Wave lengths	0.43429 $(\alpha_1 x_1)$	$\log_{10}$ E_p	E_p volts	Angle with E_{rn}	I_p amp	Angle with E_{rn}
0	0	0	4.8820	76,212	0	184.80	9.226
50	0 0167	0.0074	4.8893	77,530	6.014	188.84	15 239
100	0 0334	0.0148	4.8968	78,856	12.027	191.23	21 253
150	0 0501	0.0222	4.9042	80,209	18 041	194.50	27 227
200	0 0668	0.0296	4.9116	81,591	24 054	197.84	33 280
250	0 0835	0.0370	4.9190	82,994	30.068	201.24	39.294

For example, refer to $x_1 = 100$ miles

x_1 in terms of a wave length $= \dfrac{(100 \times 0.00209924)}{6.2831835} = 0.0334$

$0.43429(\alpha_1 x_1) = 0.43429 \times 0.000341 \times 100 = 0.0148$

$\log_{10} E_{rn} = \log_{10} 76{,}212 = 4.8820$, from which $\log_{10} E_p = 4.8820 + 0.0148 = 4.8968$

From a table of common logarithms, E_p is found to be 78,856 volts

Angle with $E_{rn} = 0.0334 \times 360 = 12.027°$. Or $100/2{,}993 \times 360 = 12.027°$

$I_p = \dfrac{78{,}856}{76{,}212} \times 184.8 = 191.23$ amp. The power factor is leading by 9.226°. Therefore, I_p leads E_{rn} by $12.027 + 9.226 = 21.253°$. Other values are found in the same manner.

The six sets of vectors in Fig. 120 may be visualized as representing the respective positions of voltage and current at 0, 50, 100, 150, 200, and 250 miles from the receiving end of the line, or a little over 0.08 of the full wave length of 2,993 miles. Counterclockwise rotation of the vectors is assumed, and it should be noted that the voltage and current vectors occupy the same relative positions at each of the given points on the line.

The electric propagation of 2,993 miles per cycle is equal to 60 × 2,993 = 179,580 mps. This is only about 96.5 percent of the velocity of light, which is approximately 186,000 mps. Were it not for the internal inductance, *i.e.*, the inductance inside the conductor, the velocity of the electric and magnetic field would be the same as the velocity of light, and the wave length would be in the order of 3,100 miles.

Transmission-line performance at critical load, as well as for other load conditions, may be solved by rigorous calculations.

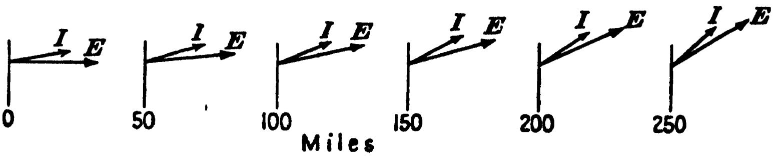

Fig. 120.—Vector representation of critical-load transmission for the 250-mile line.

Rigorously calculated values give values of voltage, current, and power factor the same as those derived from the hyperbolic theory and compiled in Table 43.

In verification of the statement that critical load is imposed on a transmission circuit at one power factor only, and that any variation therefrom will destroy the constant ratio properties of the circuit, refer to Table 44, which gives values at four different power factors derived from rigorous calculations for a load equivalent to the critical kva.

Table 44

$(Pf)_r$, percent	E_{sn}, volts	I_s, amp	Phase angle	$(Pf)_s$, percent
98.71*	82,994/30°04′05″	201.24/39°17′38″	9°13′33″	98.71*
100.00	87,652/27°17′	188.88/32°37′	5°20′	99.57
−99.00	91,341/24°42′	177.00/26°31′	1°49′	99.95
−98.00	92,760/23°46′	172.19/23°54′	0°17′	100.00 −

Minus sign indicates lagging power factor.
* Critical load power factor.

In order to construct the performance diagram, a breakdown of certain values in Table 43 is required. This is best accomplished by application of the auxiliary constants *A*, *B*, and *C*. The auxiliary constants are derived as described in Chap. V for the line at 50-mile intervals and are compiled in Table 45.

TABLE 45.—AUXILIARY CONSTANTS

Miles	$a_1 + ja_2$	$b_1 + jb_2$	$c_1 + jc_2$
50	0 994641 + *j*0.001786 0.994642$\underline{/0°06'10''}$	13.83 + *j*41 53 43 78$\underline{/70°34'55''}$	−0 00000015 + *j*0 0002574 0 0002574$\underline{/90°02'04''}$
100	0.978615 + *j*0 007107 0 97864$\underline{/0°24'58''}$	27 36 + *j*82 67 87 36$\underline{/70°40'18''}$	−0 00000123 + *j*0 000512 0 000512$\underline{/90°08'35''}$
150	0 952077 + *j*0 015846 0 95221$\underline{/0°57'13''}$	40.31 + *j*123 02 129 46$\underline{/70°52'24''}$	−0 00000411 + *j*0 0007611 0 0007615$\underline{/90°18'13''}$
200	0.915275 + *j*0 027819 0.915698$\underline{/1°44'27''}$	52 38 + *j*162 20 170 45$\underline{/71°09'31''}$	−0 00000968 + *j*0 0010021 0 0010021$\underline{/90°33'13''}$
250	0 868583 + *j*0 042737 0.869634$\underline{/2°49'00''}$	63 32 + *j*199 82 209 61$\underline{/72°25'07''}$	−0 00001871 + *j*0 0012323 0 0012323$\underline{/90°52'12''}$

Rigorous Calculations

$$E_1 = E_{rn}(a_1 + ja_2) = 66{,}197 + j\ 3{,}257 = 66{,}277\underline{/2°49'}\text{ volts}$$

$$E_2 = I_r(b_1 + jb_2) = \underline{\ 5{,}629 + j38{,}325} = 38{,}735\underline{/81°38'40''}\text{ volts}$$

$$E_{sn} = E_1 + E_2 = 71{,}826 + j41{,}582 = 82{,}994\underline{/30°04'05''}\text{ volts}$$

$$I_1 = I_r(a_1 + ja_2) = 157.17 + j\ 33.53 \text{ vector amp}$$

$$I_2 = E_{rn}(c_1 + jc_2) = \underline{-1.43 + j\ 93.92} \text{ vector amp (charging current)}$$

$$I_s = I_1 + I_2 = 155.74 + j127.45 = 201.24\underline{/39°17'38''}\text{ amp}$$

I_s leads E_{sn} by 9°13′33″, from which the power factor at the sending end of the line is 98.71 percent leading, as before.

$$(Kw)_{rn} = 76.212 \times 184.8 \times 0.9871 = 13{,}902 \text{ kw per phase}$$

$$(Kw)_{sn} = 82.994 \times 201.24 \times 0.9871 = 16{,}486 \text{ kw per phase}$$

$$\text{Transmission efficiency} = \frac{13{,}902}{16{,}486} = 0.8433 \text{ or } 84.33 \text{ percent}$$

These values are seen to correspond with the sending-end values as derived from the hyperbolic theory and provide a definite check on the work.

Figure 121 is a performance diagram for the 250-mile line and depicts by vector representation the line performance when operating at its critical load. To construct the diagram, a simple matter, proceed as follows:

Lay off vector *OR* to represent the receiver voltage E_{rn} = 76,212 volts, and let this be the vector of reference. Plot point *B* on *OR* a distance from *O* equal to 66,197 volts. Draw *BC* at right angles to *OR* and to a length of 3,257 volts to the same scale as *OR*. $OC = E_1 = E_{rn}A$ = 66,277 volts. Draw *CF* leading vector *OR* by 72°25′. This will represent the 100 percent power factor line. Draw *CD* displaced from vector *OR* by 81°38′40″ and to a length of I_rB = 38,735 volts and represents the impedance drop E_2 at critical load. *CD* leads *CF* by

$$81°38'40'' - 72°25'07'' = 9°13'33''.$$

Vector *OD* represents the sending-end voltage at critical load and is equal to 82,994 volts to neutral leading vector *OR* by 30°04′05″.

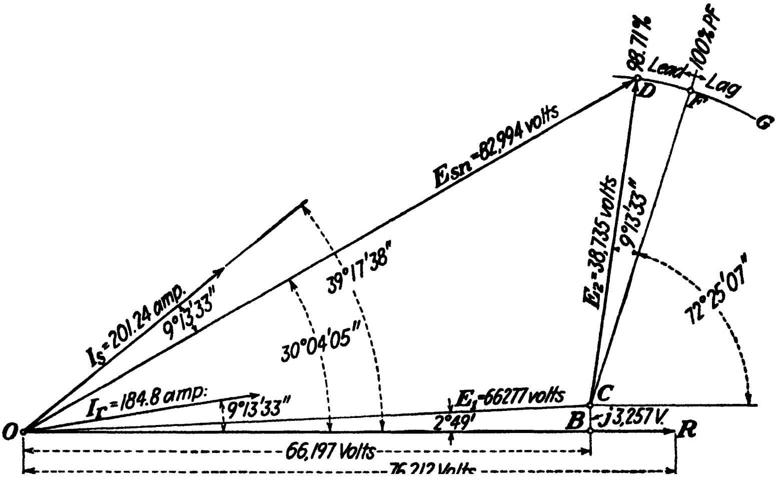

FIG. 121.—Transmission-line performance at critical load.

To construct the current diagram, draw I_r, to a suitable scale, to a length of 184.8 amp, leading *OR* by 9°13′33″. Draw I_s to a length of 201.24 amp, leading *OR* by 39°17′38″. Then I_s leads E_{sn} by 39°17′38″ − 30°04′05″ = 9°13′33″. This graphical representation of the line performance demonstrates conclusively that the power factor is uniform.

Leading and Lagging Kva of the Line.—There are three distinct properties of a transmission circuit, a graphical representation and study of which are to a certain degree essential to a verification of the statement that critical load is imposed on the circuit at leading power factor.

I^2r = energy flowing to the resistance of the circuit. This represents real power and is in phase with the voltage.

I^2X = energy flowing to the reactance of the circuit. This represents lagging reactive kva and is out of phase with the voltage producing it by 90 deg in a lagging direction.

E^2b = energy flowing to the capacity of the circuit. This represents leading reactive kva and is out of phase with the voltage producing it by 90 deg in a leading direction.

I^2X and E^2b, in this case, are functions of the properties of the line. They are independent of the character of the load and should not be confused with the leading and lagging kva, which may be required for voltage control and power-factor correction. Obviously, since the leading reactive kva E^2b is proportional to the voltage, its value is practically independent of the load. This property is relatively important, since it cannot be neutralized except by the lagging reactive kva in a loaded line. The lagging reactive component I^2X varies as the square of the current and is therefore practically non-existent in a lightly loaded line.

Unit Values at Receiving End

$(I^2r)_r = 184.8^2 \times 0.2776 \times 10^{-3} = 9.48$ kw
$(I^2x)_r = 184.8^2 \times 0.832 \times 10^{-3} = 28.41$ lagging reactive kva
$(E^2b)_r = (76.212)^2 \times 5.1569 \times 10^{-6} = 29.95$ leading reactive kva
From this, the resultant value is
$9.48 - j28.41 + j29.95 = 9.48 + j1.54 = 9.6\underline{/9^\circ 13' 33''}$ kva

Unit Values at Sending End

$(I^2r)_s = 201.24^2 \times 0.2776 \times 10^{-3} = 11.24$ kw
$(I^2x)_s = 201.24^2 \times 0.832 \times 10^{-3} = 33.70$ lagging reactive kva
$(E^2b)_s = 82.994^2 \times 5.1569 \times 10^{-6} = 35.52$ leading reactive kva
From this

$11.24 - j33.70 + j35.52 = 11.24 + j1.83 = 11.39/9^\circ 13' 33''$ kva

The above derived values for each end of the line are shown in both phase and magnitude in Fig. 122. Values at intermediate points on the line will fall on the straight-line loci as shown. This diagram makes clear the meaning of the statement that, when $E/I = \sqrt{z/y}$, it does not represent balanced out-of-

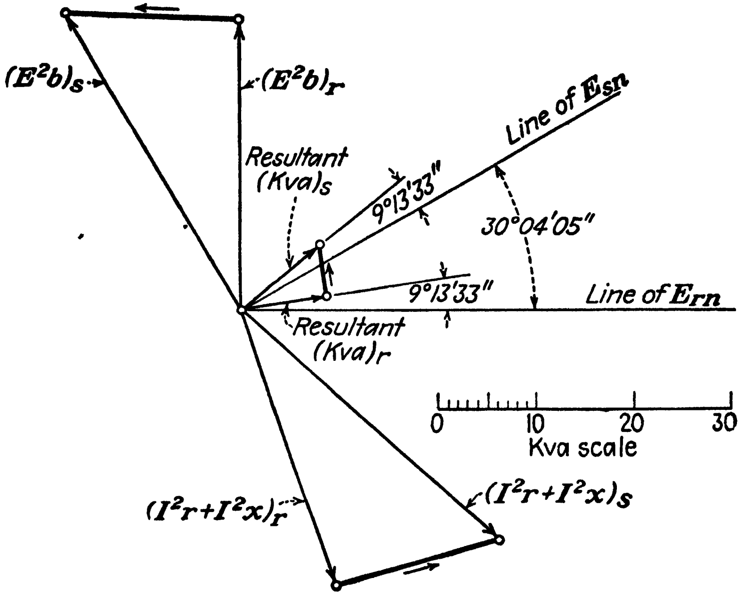

FIG. 122.—Leading and lagging reactive kva of the line

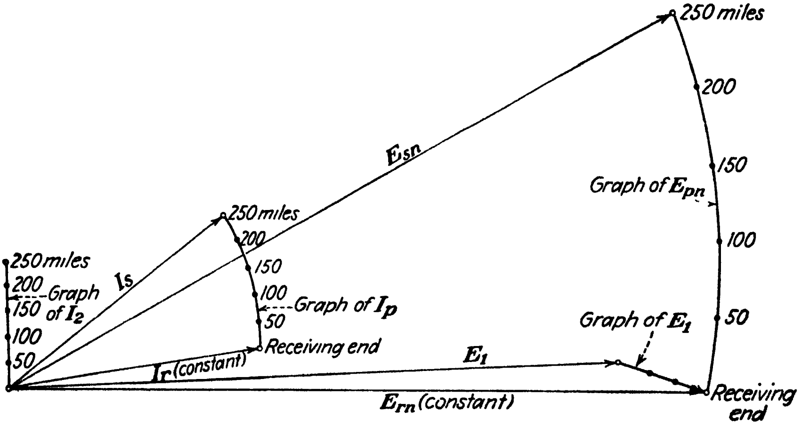

FIG. 123.—Polar diagram for transmission line at critical load with the power factor determined by the action of the line itself.

phase energy at critical load. It also makes clear that, since ohmic resistance will always be present, even though in an infinitesimal amount, the power factor will always be leading at critical load. The larger the conductor, *i.e.*, the greater the ratio of x to r, the nearer the critical-load power factor will approach unity.

A clear conception of the line performance at critical load will be obtained from a study of the polar diagram in Fig. 123. Graphs of E_1, E_p, I_p, and I_2 have been plotted to indicate the positions of the vectors at intervals of 50 miles. It is evident that, insofar as voltage drop alone is concerned, the critical load could be transmitted the full 250 miles.

A convenient method of determining the approximate critical kva, which will usually closely approximate the true critical kva for a given line voltage, may be derived from the following simplified equation:

$$\text{Critical kva} = 2.5 \times (\text{kv})^2$$

The approximate critical kva derived from this equation is based on an assumed surge impedance of 400 ohms. The values are as follows:

66 Kv	10,900 kva
110 Kv	30,250 kva
132 Kv	43,250 kva
154 Kv	59,300 kva
220 Kv	121,000 kva

In the 132-kv line that we have discussed, the critical kva is equal to 3 × 14,084 = 42,250 kva. This is just 1,000 kva below the value derived from the above simplified equation.

Effect of Dropping the Load.—The solution would not be complete without an investigation of the effect of suddenly dropping the load. The danger lies in the fact that full sustained voltage may be maintained at the sending end of the line, at least for a considerable length of time, after the load is dropped. Two things can happen, either of which may cause the voltage at the receiving end of the line to rise to a dangerously high value. Drop the entire load, including the receiving-end transformers, in which case voltage will be impressed on an open line. Drop the load exclusive of the receiving-end transformers, in which case voltage will be impressed on the line and connected transformers. Maximum voltage rise will occur in the first case, and the investigation will be made to determine the effect of full sustained voltage on the open line.

The calculations are somewhat more involved than those for critical-load conditions; however, the derived results provide not only an interesting study but a great deal of information that is valuable from a practical engineering and operating point of view.

In an open line, since there is no load at the receiving end, it is apparent that the so-called "load impedance" is infinite. There is consequently no real part to the load angle. This causes a phase rotation of 90 deg in a leading direction. This may be represented as being equal to $j\pi/2$, or $j1.5708$ circular radians. The angle is therefore equal to $(0 + j1.5708)$ circular radians, expressed in complex form. This represents the zero load angle, designated as θ_{ro}, which must be added to the line angle $x_1\alpha$ as computed for various points along the line. This will give what is known as the position angle θ_{po} at zero load. For example, the unit line angle $\alpha = \sqrt{zy}$ has been computed to be $0.002127\underline{/80°46'}$. In rectangular coordinates $= 0.002127 (\cos 80°46' + j \sin 80°46') = 0.000341 + j0.002099 = \alpha_1 + j\alpha_2$. For $x_1 = 250$ miles (sending end), the line angle

$$x_1(\alpha_1 + j\alpha_2) = 250(0.000341 + j0.002099) = 0.08525 + j0.52481 = 0.53169\underline{/80°46'}.$$

Then the combined zero load and line angles $\theta_s = \theta_1 + j\theta_2 = \theta_{ro} + x_1\alpha = (0 + j1.5708) + (0.08525 + j0.5248) = 0.08525 + j2.0956$ complex hyperbolic radians. $\theta_s = \theta_{po}$ when taken as a point on the line.

The angle $\theta_s = \theta_1 + j\theta_2$ has been referred to as the complex hyperbolic angle and may be written as (hyperbolic angle) $+ j$ (circular angle). The component θ_1 is the hyperbolic, or real, component of the angle and represents a measure of the stretch or shrinkage of the radius vector. The circular component θ_2 represents the arc, or swing, of the radius vector of fixed length. It will therefore be seen that the quadrature component θ_2 must be changed to read in terms of the circular angle. This is accomplished by multiplying θ_2 by $180/\pi = 57.2958$, the number of degrees in one circular radian.

The object of this study is to determine the distribution of voltage and current along the line at zero load. The hyperbolic functions of the position angles, as above derived, are determined from the following equations:

$$\sinh(\theta_1 + j\theta_2) = (\sinh \theta_1 \cos \theta_2 + j \cosh \theta_1 \sin \theta_2) \qquad (61)$$

$$\cosh(\theta_1 + j\theta_2) = (\cosh \theta_1 \cos \theta_2 + j \sinh \theta_1 \sin \theta_2) \qquad (62)$$

To aid in the solution of equations such as (61) and (62), exponential and hyperbolic functions have been derived for values of x in intervals of 0.001 to 0.525 and compiled in Table 50. To facilitate interpolation between the table values of hyperbolic sines and cosines the differences between consectutive functions are given. Values of x for which the functions are derived are given to only three places. For extreme accuracy, interpolation will therefore be required. Interpolation is simplified by use of the tabular differences.

In order to conserve space, the differences are given only to the number of places in which there exists an actual difference in the consecutive functions. Since the functions in Table 50 are given for intervals of 0.001, direct interpolation may be applied, the work being carried out as follows:

Refer to Table 50, from which it will be seen that the hyperbolic sine of 0.085 and 0.086 are 0.0851024 and 0.0861061. Difference = 0.0010037. Hyperbolic cosine of 0.085 and 0.086 are 1.0036147 and 1.0037003. Difference = 0.0000856. 0.25 × 0.0010037 = 0.0002509. sinh 0.08525 is therefore 0.0851024 + 0.0002509 = 0.0853533.* 0.25 × 0.0000856 = 0.0000214. cosh 0.08525 is therefore 1.0036147 + 0.0000214 = 1.0036361.*

Converting the j term to its equivalent circular angle for

$$x_1 = 250 \text{ miles} = 180/\pi \times \theta_2 = 57.2958 \times 2.0956 = 120.069° = 120°04',$$

the cosine and sine of which are −0.50104 and 0.86542, respectively. Substituting in (61) and (62) and solving, we have

$$\begin{aligned}\text{From (61)} \sinh(\theta_1 + j\theta_2) &= \sinh \theta_{p0} \\ &= (0.08535 \times -0.50104 \\ &\qquad + j1.003636 \times 0.86542) \\ &= -0.04277 + j0.86857 \\ &= \underline{0.86964\backslash 92°49'}\end{aligned}$$

---

* The error resulting from this method of interpolation will not exceed one unit in the seventh place for any value in Table 50.

$$\begin{aligned}\text{From (62) } \cosh(\theta_1 + j\theta_2) &= \cosh \theta_{p0} \\ &= (1.003636 \times -0.50104 \\ &\qquad + j0.08535 \times 0.86542) \\ &= -0.50286 + j0.07387 \\ &= 0.50826\backslash\underline{171^\circ38'}\end{aligned}$$

The complex hyperbolic angle θ_{p0}, and the hyperbolic sine and cosine of θ_{p0} at intervals of 50 miles are compiled in Table 46.

TABLE 46

x_1, miles	θ_{po}	sinh θ_{po}	cosh θ_{po}
0	$0 \quad + j1.5708$ $\theta_2 = 90^\circ$	$0 \quad + j1.0000$ $= 1.0000\,\vert\underline{90^\circ}$	0 $0\,\vert\underline{90^\circ}$
50	$0.01705 + j1.6758$ $\theta_2 = 96^\circ01'$	$-0.00179 + j0.99464$ $= 0.99464\backslash\underline{90^\circ06'}$	$-0.10483 + j0.01696$ $= 0.1062\backslash\underline{170^\circ49'}$
100	$0.03411 + j1.7807$ $\theta_2 = 102^\circ02'$	$-0.00712 + j0.97856$ $= 0.97858\backslash\underline{90^\circ25'}$	$-0.20849 + j0.03336$ $= 0.21114\backslash\underline{170^\circ55'}$
150	$0.05115 + j1.8857$ $\theta_2 = 108^\circ02'$	$-0.01585 + j0.95207$ $= 0.95220\backslash\underline{90^\circ57'}$	$-0.31013 + j0.04866$ $= 0.31393\backslash\underline{171^\circ05'}$
200	$0.06820 + j1.9906$ $\theta_2 = 114^\circ03'$	$-0.02782 + j0.91530$ $= 0.9157\backslash\underline{91^\circ43'}$	$-0.40853 + j0.06233$ $= 0.41326\backslash\underline{171^\circ20'}$
250	$0.08525 + j2.0956$ $\theta_2 = 120^\circ04'$	$-0.04277 + j0.86857$ $= 0.86964\backslash\underline{92^\circ49'}$	$-0.50286 + j0.07387$ $= 0.50826\backslash\underline{171^\circ38'}$

Let us designate the combined zero load and line angles at the sending end of the line as θ_s; and at other points along the line as θ_{p0}; the zero-load voltage at the receiving end as E_{rn0}; the current at the sending end as I_s, and at other points along the line as I_{p0}. The voltage follows the complex function (sinh θ_{p0}/sinh θ_s), and the current follows the complex function (cosh θ_{p0}/cosh θ_s.) Substituting numerical values from Table 46 for any given point on the line in the following equations and solving, we have

$$E_{rn0} = \frac{E_{sn} \times \sinh \theta_{p0}}{\sinh \theta_s} \tag{63}$$

$$I_s = \frac{E_{sn} \times \cosh \theta_s}{Z_0 \times \sinh \theta_s} \tag{64}$$

$$I_p = \frac{I_s \times \cosh \theta_{p0}}{\cosh \theta_s} \tag{65}$$

The numerical values may be expressed in rectangular coordinates or polar form as desired. The work is, however, reduced considerably by the polar expression, as follows:
From (63) for $x_1 = 0$ miles

$$E_{rn0} = \frac{82{,}994\underline{/2^\circ 49'} \times 1.000\underline{|90^\circ}}{0.86964\underline{\backslash 92^\circ 49'}} = 95{,}435\underline{/0^\circ} \text{ volts}$$

From (64) for $x_1 = 250$ miles

$$I_s = \frac{82{,}994\underline{/2^\circ 49'} \times 0.50826\underline{\backslash 171^\circ 38'}}{412.41\underline{\backslash 9^\circ 14'} \times 0.86964\underline{\backslash 92^\circ 49'}} = 117.61\underline{\backslash 90^\circ 52'} \text{ amp}$$

From (65) for $x_1 = 0$ miles

$$I_{p0} = \frac{117.61\underline{\backslash 90^\circ 52'} \times 0\underline{|90^\circ}}{0.50826\underline{\backslash 171^\circ 38'}} = 0\underline{/9^\circ 14'} \text{ amp}$$

The value of E_{rn0} and I_s may be verified as follows: Refer to Table 45 and find the auxiliary constants A and C to be 0.869634 and 0.00123, respectively, from which 82,994/0.869634 = 95,435 volts to neutral. $I_s = 95{,}435 \times 0.00123 = 117.6$ amp charging current. This checks the values derived from the hyperbolic theory.

When the polar expressions are used, it is sometimes rather difficult, especially for those who may not be familiar with this method of analysis, to know in just what manner the angles should be combined. This may readily be determined, however, by converting from polar to rectangular form and then performing the operations. For example, take the solution for I_s, Eq. (64), the numerical values of which are expressed in polar form.

The cosine and sine of 2°49′, which lies in the first quadrant, are both plus. The cosine of 171°38′ and 92°49′, which lies in the second quadrant is minus and the sine is plus. The cosine of 9°14′, which lies in the fourth quadrant, is plus and the sine is minus. From this we have below the expression for (64) in rectangular form.

$$I_s = \frac{(82{,}984 + j4{,}078)(-0.50286 + j0.07387)}{(407.07 - j66.17)(-0.04277 + j0.86857)}$$

$$= \frac{-41{,}985 + j4{,}075}{40.06 + j356.4}$$

$$\tan^{-1}\left(\frac{4{,}075}{-41{,}985}\right) = 174^\circ 27'$$

$$\tan^{-1}\left(\frac{356.4}{40.06}\right) = 83^\circ 35'$$

$$I_s = \frac{-41{,}985 + j4{,}075}{40.06 + j356.4} = \frac{42{,}182\underline{\backslash 174^\circ 27'}}{358.65\underline{/83^\circ 35'}} = 117.61\underline{\backslash 90^\circ 52'} \text{ amp}$$

The kva required to charge the line at zero load is equal to $82.994\underline{/2^\circ 49'} \times 117.61\underline{\backslash 90^\circ 52'} = 9{,}760\underline{/88^\circ 03'}$ kva per phase. The total generator capacity at zero load will therefore be 3 × 9,760 = 29,280 kva. Other values of voltage, current, and charging kva are computed for intervals of 50 miles and compiled in Table 47.

TABLE 47.—VOLTAGE, CURRENT, AND KVA AT 50-MILE INTERVALS

x_1, miles	E_{pno}	I_{po}	$(Kva)_{pno}$
0	$95{,}435\underline{/0^\circ}$	$0\underline{/9^\circ 14'}$	0
50	$94{,}923\underline{/0^\circ 02'}$	$24.57\underline{\backslash 90^\circ 02'}$	$2{,}332\underline{/89^\circ 58'}$
100	$93{,}390\underline{/0^\circ 25'}$	$48.85\underline{\backslash 90^\circ 09'}$	$4{,}652\underline{/89^\circ 44'}$
150	$90{,}873\underline{/0^\circ 57'}$	$72.64\underline{\backslash 90^\circ 19'}$	$6{,}601\underline{/89^\circ 22'}$
200	$87{,}393\underline{/1^\circ 43'}$	$95.63\underline{\backslash 90^\circ 34'}$	$8{,}387\underline{/88^\circ 51'}$
250	$82{,}994\underline{/2^\circ 49'}$	$117.61\underline{\backslash 90^\circ 52'}$	$9{,}760\underline{/88^\circ 03'}$

The solution should be considered as applying not only to a line 250 miles long, but also to a line of any length, when taken from the receiving end, up to 250 miles. For example, consider x_1 to be 100 miles, line 3, Table 47. Then, with a voltage to neutral E_p of 78,856 (see Table 43), the voltage to neutral at the receiving end of a line 100 miles long, from Eq. (63), would be 88,735 volts. Or 1.732 × 88,735 = 153,689 volts between phases.

Curves have been plotted in Fig. 124 to show the voltage and current distribution at both critical and zero loads. From them, values at intermediate points on the line may be read, Critical-load voltage and current distribution are represented by straight lines, as a result of the constant ratio properties of the line (previously discussed) at critical load. Zero-load voltage and current distribution lines are slightly curved.

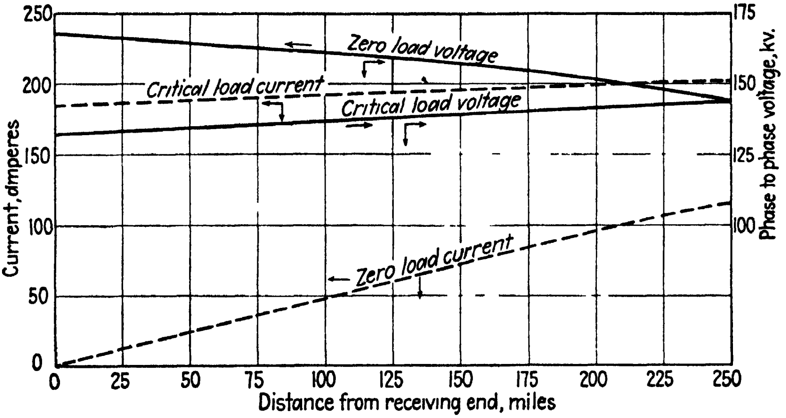

FIG. 124.—Voltage and current distribution at critical and zero loads.

The zero-load study should not be confused with the overvoltage that may be impressed on a transmission line and associated equipment from overspeeding of generators when they are suddenly relieved of their load. This study applies strictly to the rise in voltage along an unloaded line produced by the line charging current. The voltage produced by the charging current is present whether or not the line is loaded, but for a loaded line it is partly or wholly neutralized by the lagging component, or *IX* drop in the line.

CHAPTER XVI

MATHEMATICAL OPERATIONS AND CORONA-LOSS CALCULATIONS

Vector Operations.—If an engineer were deprived of the use of vectors in connection with the analysis of high-voltage transmission-line problems, it would be almost as much of a handicap as if a carpenter were deprived of the use of his saw and hammer. Either would have to work without his most useful tools. Vector analysis was resorted to in Chaps. V, XIV, and XV. In view of the importance of the operation of vectors, however, it is believed that the subject should have further treatment. Examples demonstrating the procedure with vectors expressed in complex form are therefore included. The illustrations and operations will, of course, seem elementary to more advanced engineers, but they should be of value to many engineers less experienced in the solution of problems involving the use of vectors.

The vector is extremely useful for providing a picture of certain conditions, such as voltage, current, kva, and power factor in an electric circuit. It can be made to represent, by its length, the magnitude of a given quantity, to a given scale in inches, feet, miles, etc. The position of the quantity is shown by its angular displacement, measured in degrees or radians with respect to a given line called the "vector of reference."

A vector quantity should be considered as made up of two components at right angles to each other. Thus, in Fig. 125, vector A is the resultant of the components a_1 and jb_1. a_1 has a length of 40 units in phase with, or coinciding with the X axis (axis of reals); and jb_1 has a length of 30 units coinciding with the Y axis. a_1 is taken as the real component; and b_1 is taken as the imaginary, or quadrature, component.

Addition and Subtraction of Vectors.—Let it be required to find the resultant A of the complex expression $a_1 + jb_1$; and the resultant B of the complex expression $a_2 + jb_2$. Then add the components of A and B to obtain the final resultant C (Fig. 125).

It will be seen from the figure that $A = a_1 + jb_1$, the absolute value of which $= \sqrt{a_1^2 + b_1^2}$. Since the real component $a_1 = 40$ units of length, and the quadrature component jb_1 30 units of length, $A = 40 + j30 = \sqrt{40^2 + 30^2} = 50$. Similarly, vector $B = a_2 + jb_2 = \sqrt{a_2^2 + b_2^2} = 30 - j20 = \sqrt{30^2 + 20^2} = 36.67$. Assuming counterclockwise rotation of the vectors, vector A is displaced from the X axis in a leading direction by the angle whose tangent $= 30/40 = 36°52'$. Vector B is displaced from the X axis in a lagging direction by the angle whose tangent $= -20/30 = -33°42'$. In practice, these would be written $A = 40 + j30 = 50\underline{/36°52'}$; and $B = 30 - j20 = 36.67\overline{\backslash 33°42'}$. Or $36.67\underline{/-33°42'}$. $\theta_a = \tan^{-1} 30/40 = 36°52'$. $\theta_b = \tan^{-1} -20/30 = -33°42'$.

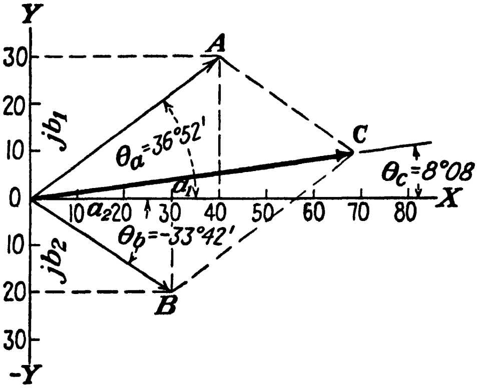

FIG. 125.

$$C = A + B = (a_1 + jb_1) + (a_2 + jb_2)$$
$$= (a_1 + a_2) + j(b_1 + b_2)$$

Substituting the known numerical values and adding, we have

$$A = 40 + j30$$
$$B = \underline{30 - j20}$$
$$C = 70 + j10 = 70.71\underline{/8°08'} \text{ as indicated in Fig. 125.}$$

The subtraction of vectors is accomplished by reversing one of the vectors and changing the signs, then adding algebraically as before. Let it be required to subtract vector B from vector

C in Fig. 126. This will give vector A and will serve as a check on the addition of vectors A and B.

$A = C - B$. Reversing vector B, as shown, we have $-B = -30 + j20$. From this

$$\begin{aligned} C &= 70 + j10 \\ B &= \underline{-30 + j20} \\ A &= 40 + j30 = 50\underline{/36^\circ 52'} \end{aligned}$$

If it had been required to subtract vector C from B, we should have

$$\begin{aligned} C &= -70 - j10 \\ B &= 30 - j20 \\ A &= -40 - j30 = 50/36^\circ 52' \text{ (third quadrant)} \end{aligned}$$

In all cases, the resultant vectors are made up of the components a_1, jb_1, a_2, and jb_2. The real and quadrature components are shown only in Fig. 125, however.

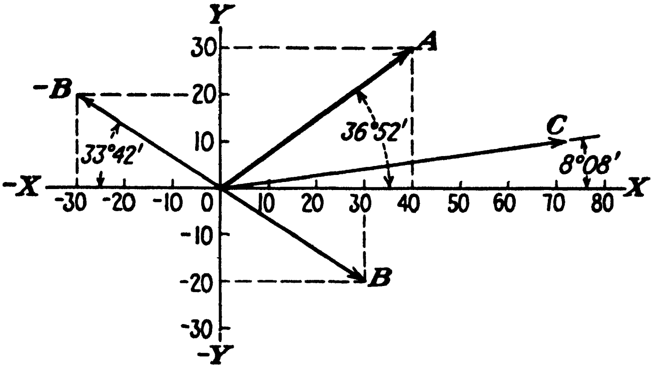

FIG. 126.

Multiplication of Vectors.—The product of two complex quantities is a complex quantity, whose length is equal to the product of the lengths of the original vectors and whose angle of displacement is the sum of the original angles.

Let it be required to find the product of vectors A and B in Fig. 127. To obtain the expression for the product of the two complex quantities $a_1 + jb_1$ and $a_2 + jb_2$, we have

$$\begin{aligned} A &= a_1 + jb_1 \\ B &= \underline{a_2 + jb_2} \\ C &= a_1a_2 + a_2jb_1 + a_1jb_2 - b_1b_2 \end{aligned}$$

A convenient method, after substituting numerical values, is to express each of the complex numbers in polar form, find the product of the two polar expressions, and then add the two angles. Thus $A = 4 + j3 = 5\underline{/36^\circ 52'}$, and $B = 3 - j2 = 3.605\overline{\backslash 33^\circ 42'}$.

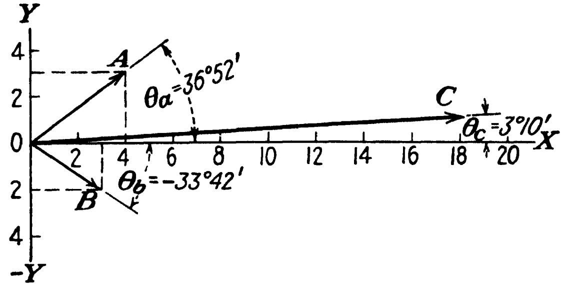

FIG. 127.—Multiplication of complex quantities.

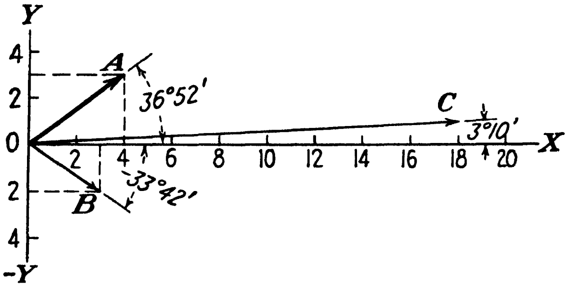

FIG. 128.—Division of complex quantities.

$C = 5\underline{/36^\circ 52'} \times 3.605\overline{\backslash 33^\circ 42'} = 18.03\underline{/3^\circ 10'}$. In rectangular coordinates, $C = 18.03(\cos 3^\circ 10' + j \sin 3^\circ 10') = 18 + j1$. When desired, the operation may be performed in the reverse order.

$$\begin{aligned} A &= 4 + j3 \\ B &= \underline{3 - j2} \\ C &= 18 + j1 = 18.03\underline{/3^\circ 10'} \end{aligned}$$

Division of Vectors.—The division of complex quantities results in a complex quantity whose magnitude is the quotient of the original quantities and whose angle of displacement is the angle of the dividend minus the angle of the divisor.

Let it be required to divide vector C by vector B in Fig. 128. In the division of complex quantities, the denominator must be cleared of its j term. This is accomplished by multiplying both numerator and denominator by the conjugate of the denominator. The conjugate of the denominator is the denominator with the sign of its j term changed.

It has previously been determined that vector C, expressed in rectangular coordinates $= 18 + j1$, from which

$$\begin{aligned} A &= \frac{C}{B} = \frac{18 + j1}{3 - j2} = \frac{(18 + j1)(3 + j2)}{(3 - j2)(3 + j2)} = \frac{52 + j39}{13} \\ &= 4 + j3 = 5\underline{/36^\circ 52'} \end{aligned}$$

A more convenient and practical method is to reduce each of the complex numbers to polar form and then make the division. Thus

$$A = \frac{18.03\underline{/3^\circ 10'}}{3.605\overline{\backslash 33^\circ 42'}} = 5\underline{/36^\circ 52'}$$

Root of a Complex Quantity.—The square root, or any root, of a complex quantity is derived just as readily as is the root of any general number. The method is known as "de Moivre's theorem" and is based on the following law:

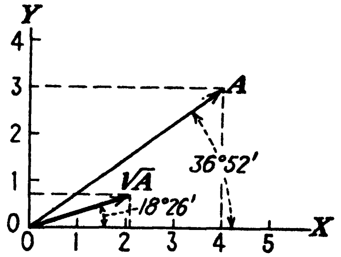

Fig. 129.

The nth root of a complex quantity is a complex quantity whose magnitude is the nth root of the original quantity and whose angle of displacement is the $1/n$th root of the original angle.

Let it be required to find the square root of vector A in Fig. 129. Based on the above law, we have

$$\begin{aligned} A &= 4 + j3 = 5\underline{/36^\circ 52'} \\ \sqrt{A} &= \sqrt{5}\underline{/0.5(36^\circ 52')} = 2.23\underline{/18^\circ 26'} = 2.12 + j0.705 \end{aligned}$$

Power of a Complex Quantity.—A complex quantity raised to a power n is a complex quantity whose magnitude is the nth power of the original quantity and whose angle of displacement is n times the original angle. This is also known as "de Moivre's theorem," and the work is carried out in the following manner:

Let it be required to find the third power of vector A in Fig. 130.

$$A = 4 + j3 = 5\underline{/36^\circ 52'} = 5(\cos 36^\circ 52' + j \sin 36^\circ 52')$$

Complying with the above law, we have

$$\begin{aligned}
A^3 &= 5^3[\cos 3(36^\circ 52') + j \sin 3(36^\circ 52')] \\
&= 5^3(\cos 110^\circ 36' + j \sin 110^\circ 36') \\
&= 125(-0.34857 + j0.93728) \\
&= -43.57 \ + j117.16 \\
&= 125\underline{\backslash 110^\circ 36'} \\
A^2 &= 5^2[\cos 2(36^\circ 52') + j \sin 2(36^\circ 52')] \\
&= 5^2(\cos 73^\circ 44' + j \sin 73^\circ 44') \\
&= 25(0.2801 + j0.96) \\
&= 7.0 + j24 = 25\underline{/73^\circ 44'}
\end{aligned}$$

Mathematical Solution for Voltage and Current at Intermediate Load.—In Chap. XV, a solution was given for the 250-mile 132-kv line, when operating at its critical load, which, as there explained, is a condition of constant voltage-current

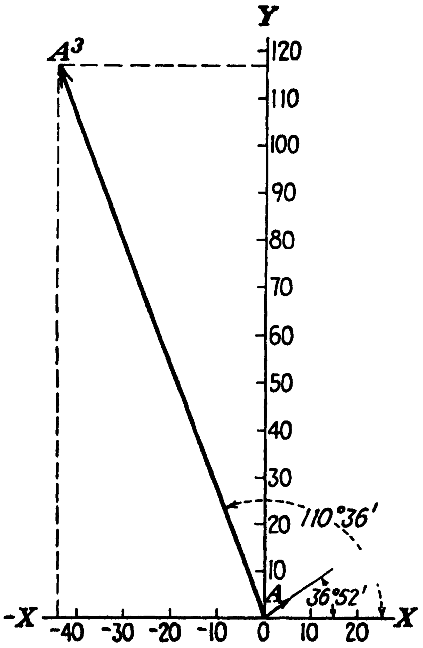

FIG. 130.

ratio. A solution was also given for determining the effect of suddenly dropping the load when full sustained voltage was maintained at the sending end of the line. The discussion therefore covered the two extremes, but it did not take into account the voltage and current distribution at intermediate loads, because the calculations are involved and might have confused the problem.

Let us now consider a case somewhere between the extremes, that of an intermediate load being transmitted over the 250-mile line that was discussed in the previous chapter. The solution in this case differs materially from that at zero load (open line) in that the line angle contains both a real and a quadrature component. It also differs from the zero-load condition, since the receiver voltage must necessarily be maintained at a practically constant value, and both the line and load impedance must be taken to account.

Assume a load of 5,000 kva per phase. The load impedance Z_L varies as the square of the receiving-end voltage, from which

$$Z_L = \frac{(76{,}212)^2}{5{,}000{,}000} = 1{,}162\underline{/36°52'} \text{ ohms load impedance}$$

The surge impedance of the line (previously calculated)

$$= 412.41\overline{\backslash 9°14'} \text{ ohms}$$

$$\tanh \theta_r = \frac{1162\underline{/36°52'}}{412.41\overline{\backslash 9°14'}} = 2.8175\underline{/46°06'} = 1.9537 + j2.03$$

$$\theta_r = \tanh^{-1}(\theta_1 + j\theta_2) = \tanh^{-1}(1.9537 + j2.03)$$

$$\theta_r = 1.1513 \log_{10} \sqrt{\frac{(1+\theta_1)^2 + \theta_2^2}{(1-\theta_1)^2 + \theta_2^2}}$$

$$+ j\left[\frac{180° - \tan^{-1}\left(\dfrac{\theta_1 + 1}{\pm\theta_2}\right) + \tan^{-1}\left(\dfrac{\theta_1 - 1}{\pm\theta_2}\right)}{2}\right] \qquad (66)$$

Substituting in (66) and solving, we have
The real component of θ_r

$$= \sqrt{\frac{(1 + 1.9537)^2 + 2.03^2}{(1 - 1.9537)^2 + 2.03^2}} = \sqrt{\frac{12.845144}{5.030344}} = \sqrt{2.553532}$$

$$= 1.5979774$$

$$\log_{10} 1.5979774 = 0.2035706 \quad \text{(by interpolation)}$$

The real component of $\theta_r = 1.1513 \times 0.2035706 = 0.23437$

$$j\left[\frac{180° - \tan^{-1}\left(\dfrac{1.9537 + 1}{2.03}\right) + \tan^{-1}\left(\dfrac{1.9537 - 1}{2.03}\right)}{2}\right]$$

$$= j\,\frac{(180° - 55°30' + 25°10')}{2} = 74°50' = 74.8333°$$

The quadrature component of $\theta_r = \dfrac{74.8333}{57.2958} = 1.3061$

$$\theta_r = 0.23437 + j1.3061 = 1.327 \text{ radians}$$

The receiving-end angle θ_r must be added to the angle of the line θ (previously derived), which for

$$x_1 = 250 \text{ miles} = 0.08525 + j0.5248.$$

Thus

$$\begin{aligned} \text{Receiving-end angle } \theta_r &= 0.23437 + j1.3061 \\ \text{Line angle } \theta &= \underline{0.08525 + j0.5248} \\ \theta_s = \theta + \theta_r &= 0.31962 + j1.8309 \end{aligned}$$

The hyperbolic functions, sinh and cosh, are derived as explained in Chap. XV and are as follows:

θ_r $0.23437 + j1.3061$ $\theta_2 = 74°50'$	sinh θ_r $0\ 0619 + j0\ 9918$ $= 0\ 9937\underline{/86°26'}$	cosh θ_r $0\ 2688 + j0\ 2283$ $= 0\ 3527\underline{/40°20'}$
θ_s $0.3196 + j1.8309$ $\theta_2 = 104°54'$	sinh θ_s $-0.0836 + j1.0161$ $= 1\ 0196\backslash\underline{94°42'}$	cosh θ_s $-0\ 2704 + j0.3141$ $= 0.4145\backslash\underline{130°44'}$

$$I_r = \frac{E_{rn}}{Z_L} = \text{current at receiving end of line} \qquad (67)$$

$$E_{sn} = \frac{E_{rn} \times \sinh\theta_s}{\sinh\theta_r} = \text{voltage at sending end} \qquad (68)$$

$$I_s = \frac{I_r \times \cosh\theta_s}{\cosh\theta_r} = \text{current at sending end of line} \qquad (69)$$

From (67) $I_r = \dfrac{76{,}212\underline{/0°}}{1162\underline{/36°52'}} = 65.6\overline{\backslash 36°52'}$ amp

the quadrature components of which $= 52.47 - j39.35$ amp

$$\text{From (68) } E_{sn} = \frac{76{,}212\underline{/0^\circ} \times 1.0196\underline{\backslash 94^\circ 42'}}{0.9937\underline{/86^\circ 26'}} = 78{,}198\underline{/8^\circ 16'} \text{ volts}$$

the quadrature components of which = $77{,}385 + j11{,}243$ volts

$$\text{From (69) } I_s = \frac{65.6\overline{\backslash 36^\circ 52'} \times 0.4145\underline{\backslash 130^\circ 44'}}{0.3527\underline{/40^\circ 20'}} = 77.1\underline{/53^\circ 32'}$$

the quadrature components of which = $45.82 + j62.00$ amp

The constant of 1.1513 in Eq. (66) is derived as follows:

Ordinarily the first part of Eq. (66) would be given in terms of natural logarithms to the base e, for example, $\frac{1}{2}(\log_e \sqrt{x})$ where x represents the value of the expression under the radical. In terms of common logarithms

$$\begin{aligned} \theta_r &= \tfrac{1}{2}(2.302585) \times \log_{10} \sqrt{x} \\ &= 1.1513 \times 0.2035706 = 0.23437 \end{aligned}$$

This simplifies the work and makes the use of a table of natural logarithms unnecessary.

An important feature of the above method of solution is the fact that it gives properties of the line distinct from the properties of the load and then combines them to give values from which the voltage and current distribution may be derived.

Effect of Corona Formation.—If the transmission of electric power at high voltage were dependent only upon a conductor of sufficient size to carry the line current at a permissible voltage drop, the design would be comparatively simple. Unfortunately, however, other factors often affect the size of conductor that should be installed.

With low potential difference between conductors, air is practically a perfect insulator. If, however, the impressed voltage is alternating and is increased until what is known as the "disruptive critical voltage" or potential gradient is reached, the air immediately surrounding the conductor becomes ionized and forms a conducting medium. As the potential is further increased to what is known as the "visual critical voltage," the conductor, when viewed in the dark, will give off a violet glow. This phenomenon is known as "corona."

The diameter and roughness of a conductor have an important bearing on the formation of corona, as do the spacing, weather, temperature, and altitude. It is therefore evident, since the designer has no control over any of these items except conductor size and spacing, unless the given line is comparatively short and approximately the same elevation throughout its length, that corona-loss calculations will give only roughly approximate results.

The result of corona formation is a power loss. In the design of high-voltage lines, the conductor should generally be large enough in diameter to prevent any appreciable loss from this source during normal operation. Equations and formulas have been set up by various investigators for computing the critical voltage and power loss produced by the formation of corona on transmission-line conductors. Probably the following equations, which were derived by the late F. W. Peek, Jr., are the most widely used for this purpose:

TABLE 48.—ALTITUDE AND TEMPERATURE CORRECTION FACTORS

Altitude, ft	Barometer, in. of mercury	a_f at 25°C (77°F)	Altitude, ft	Barometer, in. of mercury	a_f at 25°C (77°F)
0	30.00	1.000	3,000	26.73	0.893
100	29.89	998	3,500	26 25	.877
200	29 77	994	4,000	25.75	860
400	29.54	987	4,500	25.26	844
600	29.33	979	5,000	24.76	.827
800	29.10	0.972	6,000	23.84	0.796
1,000	28.88	.964	7,000	22.97	767
1,250	28 60	955	8,000	22.11	738
1,500	28.33	.946	9,000	21.24	.704
2,000	27.79	.928	10,000	20.44	.683
2,500	27.28	.911	12,000	19.00	.633

$$e_0 = 2.303 m_0 g_0 a_f r \log_{10}\left(\frac{D}{r}\right) \qquad (70)$$

$$P = \left(\frac{390}{a_f}\right)(f + 25)\sqrt{\frac{r}{D}}\,(e - e_0)^2 \times 10^{-5} \qquad (71)$$

where e_0 disruptive critical voltage kv to neutral.
m_0 coefficient of roughness for stranded conductor (usually taken as 0.87).
g_0 = disruptive gradient of air (53.6 kv per in.).
a_f = altitude correction factor (see Table 48).

D = spacing of conductors, in.
r = radius of conductor, in.
P = loss, kw per mile for one conductor.
f = frequency, cycles per sec.
e = effective voltage kv to neutral.

$$a_f = \frac{17.9 \times b}{459 + t} \tag{72}$$

where b = barometer reading, in. of mercury.
t = temperature, deg F.

The correction factor a_f for temperatures other than 25°C, the table values, may be derived from (72), as follows:

$$\text{At 1,000 ft } 32^\circ \; a_f = \frac{17.9 \times 28.88}{459 + 32} = 1.053$$

$$\text{At 1,000 ft } 100^\circ \; a_f = \frac{17.9 \times 28.88}{459 + 100} = 0.925$$

In order to demonstrate the procedure in determining the critical voltage and power loss from corona formation for a given condition, let us take the problem used in our critical-load study in Chap. XIV; that of a 132-kv 4/0 seven-strand copper, 250-mile line, equivalent spacing 15 ft, average elevation assumed as 1,000 ft at a temperature of 25°C (77°F).

For an equivalent spacing of 15 ft, the flat spacing will be $15/1.26 = 11.9$ ft. The radius of the conductor is 0.261 in. For substitution in (70), $\log_{10}(D/r) = \log_{10} (11.9 \times 12)/0.261 = \log 547.13 = 2.738$. For substitution in (71), $\sqrt{r/D} = \sqrt{0.261/142.8} = 0.0428$.

From (70), critical voltage

$$\begin{aligned} e_0 &= 2.303 \times 0.87 \times 53.6 \times 0.964 \times 0.261 \times 2.738 \\ &= 27.02 \times 2.738 = 74.0 \text{ kv to neutral} \end{aligned}$$

The critical voltage for the center conductor, flat spacing, will be in the order of 96 percent. That for the outer conductors will be 106 percent of the critical voltage for each conductor with triangular (delta) spacing equal to the flat spacing.[1] Let the conductors be designated a, b, and c (see Fig. 131) and the critical voltages be e_{oa}, e_{ob} and e_{oc}, respectively. The critical voltage e_{ob} will then be $0.96 \times 74.0 = 71.0$ kv to neutral.

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[1] See Peek, F. W., Jr., "Dielectric Phenomena in High Voltage Engineering," p. 208, McGraw-Hill Book Company, Inc., New York, 1915.

$$e_{oa} = e_{oc} = 1.06 \times 74.0 = 78.4 \text{ kv.}$$

From Table 43, the voltage at $x_1 = 0$ miles is seen to be 76.2 kv to neutral, from which $e - e_{ob} = 76.2 - 71.0 = 5.2$ kv. For the outer conductors, $e - e_{oa} = 76.2 - 78.4 = -2.2$ kv. It will therefore be seen that, since the critical voltage of 78.4 kv for conductor a and c is higher than the operating voltage of 76.2 kv, there will be no corona formation on a and c at this point.

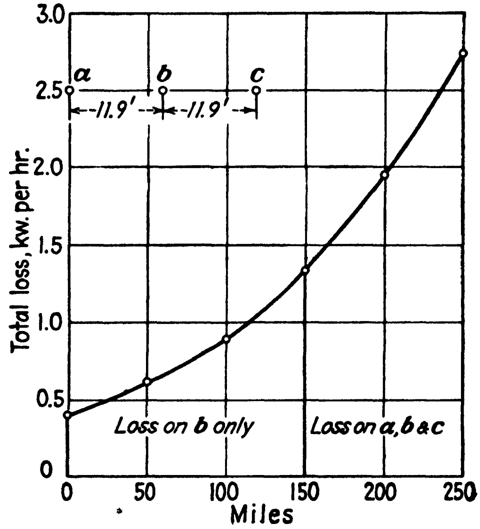

FIG. 131.—Corona-loss curve for 132-kv line

From (71), loss at x_1 = zero miles

$$\begin{aligned} P_b &= \left(\frac{390}{0.964}\right)(60 + 25) \times 0.0428 \times 10^{-5}(76.2 - 71.0)^2 \\ &= 0.0147 \times 5.2^2 = 0.40 \text{ kw per mile} \\ P_a &= P_c = \text{zero kw per mile} \end{aligned}$$

This problem affords an excellent example of the manner in which the excess of the operating voltage over the critical voltage and power loss are affected by variation in voltage along the line. The values in Table 49 represent the fair-weather corona loss at points in the line 50 miles apart. By plotting a kw loss curve

TABLE 49.—CALCULATED CORONA LOSS PER MILE FOR EACH CONDUCTOR AT 50-MILE INTERVALS

x_1, miles	e $(Kv)_n$	$e - e_{ob}$ (Kv)	$e - e_{oa}$ (Kv)	P_b (Kw)	$P_a + P_c$ (Kw)	Total (Kw)
0	76.2	5.2	−2.2	0.40		0.40
50	77.5	6.5	−0.9	0.62		0.62
100	78.9	7.8	0.5	0.89	0+	0.89
150	80.2	9.2	1.8	1.24	0.10	1.34
200	81.6	10.6	3.2	1.65	0.30	1.95
250	83.0	12.0	4.6	2.12	0.62	2.74

and measuring the area under the curve, the total kw loss per hour for the assumed condition is obtained (see Fig. 131).

The critical storm-weather voltage has been found by Peek to be in the order of 80 percent of the fair-weather value. Since the loss from corona formation varies as the square of the operating voltage e over the critical voltage e_o, the losses will be enormously increased during stormy weather. This should not, however, be taken too seriously, since the total time throughout the year in which storms occur will generally comprise a comparatively small portion of the time and be so localized that possibly only comparatively short sections of a given line will come under the influence of a particular storm. Further, during storm periods, the temperature usually drops in the storm area and therefore minimizes the losses. After the critical voltage is determined for storm-weather conditions, the corona loss is calculated in the same manner as above.

When the construction of a new high-voltage line is under consideration, the approximate maximum operating voltage will be known, and a conductor should generally be specified of such diameter as to prevent the formation of corona under normal operating conditions during fair weather. Whether the conductor should be of sufficient diameter to prevent corona formation during storm periods will depend on the economics of the situation, which can be determined only from a knowledge of local conditions.

The determination of corona loss is an important consideration when it becomes necessary to increase the operating voltage of a given line, it being desired to leave the existing conductor in place. Such situations should generally be studied carefully and the final results derived analytically from field data, since it will rarely be feasible to attempt corona-loss tests in the field.

Deriving the Auxiliary Constants from Hyperbolic Functions.

$$\begin{aligned}
A &= \cosh \sqrt{ZY} \\
a_1 + ja_2 &= (\cosh \theta_1 \cos \theta_2 + j \sinh \theta_1 \sin \theta_2) && (73) \\
B &= Z_0 \sinh \sqrt{ZY} \\
b_1 + jb_2 &= Z_0(\sinh \theta_1 \cos \theta_2 + j \cosh \theta_1 \sin \theta_2) && (74) \\
C &= \frac{1}{Z_0 \sinh \sqrt{ZY}} \\
c_1 + jc_2 &= \frac{1}{Z_0(\sinh \theta_1 \cos \theta_2 + j \cosh \theta_1 \sin \theta_2)} && (75)
\end{aligned}$$

The $\sqrt{ZY}$ (previously determined) $= 0.08525 + j0.5248 = \theta_1 + j\theta_2$. From Table 50, by interpolation, $\cosh \theta_1 = 1.003636$. $\sinh \theta_1 = 0.08535$. θ_2, expressed in terms of its circular angle, $= 57{,}2958 \times 0.5248 = 30.0688 = 30^{\circ}04'$, the cosine and sine of which are 0.86544 and 0.501.

$$\begin{aligned}
\text{From (73) } a_1 + ja_2 &= 1.003636 \times 0.86544 + j0.08535 \times 0.501 \\
&= 0.86859 + j0.04276 \\
A &= 0.86964\underline{/2^{\circ}49'}
\end{aligned}$$

From (74) $Z_0 = \sqrt{\dfrac{Z}{Y}} = 412.41\overline{\diagdown 9^{\circ}14'} = 407.07 - j66.2$ vector ohms

$(0.08535 \times 0.86544 + j1.003636 \times 0.501) = 0.07386 + j0.50283$

$$\begin{array}{rr}
 & 0.07386 + j0.50283 \\
z_0 = & \underline{\;407.07 - j66.2\quad\;} \\
 & 30.07 + j204.69 \\
 & 33.29 - j\;4.89 \\
b_1 + jb_2 = & 66.36 + j199.8 \\
B = & 209.6\underline{/72^{\circ}25'}
\end{array}$$

From (75) $\dfrac{1}{Z_0} = 0.002393 + j0.000389$

$$\begin{array}{rl}
 & 0.07386 \quad + j0.50283 \\
 & 0.002393 \quad + j0.000389 \\
 & 0.0001768 + j0.0012033 \\
- & 0.0001956 + j0.0000287
\end{array}$$

$$c_1 + jc_2 = -0.0000188 + j0.001232$$
$$C = 0.001233\backslash\underline{90°52'}$$

Exponential and Hyperbolic Functions.—Analyzing the performance of long high-voltage transmission lines, such as the one discussed in Chap. XV, is probably best performed by application of the hyperbolic theory. The method has been used only to a limited extent, however, largely because adequate tables of hyperbolic functions are generally not available. Some handbooks contain tables of hyperbolic functions, but usually such tables do not give values at sufficiently small intervals to be of any particular value, since straight-line interpolation cannot generally be satisfactorily employed.

Table 50 is compiled to give the exponential functions e^x and e^{-x} and the hyperbolic sine (sinh) and cosine (cosh) for x at intervals of 0.001 to and including 0.525. The table gives the sinh and cosh functions to a sufficient range to cover the requirements for most transmission-line problems. Functions for values of x outside the table may, if desired, be derived as follows:

$$e^x = \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{5{,}040} + \right)$$

$$e^{-x} = \frac{1}{e^x} \qquad \sinh x = 0.5(e^x - e^{-x}) \qquad \cosh x = 0.5(e^x + e^{-x})$$

$$\tanh x = \frac{\sinh x}{\cosh x} \quad \text{(not given in the table)}$$

The required number of terms in the series will depend on the size of x and the number of places required in the function. The larger the value of x, the more terms will be required for a given number of places. For values of x to about 0.04, only two terms, beyond the x term, will be required to give seven decimal places. For value of x in excess of about 0.04, it is probably more convenient to resort to logarithms, provided, however, that a table of common logarithms to the required number of places in the mantissa is available.

The base of the natural logarithm is here represented by the small letter e and, to seven places of decimals, is equal to 2.7182818.

By application of the above series, let us compute the exponential and hyperbolic functions, correct to seven decimal places, for $x = 0.02$.

$$\begin{aligned} e^x &= 1 + 0.02 + \frac{(0.02)^2}{2} + \frac{(0.02)^3}{6} + \frac{(0.02)^4}{24} + \cdots \\ &= 1 + 0.02 + 0.0002 + 0.00000133 + 0.00000000 \\ &\qquad = 1.0202013 \end{aligned}$$

$$\begin{aligned} e^{-x} &= \frac{1}{1.0202013} = 0.9801987 \\ \sinh x &= 0.5(1.0202013 - 0.9801987) = 0.0200013 \\ \cosh x &= 0.5(1.0202013 + 0.9801987) = 1.0002000 \\ \tanh x &= \frac{0.0200013}{1.0002000} = 0.0199973 \end{aligned}$$

Take a somewhat larger value of x, say, 0.25, and compute by logarithmic conversion and interpolation the exponential function e^x to seven decimal places.

$$\begin{aligned} e^x &= \text{the number belonging to the log of } x(\log_{10} e) \\ \log_{10} e &= 0.43429448 \\ x(\log_{10} e) &= 0.25 \times 0.43429448 = 0.1085736 \end{aligned}$$

From a seven place table of common logarithms, the number belonging to the mantissa 0.1085736 may be determined as follows: We find the next smaller mantissa 0.1085650 at the number 12840, and the next larger mantissa 0.1085988 at the number 12841. $0.1085988 - 0.1085650 = 338$. The difference in the given and next smaller mantissa $= 0.1085736 - 0.1085650 = 086$. Therefore to seven places the exponential function $e^x = 1.2840 + 0.086/0.338 = 1.2840254$.

The functions in Table 50 have been calculated to nine places and rounded to seven places. The differences in the fifth and seventh columns are taken from nine place functions and are not always the difference in the table values of consecutive functions. This makes the table more accurate and the differences more consistent.

The usefulness of a table lies in its adaptability to the problem at hand. Even though seven places will not always be required, a table with fewer places will not be universally applicable.

TABLE 50.—EXPONENTIAL AND HYPERBOLIC FUNCTIONS

x	e^x	e^{-x}	sinh x	Diff.	cosh x	Diff.
0.001	1.0010005	0.9990005	0.0010000		1.0000005	
.002	.0020020	.9980020	.0020000	10000	.0000020	15
.003	.0030045	.9970045	.0030000	10000	.0000045	25
.004	.0040080	.9960080	.0040000	10000	.0000080	35
.005	.0050125	.9950125	.0050000	10000	.0000125	45
0.006	1.0060180	0.9940180	0.0060000	10000	1.0000180	55
.007	.0070246	.9930244	.0070001	10000	.0000245	65
.008	.0080321	.9920319	.0080001	10000	.0000320	75
.009	.0090406	.9910404	.0090001	10000	.0000405	85
.010	.0100502	.9900498	.0100002	10000	.0000500	95
0.011	1.0110607	0.9890603	0.0110002	10000	1.0000605	105
.012	.0120723	.9880717	.0120003	10001	.0000720	115
.013	.0130849	.9870841	.0130004	10001	.0000845	125
.014	.0140985	.9860975	.0140005	10001	.0000980	135
.015	.0151131	.9851119	.0150006	10001	.0001125	145
0.016	1.0161287	0.9841273	0.0160007	10001	1.0001280	155
.017	.0171453	.9831437	.0170008	10001	.0001445	165
.018	.0181630	.9821610	.0180010	10002	.0001620	175
.019	.0191816	.9811794	.0190011	10002	.0001805	185
.020	.0202013	.9801987	.0200013	10002	.0002000	195
0.021	1.0212221	0.9792190	0.0210015	10002	1.0002205	205
.022	.0222438	.9782402	.0220018	10002	.0002420	215
.023	.0232665	.9772625	.0230020	10002	.0002645	225
.024	.0242903	.9762857	.0240023	10003	.0002880	235
.025	.0253151	.9753099	.0250026	10003	.0003125	245
0.026	1.0263409	0.9743351	0.0260029	10003	1.0003380	255
.027	.0273678	.9733612	.0270033	10004	.0003645	265
.028	.0283957	.9723884	.0280037	10004	.0003920	275
.029	.0294246	.9714165	.0290041	10004	.0004205	285
.030	.0304545	.9704455	.0300045	10004	.0004500	295
0.031	1.0314855	0.9694756	0.0310050	10005	1.0004805	305
.032	.0325175	.9685066	.0320055	10005	.0005120	315
.033	.0335505	.9675386	.0330060	10005	.0005445	325
.034	.0345846	.9665715	.0340066	10006	.0005781	335
.035	.0356197	.9656054	.0350071	10006	.0006126	345

TABLE 50.—EXPONENTIAL AND HYPERBOLIC FUNCTIONS.—(*Continued*)

x	e^x	e^{-x}	sinh x	Diff.	cosh x	Diff.
0.036	1.0366558	0.9646403	0.0360078	10006	1.0006481	355
.037	.0376930	.9636761	.0370084	10007	.0006846	365
.038	.0387312	.9627129	.0380091	10007	.0007221	375
.039	.0397705	.9617507	.0390099	10007	.0007606	385
.040	.0408108	.9607894	.0400107	10008	.0008001	395
0.041	1.0418521	0.9598291	0.0410115	10008	1.0008406	405
.042	.0428945	.9588698	.0420123	10009	.0008821	415
.043	.0439379	.9579114	.0430133	10009	.0009246	425
.044	.0449824	.9569540	.0440142	10009	.0009682	435
.045	.0460279	.9559975	.0450152	10010	.0010127	445
0.046	1.0470744	0.9550420	0.0460162	10010	1.0010582	455
.047	.0481220	.9540874	.0470173	10011	.0011047	465
.048	.0491707	.9531338	.0480184	10011	.0011522	475
.049	.0502234	.9521811	.0490196	10012	.0012007	485
.050	.0512711	.9512294	.0500208	10012	.0012503	495
0.051	1.0523229	0.9502787	0.0510221	10013	1.0013008	505
.052	.0533757	.9493289	.0520234	10013	.0013523	515
.053	.0544296	.9483800	.0530248	10014	.0014048	525
.054	.0554845	.9474321	.0540262	10014	.0014584	535
.055	.0565406	.9464851	.0550277	10015	.0015129	545
0.056	1.0575977	0.9455391	0.0560293	10015	1.0015684	555
.057	.0586558	.9445941	.0570309	10016	.0016249	565
.058	.0597150	.9436499	.0580325	10017	.0016825	575
.059	.0607752	.9427068	.0590342	10017	.0017410	585
.060	.0618365	.9417645	.0600360	10018	.0018005	595
0.061	1.0628989	0.9408232	0.0610378	10018	1.0018611	605
.062	.0639623	.9398829	.0620397	10018	.0019226	615
.063	.0650268	.9389435	.0630417	10020	.0019852	625
.064	.0660924	.9380050	.0640437	10020	.0020487	635
.065	.0671590	.9370675	.0650458	10021	.0021132	645
0.066	1.0682267	0.9361309	0.0660479	10021	1.0021788	655
.067	.0692955	.9351952	.0670501	10022	.0022453	665
.068	.0703653	.9342605	.0680524	10023	.0023129	676
.069	.0714362	9333267	.0690548	10023	.0023814	686
.070	.0725082	.9323938	.0700572	10024	.0034510	696

TABLE 50.—EXPONENTIAL AND HYPERBOLIC FUNCTIONS.—(*Continued*)

x	e^x	e^{-x}	sinh x	Diff.	cosh x	Diff.
0.071	1.0735812	0.9314619	0.0710597	10025	1.0025216	706
.072	.0746553	.9305309	.0720622	10026	.0025931	716
.073	.0757305	.9296008	.0730649	10026	.0026657	726
.074	.0768068	.9286717	.0740676	10027	.0027392	736
.075	.0778842	.9277435	.0750703	10028	.0028138	746
0.076	1.0789626	0.9268162	0.0760732	10029	1.0028894	756
.077	.0800421	.9258899	.0770761	10029	.0029660	766
.078	.0811227	.9249644	.0780791	10030	.0030435	776
.079	.0822043	.9240399	.0790822	10031	.0031221	786
.080	.0832871	.9231163	.0800854	10032	.0032017	796
0.081	1.0843709	0.9221937	0.0810886	10032	1.0032823	806
.082	.0854558	.9212720	.0820919	10033	.0033639	816
.083	.0865418	.9203511	.0830953	10034	.0034465	826
.084	.0876289	.9194313	.0840988	10035	.0035301	836
.085	.0887171	.9185123	.0851024	10036	.0036147	846
0.086	1.0898063	0.9175942	0.0861060	10037	1.0037003	856
.087	.0908967	.9166771	.0871098	10037	.0037869	866
.088	.0919881	.9157609	.0881136	10038	.0038745	876
.089	.0930807	.9148456	.0891175	10039	.0039631	886
.090	.0941743	.9139312	.0901215	10040	.0040527	896
0.091	1.0952690	0.9130177	0.0911256	10041	1.0041434	906
.092	.0963648	.9121051	.0921298	10042	.0042350	916
.093	.0974617	.9111935	.0931341	10043	.0043276	926
.094	.0985597	.9102828	.0941385	10044	.0044213	936
.095	.0996589	.9093729	.0951430	10045	.0045159	946
0.096	1.1007591	0.9084640	0.0961475	10046	1.0046115	956
.097	.1018604	.9075560	.0971522	10047	.0047082	967
.098	.1029628	.9066489	.0981569	10048	.0048058	977
.099	.1040663	.9057427	.0991618	10049	.0049045	987
.100	.1051709	.9048374	.1001668	10050	.0050042	997
0.101	1.1062766	0.9039330	0.1011718	10050	1.0051048	1007
.102	.1073835	.9030296	.1021770	10052	.0052065	1017
.103	.1084914	.9021270	.1031822	10053	.0053092	1027
.104	.1096005	.9012253	.1041876	10054	.0054129	1037
.105	.1107106	.9003245	.1051930	10055	.0055176	1047

TABLE 50.—EXPONENTIAL AND HYPERBOLIC FUNCTIONS.—(*Continued*)

x	e^x	e^{-x}	sinh x	Diff.	cosh x	Diff.
0.106	1.1118219	0.8994246	0.1061986	10056	1.0056233	1057
.107	.1129343	.8985257	.1072043	10057	.0057300	1067
.108	.1140477	.8976276	.1082101	10058	.0058377	1077
.109	1151624	.8967304	.1092160	10059	.0059464	1087
.110	.1162781	.8958341	.1102220	10060	.0060561	1097
0.111	1.1173949	0.8949387	0.1112281	10061	1.0061668	1107
.112	.1185129	.8940443	.1122343	10062	.0062786	1117
.113	.1196319	.8931507	.1132406	10063	.0063913	1127
.114	.1207521	.8922580	.1142471	10065	.0065050	1137
.115	.1218734	.8913661	.1152536	10066	.0066198	1148
0.116	1.1229959	0.8904752	0.1162603	10067	1.0067355	1158
.117	.1241194	.8895852	1172671	10068	.0068523	1168
.118	.1252441	.8886961	.1182740	10069	.0069701	1178
.119	.1263699	.8878078	.1192811	10070	.0070889	1188
.120	.1274969	.8869204	.1202882	10071	.0072086	1198
0.121	1.1286249	0 8860340	0.1212955	10073	1.0073294	1208
.122	.1297541	8851484	.1223029	10074	.0074512	1218
.123	.1308844	.8842637	.1233104	10075	.0075740	1228
.124	.1320159	8833798	.1243180	10076	.0076979	1238
.125	.1331485	8824969	.1253258	10078	.0078227	1248
0.126	1.1342822	0.8816148	0.1263337	10079	1.0079485	1258
.127	.1354170	.8807337	.1273417	10080	.0080753	1268
.128	1365530	.8798534	.1283498	10081	.0082032	1278
.129	.1376901	.8789740	.1293581	10083	.0083320	1288
.130	.1388284	.8780954	.1303665	10084	.0084619	1299
0.131	1.1399678	0.8772178	0.1313750	10085	1.0085928	1309
.132	.1411083	.8763410	.1323837	10087	.0087247	1319
.133	.1422500	.8754651	.1333925	10088	.0088575	1329
.134	.1433928	.8745901	.1344014	10089	.0089914	1339
.135	.1445368	.8737159	.1354104	10090	.0091263	1349
0.136	1.1456819	0.8728426	0.1364196	10092	1.0092623	1359
.137	.1468281	.8719702	.1374290	10093	.0093992	1369
.138	.1479756	.8710987	.1384384	10095	.0095371	1379
.139	.1491241	.8702280	.1394480	10096	.0096761	1389
.140	.1502738	.8693582	.1404578	10098	.0098160	1399

TABLE 50.—EXPONENTIAL AND HYPERBOLIC FUNCTIONS.—(*Continued*)

x	e^x	e^{-x}	sinh x	Diff.	cosh x	Diff.
0.141	1.1514246	0.8684893	0.1414677	10099	1.0099570	1410
.142	.1525766	.8676213	.1424777	10100	.0100990	1420
.143	.1537298	.8667541	.1434879	10102	.0102419	1430
.144	.1548841	.8658877	.1444982	10103	.0103859	1440
.145	.1560396	.8650223	.1455086	10105	.0105309	1450
0.146	1.1571962	0.8641577	0.1465192	10106	1.0106769	1460
.147	.1583540	.8632940	.1475300	10108	.0108240	1470
.148	.1595129	.8624311	.1485409	10109	.0109720	1480
.149	.1606730	.8615691	.1495519	10110	.0111211	1491
.150	.1618342	.8607080	.1505631	10112	.0112711	1501
0.151	1.1629967	0.8598477	0.1515745	10113	1.0114222	1511
.152	.1641602	.8589883	.1525860	10115	.0115743	1521
.153	.1653250	.8581297	.1535976	10116	.0117274	1531
.154	.1664909	8572720	.1546094	10118	.0118815	1541
.155	.1676580	.8564152	.1556214	10120	.0120366	1551
0 156	1.1688262	0.8555592	0.1566335	10121	1.0121927	1561
.157	.1699956	8547041	.1576458	10123	.0123498	1571
.158	.1711662	.8538498	.1586582	10124	.0125080	1582
.159	.1723379	.8529964	.1596708	10126	.0126672	1592
.160	.1735109	.8521438	.1606835	10127	.0128273	1602
0.161	1.1746850	0 8512921	0.1616964	10129	1.0129885	1612
.162	.1758602	8504412	.1627095	10131	.0131507	1622
.163	.1770367	.8495912	.1637228	10132	.0133139	1632
.164	.1782143	.8487420	.1647361	10134	.0134782	1642
.165	.1793931	.8478937	.1657497	10136	.0136434	1652
0.166	1.1805731	0.8470462	0.1667634	10137	1.0138097	1663
.167	.1817543	.8461996	.1677773	10139	.0139769	1673
.168	.1829366	.8453538	.1687914	10141	.0141452	1683
.169	.1841201	.8445089	.1698056	10142	.0143145	1693
.170	.1853049	.8436648	.1708200	10144	.0144848	1703
0.171	1.1864907	0.8428216	0.1718346	10146	1.0146562	1713
.172	.1876778	.8419792	.1728493	10147	.0148285	1723
.173	.1888661	.8411376	.1738642	10149	.0150019	1734
.174	.1900556	.8402969	.1748793	10151	.0151762	1744
.175	.1912462	.8394570	.1758946	10153	.0153516	1754

TABLE 50.—EXPONENTIAL AND HYPERBOLIC FUNCTIONS.—(*Continued*)

x	e^x	e^{-x}	sinh x	Diff.	cosh x	Diff.
0.176	1.1924381	0.8386180	0.1769100	10154	1.0155280	1764
.177	.1936311	.8377798	.1779257	10156	.0157054	1774
.178	.1948253	.8369424	.1789414	10158	.0158839	1784
.179	.1960207	.8361059	.1799574	10160	.0160633	1794
.180	.1972174	.8352702	.1809736	10162	.0162438	1805
0.181	1.1984152	0.8344354	0.1819899	10163	1.0164253	1815
.182	.1996142	.8336013	.1830064	10165	.0166078	1825
.183	.2008144	.8327682	1840231	10167	.0167913	1835
.184	.2020158	.8319358	.1850400	10169	.0169758	1845
.185	.2032184	.8311043	.1860571	10171	.0171614	1855
0.186	1.2044223	0.8302736	0.1870743	10173	1.0173479	1866
.187	.2056273	.8294437	.1880918	10174	.0175355	1876
.188	.2068335	.8286147	.1891094	10176	.0177241	1886
.189	.2080410	.8277865	.1901272	10178	.0179137	1896
.190	.2092496	.8269591	.1911452	10180	.0181044	1906
0.191	1.2104595	0.8261326	0.1921634	10182	1.0182960	1916
.192	.2116705	.8253069	.1931818	10184	.0184887	1927
.193	.2128828	.8244820	1942004	10186	.0186824	1937
.194	.2140963	.8236579	.1952192	10188	.0188771	1947
.195	.2153110	.8228347	.1962382	10190	.0190728	1957
0.196	1.2165269	0.8220122	0.1972573	10192	1.0192696	1967
.197	.2177440	.8211906	.1982767	10194	.0194673	1978
.198	.2189624	.8203699	.1992963	10196	.0196661	1988
.199	.2201820	.8195499	.2003160	10198	.0198659	1998
.200	.2214028	.8187308	.2013360	10200	.0200668	2008
0.201	1.2226248	0.8179124	0.2023562	10202	1.0202686	2018
.202	.2238480	.8170949	.2033765	10204	.0204715	2029
.203	.2250725	.8162782	.2043971	10206	.0206754	2039
.204	.2262982	.8154624	.2054179	10208	.0208803	2049
.205	.2275251	.8146473	.2064389	10210	.0210862	2059
0.206	1.2287532	0.8138331	0.2074601	10212	1.0212931	2069
.207	.2299826	.8130196	.2084815	10214	.0215011	2080
.208	.2312132	.8122070	.2905031	10216	.0217101	2090
.209	.2324450	.8113952	.2105249	10218	.0219201	2100
.210	.2336781	.8105842	.2115469	10220	.0221312	2110

TABLE 50.—EXPONENTIAL AND HYPERBOLIC FUNCTIONS.—(*Continued*)

x	e^x	e^{-x}	sinh x	Diff.	cosh x	Diff.
0.211	1.2349124	0.8097741	0.2125691	10222	1.0223432	2121
.212	.2361479	.8089647	.2135916	10225	.0225563	2131
.213	.2373847	.8081561	.2146143	10227	.0227704	2141
.214	.2386227	.8073484	.2156371	10229	.0229855	2151
.215	.2398619	.8065414	.2166602	10231	.0232017	2162
0.216	1.2411024	0.8057353	0.2176835	10233	1.0234188	2172
.217	.2423441	.8049300	.2187071	10235	.0236370	2182
.218	.2435871	.8041254	.2197308	10237	.0238563	2192
.219	.2448313	.8033217	.2207548	10240	.0240765	2202
.220	.2460767	.8025188	.2217790	10242	.0242978	2213
0.221	1.2473234	0.8017167	0.2228034	10244	1.0245201	2223
.222	.2485714	.8009154	.2238280	10246	.0247434	2233
.223	.2498206	.8001148	.2248529	10249	.0249677	2243
.224	.2510710	.7993151	.2258779	10251	.0251931	2254
.225	.2523227	.7985162	.2269032	10253	.0254195	2264
0.226	1.2535757	0.7977181	0.2279288	10256	1.0256469	2274
.227	.2548299	.7969208	.2289545	10258	.0258753	2284
.228	.2560853	.7961243	.2299805	10260	.0261048	2295
.229	.2573420	.7953285	.2310068	10262	.0263353	2305
.230	.2586000	.7945336	.2320332	10264	.0265668	2315
0.231	1.2598592	0.7937395	0.2330599	10267	1.0267994	2325
.232	.2611197	.7929461	.2340868	10269	.0270329	2336
.233	.2623815	.7921536	.2351140	10272	.0272675	2346
.234	.2636445	.7913618	.2361413	10274	.0275032	2356
.235	.2649088	.7905708	.2371690	10276	.0277398	2366
0.236	1.2661743	0.7897807	0.2381968	10279	1.0279775	2377
.237	.2674411	.7889913	.2392249	10281	.0282162	2387
.238	.2687092	.7882027	.2402533	10283	.0284559	2397
.239	.2699785	.7874149	.2412818	10286	.0286967	2408
.240	.2712492	.7866279	.2423106	10288	.0289385	2418
0.241	1.2725210	0.7858416	0.2433397	10291	1.0291813	2428
.242	.2737942	.7850562	.2443690	10293	.0294252	2439
.243	.2750686	.7842715	.2453986	10295	.0296701	2449
.244	.2763443	.7834876	.2464283	10298	.0299160	2459
.245	.2776213	.7827045	.2474584	10300	.0301629	2469

TABLE 50.—EXPONENTIAL AND HYPERBOLIC FUNCTIONS.—(*Continued*)

x	e^x	e^{-x}	sinh x	Diff.	cosh x	Diff.
0.246	1.2788996	0.7819222	0.2484887	10303	1.0304109	2480
.247	.2801791	.7811407	.2495192	10305	.0306599	2490
.248	.2814599	.7803599	.2505500	10308	.0309099	2500
.249	.2827420	.7795800	.2515810	10310	.0311610	2511
.250	.2840254	.7788008	.2526123	10313	.0314131	2521
0.251	1.2853101	0.7780224	0.2536439	10315	1.0316662	2531
.252	.2865960	.7772447	.2546756	10318	.0319204	2542
.253	.2878833	.7764679	.2557077	10320	.0321756	2552
.254	.2891718	.7756918	.2567400	10323	.0324318	2562
.255	.2904616	.7749165	.2577726	10326	.0326891	2573
0.256	1.2917527	0.7741420	0.2588054	10328	1.0329473	2583
.257	.2930451	.7733682	.2598385	10331	.0332067	2593
.258	.2943388	.7725952	.2608718	10333	.0334670	2604
.259	.2956338	.7718230	.2619054	10336	.0337284	2614
.260	.2969301	.7710516	.2629393	10339	.0339908	2624
0.261	1.2982277	0.7702809	0.2639734	10341	1.0342543	2635
.262	.2995265	.7695110	.2650078	10344	.0345188	2645
.263	.3008267	.7687419	2660424	10346	.0347843	2655
.264	.3021282	.7679735	.2670773	10349	.0350509	2666
.265	.3034310	.7672059	.2681125	10352	.0353185	2676
0.266	1.3047351	0.7664391	0 2691480	10355	1.0355871	2686
.267	.3060404	7656731	.2701837	10357	.0358568	2697
.268	.3073471	.7649078	.2712197	10360	.0361275	2707
.269	.3086551	.7641433	.2722559	10363	.0363992	2717
.270	.3099645	.7633795	.2732925	10365	.0366720	2728
0.271	1.3112751	0.7626165	0.2743293	10368	1.0369458	2738
.272	.3125870	.7618543	.2753664	10371	.0372206	2748
.273	.3139002	.7610928	.2764037	10374	.0374965	2759
.274	.3152148	.7603321	.2774414	10376	.0377734	2769
.275	.3165307	.7595721	.2784793	10379	.0380514	2780
0.276	1.3178479	0.7588129	0.2795175	10382	1.0383304	2790
.277	.3191664	.7580545	.2805559	10385	.0386104	2800
.278	.3204862	.7572968	.2815947	10388	.0388915	2811
.279	.3218073	.7565399	.2826337	10390	.0391736	2821
.280	.3231298	.7557837	.2836730	10393	.0394568	2832

Table 50.—Exponential and Hyperbolic Functions.—(*Continued*)

x	e^x	e^{-x}	sinh x	Diff.	cosh x	Diff.
0.281	1.3244536	0.7550283	0.2847126	10396	1.0397410	2842
.282	.3257787	.7542737	.2857525	10399	.0400262	2852
.283	.3271052	.7535198	.2867927	10402	.0403125	2863
.284	.3284329	.7527666	.2878331	10404	.0405998	2873
.285	.3297620	.7520143	.2888739	10408	.0408881	2883
0.286	1.3310925	0.7512626	0.2899149	10410	1.0411775	2894
.287	.3324242	.7505117	.2909562	10413	.0414680	2904
.288	.3337573	.7497616	.2919979	10416	.0417594	2915
.289	.3350917	.7490122	.2930398	10419	.0420520	2925
.290	.3364275	.7482636	.2940820	10422	.0423455	2936
0.291	1.3377646	0.7475157	0.2951245	10425	1.0426401	2946
.292	.3391030	.7467685	.2961672	10428	.0429358	2956
.293	.3404428	.7460221	.2972103	10431	.0432325	2967
.294	.3417839	.7452765	.2982537	10434	.0435302	2977
.295	.3431264	.7445316	.2992974	10437	.0438290	2988
0.296	1.3444702	0.7437874	0.3003414	10440	1.0441288	2998
.297	.3458153	.7430440	.3013856	10443	.0444297	3009
.298	.3471618	.7423013	.3024302	10446	.0447316	3019
.299	.3485096	.7415594	.3034751	10449	.0450345	3029
.300	.3498588	.7408182	.3045203	10452	.0453385	3040
0.301	1.3512093	0.7400778	0.3055658	10455	1.0456436	3050
.302	.3525612	.7393381	.3066116	10458	.0459496	3061
.303	.3539145	.7385991	.3076577	10461	.0462568	3071
.304	.3552691	.7378609	.3087041	10464	.0465650	3082
.305	.3566250	.7371234	.3097508	10467	.0468742	3092
0.306	1.3579823	0.7363866	0.3107978	10470	1.0471845	3103
.307	.3593410	.7356506	.3118452	10473	.0474958	3113
.308	.3607010	.7349153	.3128928	10476	.0478082	3124
.309	.3620624	.7341808	.3139408	10480	.0481216	3134
.310	.3634251	.7334470	.3149891	10483	.0484360	3145
0.311	1.3647892	0.7327139	0.3160377	10486	1.0487515	3155
.312	.3661547	.7319815	.3170866	10489	.0490681	3166
.313	.3675215	.7312499	.3181358	10492	.0493857	3176
.314	.3688897	.7305190	.3191854	10495	.0497044	3187
.315	.3702593	.7297889	.3202352	10499	.0500241	3197

TABLE 50.—EXPONENTIAL AND HYPERBOLIC FUNCTIONS.—(*Continued*)

x	e^x	e^{-x}	sinh x	Diff.	cosh x	Diff.
0.316	1.3716303	0.7290595	0.3212854	10502	1.0503449	3208
.317	.3730026	.7283308	.3223359	10505	.0506667	3218
.318	.3743763	.7276028	.3233867	10508	.0509895	3229
.319	.3757513	.7268755	.3244379	10512	.0513134	3239
.320	.3771278	.7261490	.3254894	10515	.0516384	3250
0.321	1.3785056	0.7254233	0.3265412	10518	1.0519644	3260
.322	.3798848	.7246982	.3275933	10521	.0522915	3271
.323	.3812654	.7239739	.3286457	10525	.0526196	3281
.324	.3826473	.7232502	.3296985	10528	.0529488	3292
.325	.3840306	.7225274	3307516	10531	.0532790	3302
0.326	1.3854154	0.7218052	0.3318051	10534	1.0536103	3313
.327	.3868015	.7210837	.3328589	10538	.0539426	3323
.328	3881890	.7203630	.3339130	10541	.0542760	3334
.329	.3895779	.7196430	.3349674	10544	.0546104	3344
.330	.3909681	.7189237	.3360222	10548	.0549459	3355
0.331	1.3923598	0.7182052	0.3370773	10551	1.0552825	3366
.332	.3937528	.7174873	.3381328	10555	.0556201	3376
.333	.3951473	.7167702	.3391886	10558	.0559587	3387
.334	.3965431	.7160538	.3402447	10561	.0562985	3397
.335	.3979404	.7153381	.3413011	10565	.0566392	3408
0.336	1.3993390	0.7146231	0.3423580	10568	1.0569811	3418
.337	.4007391	.7139088	.3434151	10572	.0573240	3429
.338	.4021405	.7131953	.3444726	10575	.0576679	3439
.339	.4035433	.7124824	.3455304	10578	.0580129	3450
.340	.4049476	.7117703	.3465886	10582	.0583590	3461
0.341	1.4063532	0.7110589	0.3476472	10585	1.0587061	3471
.342	.4077603	.7103482	.3487060	10589	.0590543	3482
.343	.4091688	.7096382	.3497653	10592	.0594035	3492
.344	.4105786	.7089289	.3508249	10596	.0597538	3503
.345	.4119899	.7082204	.3518848	10599	.0601051	3514
0.346	1.4134026	0.7075125	0.3529451	10603	1.0604576	3524
.347	.4148167	.7068053	.3540057	10606	.0608110	3535
.348	.4162322	.7060989	.3550667	10610	.0611656	3545
.349	.4176492	.7053931	.3561280	10613	.0615212	3556
.350	.4190675	.7046881	.3571897	10617	.0618778	3567

Table 50.—Exponential and Hyperbolic Functions.—(*Continued*)

x	e^x	e^{-x}	sinh x	Diff.	cosh x	Diff.
0.351	1.4204873	0.7039838	0.3582518	10621	1.0622355	3577
.352	.4219085	.7032801	3593142	10624	.0625943	3588
.353	.4233311	.7025772	.3603770	10628	.0629542	3598
.354	.4247552	.7018750	.3614401	10631	.0633151	3609
.355	.4261807	.7011734	.3625036	10635	.0636770	3620
0.356	1.4276075	0.7004726	0.3635675	10639	1.0640401	3630
.357	.4290359	.6997725	.3646317	10642	.0644042	3641
.358	.4304656	.6990731	.3656963	10646	.0647693	3652
.359	.4318968	.6983744	.3667612	10649	.0651356	3662
.360	.4333294	.6976763	.3678265	10653	.0655029	3673
0.361	1.4347635	0.6969790	0.3688922	10657	1.0658712	3684
.362	.4361989	.6962824	.3699583	10661	.0662407	3694
.363	.4376359	.6955864	.3710247	10664	.0666111	3705
.364	.4390742	.6948912	.3720915	10668	.0669827	3716
.365	.4405140	.6941967	.3731587	10672	.0673553	3726
0.366	1.4419552	0.6935028	0.3742262	10675	1.0677290	3737
.367	.4433979	.6928096	.3752941	10679	.0681038	3748
.368	.4448420	.6921172	.3763624	10683	.0684796	3758
.369	.4462876	.6914254	.3774311	10687	.0688565	3769
.370	.4477346	.6907343	.3785001	10690	.0692345	3780
0.371	1.4491831	0.6900439	0.3795696	10694	1.0696135	3790
.372	.4506330	.6893542	.3806394	10698	.0699936	3801
.373	.4520843	.6886652	.3817096	10702	.0703748	3812
.374	.4535372	.6879769	.3827801	10706	.0707570	3822
.375	.4549914	.6872893	.3838511	10709	.0711403	3833
0.376	1.4564471	0.6866023	0.3849224	10713	1.0715247	3844
.377	.4579043	.6859161	.3859941	10717	.0719102	3855
.378	.4593629	.6852305	.3870662	10721	.0722967	3865
.379	.4608230	.6845456	.3881387	10725	.0726843	3876
.380	.4622846	.6838614	.3892116	10729	.0730730	3887
0.381	1.4637476	0.6831779	0.3902849	10733	1.0734627	3897
.382	.4652121	.6824951	.3913585	10737	.0738536	3908
.383	.4666780	.6818129	.3924326	10741	.0742455	3919
.384	.4681454	.6811314	.3935070	10744	.0746384	3930
.385	.4696143	.6804506	.3945818	10748	.0750325	3940

TABLE 50.—EXPONENTIAL AND HYPERBOLIC FUNCTIONS.—(*Continued*)

x	e^x	e^{-x}	sinh x	Diff.	cosh x	Diff.
0.386	1.4710847	0.6797705	0.3956571	10752	1.0754276	3951
.387	.4725565	.6790911	.3967327	10756	.0758238	3962
.388	.4740298	.6784123	.3978087	10760	.0762211	3973
.389	.4755046	.6777343	.3988851	10764	.0766194	3983
.390	.4769808	.6770569	.3999620	10768	.0770188	3994
0.391	1.4784585	0.6763802	0.4010392	10772	1.0773194	4005
.392	.4799377	.6757041	.4021168	10776	.0778209	4016
.393	.4814184	.6750287	.4031948	10780	.0782236	4027
.394	.4829005	.6743541	.4042732	10784	.0786273	4037
.395	.4843842	.6736800	.4053521	10788	.0790321	4048
0.396	1.4858693	0.6730067	0.4064313	10792	1.0794380	4059
.397	.4873559	.6723340	.4075110	10796	.0798450	4070
.398	.4888440	.6716620	.4085910	10800	.0802530	4080
.399	.4903336	.6709907	.4096715	10805	.0806622	4091
.400	.4918247	.6703200	.4107523	10809	.0810724	4102
0.401	1.4933173	0.6696501	0.4118336	10813	1.0814837	4113
.402	.4948113	.6689807	.4129153	10817	.0818960	4124
.403	.4963069	.6683121	.4139974	10821	.0823095	4135
.404	.4978039	.6676441	.4150799	10825	.0827240	4145
.405	.4993025	.6669768	.4161628	10829	.0831397	4156
0.406	1.5008026	0.6663102	0.4172462	10834	1 0835564	4167
.407	.5023041	.6656442	.4183300	10838	.0839741	4178
.408	.5038072	.6649789	.4194141	10842	.0843930	4189
.409	.5053117	.6643142	.4204987	10846	.0848130	4200
.410	.5068178	.6636503	.4215838	10850	.0852340	4210
0.411	1.5083254	0.6629869	0.4226692	10854	1.0856561	4221
.412	.5098344	.6623243	.4237551	10859	.0860794	4232
.413	.5113450	.6616623	.4248414	10863	.0865037	4243
.414	.5128571	.6610010	.4259281	10867	.0869290	4254
.415	.5143707	.6603403	.4270152	10871	.0873555	4265
0.416	1.5158859	0.6596803	0.4281028	10876	1.0877831	4276
.417	.5174025	.6590209	.4291908	10880	.0882117	4286
.418	.5189207	.6583622	.4302792	10884	.0886415	4297
.419	.5204404	.6577042	.4313681	10889	.0890723	4308
.420	.5219616	.6570468	.4324574	10893	.0895042	4319

TABLE 50.—EXPONENTIAL AND HYPERBOLIC FUNCTIONS.—(*Continued*)

x	e^x	e^{-x}	sinh x	Diff.	cosh x	Diff.
0.421	1.5234843	0.6563901	0.4335471	10897	1.0899372	4330
.422	.5250085	.6557340	.4346372	10902	.0903713	4341
.423	.5265343	.6550786	.4357278	10906	.0908065	4352
.424	.5280616	.6544239	.4368189	10910	.0912427	4363
.425	.5295904	.6537698	.4379103	10915	.0916801	4374
0.426	1 5311208	0.6531163	0.4390022	10919	1.0921186	4385
.427	.5326527	.6524636	.4400946	10923	.0925581	4395
.428	5341861	.6518114	.4411873	10928	.0929987	4406
.429	.5357210	.6511599	.4422806	10932	.0934405	4417
.430	.5372575	.6505091	.4433742	10937	.0938833	4428
0.431	1.5387955	0.6498589	0.4444683	10941	1.0943272	4439
.432	.5403351	.6492094	.4455629	10945	.0947722	4450
.433	.5418762	.6485605	.4466579	10950	.0952184	4461
.434	.5434189	.6479123	.4477533	10954	.0956656	4472
.435	.5449631	.6472647	.4488492	10959	.0961139	4483
0.436	1.5465088	0.6466177	0.4499455	10963	1.0965633	4494
.437	.5480561	.6459714	.4510423	10968	.0970138	4505
.438	.5496049	6453258	.4521396	10972	.0974653	4516
.439	.5511553	.6446808	.4532373	10977	.0979180	4527
.440	.5527072	.6440364	.4543354	10981	.0983718	4538
0.441	1.5542607	0.6433927	0.4554340	10986	1.0988267	4549
.442	.5558157	.6427496	.4565331	10991	.0992827	4560
.443	.5573723	.6421072	.4576326	10995	.0997398	4571
.444	.5589305	.6414654	.4587325	11000	.1001980	4582
.445	5604902	.6408243	.4598330	11004	.1006572	4593
0.446	1.5620515	0.6401838	0.4609338	11009	1.1011176	4604
.447	.5636143	.6395439	.4620352	11013	.1015791	4615
.448	.5651787	.6389047	.4631370	11018	.1020417	4626
.449	.5667447	.6382661	.4642393	11023	.1025054	4637
.450	.5683122	.6376282	.4653420	11027	.1029702	4648
0.451	1.5698813	0.6369908	0.4664452	11032	1.1034361	4659
.452	.5714519	.6363542	.4675489	11037	.1039031	4670
.453	.5730242	.6357181	.4686530	11041	.1043712	4681
.454	.5745980	.6350827	.4697576	11046	.1048404	4692
.455	.5761734	.6344480	.4708627	11051	.1053107	4703

TABLE 50.—EXPONENTIAL AND HYPERBOLIC FUNCTIONS.—(*Continued*)

x	e^x	e^{-x}	sinh x	Diff.	cosh x	Diff.
0.456	1.5777503	0.6338138	0.4719683	11055	1.1057821	4714
.457	.5793289	.6331803	.4730743	11060	.1062546	4725
.458	.5809090	.6325475	.4741808	11065	.1067282	4736
.459	.5824907	.6319152	.4752877	11070	.1072030	4747
.460	.5840740	.6312836	.4763952	11074	.1076788	4758
0.461	1.5856589	0.6306527	0.4775031	11079	1.1081558	4769
.462	.5872453	.6300223	.4786115	11084	.1086338	4781
.463	.5888333	6293926	.4797204	11089	.1091130	4792
.464	.5904230	.6287636	.4808297	11093	.1095933	4803
.465	5920142	.6281351	.4819395	11098	.1100746	4814
0.466	1.5936070	0.6275073	0.4830499	11103	1.1105571	4825
.467	.5952014	.6268801	.4841607	11108	.1110407	4836
468	.5967974	6262535	.4852719	11113	.1115255	4847
.469	.5983950	.6256276	.4863837	11118	.1120113	4858
.470	.5999942	6250023	.4874960	11123	.1124982	4869
0.471	1.6015950	0.6243776	0.4886087	11127	1.1129863	4881
472	6031974	.6237535	.4897219	11132	.1134754	4892
.473	.6048014	.6231301	.4908357	11137	.1139657	4903
.474	.6064070	.6225073	.4919499	11142	.1144571	4914
.475	.6080142	.6218851	.4930646	11147	.1149496	4925
0.476	1.6096230	0.6212635	0 4941798	11152	1.1154432	4936
.477	.6112334	.6206425	.4952955	11157	.1159380	4947
.478	.6128455	.6200222	.4964116	11162	.1164338	4958
.479	.6144591	.6194025	.4975283	11167	.1169308	4970
.480	.6160744	.6187834	.4986455	11172	.1174289	4981
0.481	1.6176913	0.6181649	0 4997632	11177	1.1179281	4992
.482	.6193098	.6175471	.5008814	11182	.1184284	5003
.483	.6209299	.6169298	.5020000	11187	.1189299	5014
.484	.6225516	.6163132	.5031192	11192	1194324	5026
.485	.6241750	.6156972	.5042389	11197	.1199361	5037
0.486	1.6258000	0.6150818	0.5053591	11202	1.1204409	5048
.487	.6274266	.6144670	.5064798	11207	.1209468	5059
.488	.6290548	.6138529	.5076010	11212	.1214539	5070
.489	.6306847	.6132393	.5087227	11217	.1219620	5082
.490	.6323162	.6126264	.5098449	11222	.1224713	5093

TABLE 50.—EXPONENTIAL AND HYPERBOLIC FUNCTIONS.—(*Continued*)

x	e^x	e^{-x}	sinh x	Diff.	cosh x	Diff.
0.491	1.6339494	0.6120141	0.5109676	11227	1.1229817	5104
.492	.6355841	.6114024	.5120909	11232	.1234932	5115
.493	.6372205	.6107913	.5132146	11237	.1240059	5127
.494	.6388586	.6101808	.5143389	11243	.1245197	5138
.495	.6404982	.6095709	.5154637	11248	.1250346	5149
0 496	1.6421396	0.6089616	0.5165890	11253	1.1255506	5160
.497	.6437825	.6083530	.5177148	11258	.1260678	5172
.498	.6454271	.6077449	.5188411	11263	.1265860	5183
.499	.6470734	.6071375	.5199679	11268	.1271054	5194
.500	.6487213	.6065307	.5210953	11274	.1276260	5205
0.501	1.6503708	0.6059244	0.5222232	11279	1.1281476	5217
.502	.6520220	.6053188	.5233516	11284	.1286704	5228
.503	.6536749	.6047138	.5244805	11289	.1291943	5239
.504	.6553294	.6041094	.5256100	11295	.1297194	5250
.505	.6569855	.6035056	.5267400	11300	.1302455	5262
0.506	1.6586433	0.6029024	0.5278705	11305	1.1307729	5273
.507	.6603028	.6022998	.5290015	11310	.1313013	5284
.508	.6619639	.6016978	.5301331	11316	.1318309	5296
.509	.6636267	.6010964	.5312652	11321	.1323616	5307
.510	.6652912	.6004956	.5323978	11326	.1328934	5318
0.511	1.6669573	0.5998954	0.5335310	11332	1 1334264	5330
.512	.6686251	.5992958	.5346647	11337	.1339604	5341
.513	.6702946	.5986968	.5357989	11342	.1344957	5352
514	.6719657	.5980984	5369337	11348	.1350320	5364
.515	.6736385	.5975006	.5380690	11353	.1355695	5375
0.516	1.6753130	0.5969034	0.5392048	11358	1.1361082	5386
.517	.6769891	.5963068	.5403412	11364	.1366480	5398
.518	.6786670	.5957108	.5414781	11369	.1371889	5409
.519	.6803465	.5951154	.5426155	11375	.1377309	5420
.520	.6820276	.5945205	.5437536	11380	.1382741	5432
0.521	1.6837105	0.5939263	0.5448921	11385	1.1388184	5443
.522	.6853951	.5933327	.5460312	11391	.1393639	5455
.523	.6870813	.5927397	.5471708	11396	.1399105	5466
.524	.6887692	.5921472	.5483110	11402	.1404582	5477
.525	.6904588	.5915554	.5494517	11407	.1410071	5489

TABLE 51.—CONVERSION OF DEGREES, MINUTES, AND SECONDS

Degrees to minutes and seconds						Minutes to degrees		Seconds to degrees	
° =	′	″	° =	′	″	′ =	°	″ =	°
0.001	00	03.6	0.006	00	21.6	01	0.01667	01	0 00028
.002	00	07.2	.007	00	25.2	02	.03333	02	.00056
.003	00	10.8	.008	00	28.8	03	.05000	03	.00083
.004	00	14.4	.009	00	32.4	04	.06667	04	.00111
.005	00	18.0	.010	00	36.0	05	.08333	05	.00139
0.01	00	36	0.51	30	36	06	0.1000	06	0.00167
.02	01	12	.52	31	12	07	.1167	07	.00194
.03	01	48	.53	31	48	08	.1333	08	.00222
.04	02	24	.54	32	24	09	.1500	09	.00250
.05	03	00	.55	33	00	10	.1667	10	.00278
0.06	03	36	0.56	33	36	11	0.1833	11	0.00306
.07	04	12	.57	34	12	12	.2000	12	.00333
08	04	48	.58	34	48	13	.2167	13	.00361
.09	05	24	.59	35	24	14	.2333	14	.00389
.10	06	00	.60	36	00	15	2500	15	.00417
0.11	06	36	0.61	36	36	16	0 2667	16	0.00444
.12	07	12	.62	37	12	17	.2833	17	.00472
.13	07	48	.63	37	48	18	.3000	18	.00500
.14	08	24	.64	38	24	19	.3167	19	.00528
.15	09	00	.65	39	00	20	.3333	20	.00556
0.16	09	36	0.66	39	36	21	0.3500	21	0.00583
.17	10	12	.67	40	12	22	3667	22	.00611
.18	10	48	.68	40	48	23	.3833	23	.00639
.19	11	24	.69	41	24	24	.4000	24	.00667
.20	12	00	.70	42	00	25	.4167	25	.00694
0.21	12	36	0.71	42	36	26	0.4233	26	0.00722
.22	13	12	.72	43	12	27	.4500	27	.00750
.23	13	48	.73	43	48	28	.4667	28	.00778
.24	14	24	.74	44	24	29	.4833	29	.00806
.25	15	00	.75	45	00	30	.5000	30	.00833

TABLE 51.—CONVERSION OF DEGREES, MINUTES, AND SECONDS.—(*Continued*)

Degrees to minutes and seconds						Minutes to degrees		Seconds to degrees	
° =	′	″	° =	′	″	′ =	°	″ =	°
0.26	15	36	0.76	45	36	31	0.5167	31	0.00861
.27	16	12	.77	46	12	32	.5333	32	.00889
.28	16	48	.78	46	48	33	.5500	33	.00917
.29	17	24	.79	47	24	34	.5667	34	.00944
.30	18	00	.80	48	00	35	.5833	35	.00972
0.31	18	36	0.81	48	36	36	0.6000	36	0.0100
.32	19	12	.82	49	12	37	.6167	37	.0103
.33	19	48	.83	49	48	38	.6333	38	.0106
.34	20	24	.84	50	24	39	.6500	39	.0108
.35	21	00	.85	51	00	40	.6667	40	.0111
0.36	21	36	0.86	51	36	41	0.6833	41	0.0114
.37	22	12	.87	52	12	42	.7000	42	.0117
.38	22	48	.88	52	48	43	.7167	43	.0119
.39	23	24	.89	53	24	44	.7333	44	.0122
.40	24	00	.90	54	00	45	.7500	45	.0125
0.41	24	36	0.91	54	36	46	0.7667	46	0.0128
.42	25	12	.92	55	12	47	.7833	47	.0131
.43	25	48	.93	55	48	48	.8000	48	.0133
.44	26	24	.94	56	24	49	.8167	49	.0136
.45	27	00	.95	57	00	50	.8333	50	.0139
0.46	27	36	0.96	57	36	51	0.8500	51	0.0142
.47	28	12	.97	58	12	52	.8667	52	.0144
.48	28	48	.98	58	48	53	.8833	53	.0147
.49	29	24	.99	59	24	54	.9000	54	.0150
.50	30	00	1.00	60	00	55	.9167	55	.0153
						56	0.9333	56	0.0156
						57	.9500	57	.0158
						58	.9667	58	.0161
						59	.9833	59	.0164
						60	1.0000	60	.0167

Examples

0.005° = 0°00′18″. 0.64° = 0°38′24″.
0°14′00″ = 0.2333°. 0°00′36″ = 0.01°.

TABLE 52.—SUBDIVISION OF A RADIAN

Radians
to
degrees, minutes, and seconds

Radians =	°	′	″ =	Degrees	Radians =	°	′	″	Degrees
0.0001	0	0	21	0.00573	0.080	4	35	1	4.58366
.0002	0	0	41	.01146	.090	5	9	24	5.15662
.0003	0	1	2	.01719	.100	5	43	46	5.72958
.0004	0	1	23	.02292	.200	11	27	33	11.45916
.0005	0	1	43	.02865	300	17	11	19	17.18873
0.0006	0	2	4	0.03438	0.400	22	55	6	22.91831
.0007	0	2	24	.04011	.500	28	38	52	28.64789
.0008	0	2	45	.04584	.600	34	22	29	34 37747
.0009	0	3	6	.05157	.700	40	6	25	40.10705
.0010	0	3	26	05730	.800	45	50	12	45.83662
0.002	0	6	53	0.11459	0.9	51	33	58	51.56620
.003	0	10	19	.17189	1.0	57	17	45	57.29578
.004	0	13	45	.22918	2.0	114	35	30	114.59156
.005	0	17	11	.28648	3.0	171	53	14	171.88734
.006	0	20	38	.34378	4.0	229	10	59	229.18312
0.007	0	24	4	0.40107	5.0	286	28	44	286.47890
.008	0	27	30	.45837	6.0	343	46	29	343.77468
.009	0	30	56	.51566	6.2831853				360.00000
.010	0	34	23	.57296					
.020	1	8	45	1.14592					
0.030	1	43	8	1.71887					
.040	2	17	31	2.29183					
.050	2	51	53	2.86479					
.060	3	26	16	3.43775					
.070	4	0	39	4.01071					

Example. Let it be required to convert the angle of 2.0956 radians to degrees with minutes and seconds expressed in decimal form.

Radians	°	′	″	Degrees
2.0000	114	35	30	114.59156
.0900	5	9	24	5.15662
.0050	0	17	11	.28648
.0006	0	2	4	.03438
2.0956	120°	04′	09″	120.06904

INDEX

A

- Anchor setting template, 220
- Angle construction chart, 168
  - use of, 167
- Angles in line, 99
  - stresses due to, 101
  - transverse load due to, 101
- Annual I^2R losses, 12
  - method of determination, 12
- Approximate critical kva at various line voltages, 252
- Auxiliary constants, 65–68
  - convergent series method of solution, 67–68
  - hyperbolic method of solution, 272–273
  - for 125 mile line, 68, 230
  - for 250 mile line, 248, 272–273

B

- Blasting rock holes, 199
- Building towers straight up, 225–226

C

- Calculations for, angle $\tanh^{-1} (\theta_1 + j\theta_2)$, 265–266
  - annual cost of conductor, 50
  - charging current, 60, 70, 248
  - complex hyperbolic angles (position angles) θ_{po}, 253, 266
  - corona loss, 269–270
    - effect of conductor surface, weather and temperature, 267–268, 271
  - critical load performance, 245–248
  - crossarm loading, 95–99, 107
  - disruptive critical voltage, 268–270
- Calculations for, disruptive critical voltage, effect of spacing and configuration, 269–270
  - equivalent hours, 50
  - heating factor, H_f, 49
  - inductive reactance, 19, 34
  - insulator swing, 170–171
  - load impedance, 265
  - long transmission lines, 62–65
  - mean annual current, 50
  - medium length lines, 58–62
  - ohmic resistance, 13
  - performance at intermediate loads, 265–267
  - reactance of steel conductors, 34
  - sag in a 500-ft level span, 111–119
  - sag in 2,000-ft level span, 120–135
  - sag in an unsymmetrical span, 142–145
  - short lines, performance of, 35
  - sinh and cosh $(\theta_1 + j\theta_2)$, 254–255, 266
  - stresses in poles, 93–94
  - surge impedance, 245
  - table values, 37, 274
  - tension in guys, 102–104
  - vertical and horizontal conductor loading, 87
  - voltage and current at zero load, 232, 256
  - voltage drop, 38, 41–43, 60, 231–232
  - voltage rise in unloaded line, 256–258
- Clearing right of way, 190–194
  - factors effecting cost of, 193–194
- Clipping in conductors, 212, 217–218
- Comparisons of, electric propogation and velocity of light, 247
  - heavy and medium loading, 80

Comparisons of, length of cable, mean-length method and catenary, 139
  sag, and length, mean-length method and catenary, 138
    mean-length method and parabola, 136–138
  simplified and rigorous solutions, 59–61
  single-phase and three-phase circuits, 12–15
Complex algebraic equations, 63–64
Construction of, H-frame lines, 203–208
  performance diagrams, 61–62, 73, 233, 235–238, 249
  steel-tower lines, 219–227
  wood-pole lines, 189–208
Copper equivalent of a.c.s.r. and copperweld-copper, 9, 44
Corona defined, 267
Critical load defined, 240
Crossarms, double, dead ending on, effect of spacing, strengthening, 99
  factor of safety, 98–99
  fiber stress, 98–99
Current density of a.c.s.r., effect of, 44–45
Curves of, cable lengths, 139
  catenary, 124
  circular mils per ampere, 51–54
  corona loss, 270
  correct for load and voltage drop, 38–45
  correct for ratio of *D'/D*, 27
  daily load, 47
  parabola, 123
  resistance correction factors, 22
  sag in unsymmetrical spans, 141–143
  sag template, 160–161
  sag tension and stretch, 114, 133, 136
  stress and strain, 215
  stresses due to angles, 99–102
  stringing sag and tension, 118
Curves of, voltage and current, 258
  wind velocity, 87

D

Danger trees, definition of, 192
Design of H-frame lines, 104–107
Difference in new and old safety codes, 78–79
Digging pole holes, 197–199
Distributing line materials, 196–197

E

Early stages of line construction, 189
Economics of power interchange, 2
Effect of, corona formation, 267–272
  dropping the load, 252–258
Elastic and non-elastic properties of conductors, 212–213
Electric propogation, 247
Emergency load, 236
Energy losses, 11–12
Equations of, the catenary, 124, 139
  critical kva, 242–243
  Kelvin's law, 52–53
  the parabola, 111–113, 124, 126, 140
    modified, 126–128, 131, 140
  the transmission circuit, 69, 230
Equipment required for various operations, 191, 205, 208, 217
Equivalent circuit diagrams, 64, 244
Equivalent line length at 60 cycles, 10–11
Erecting poles, 200
  men required for, 201
  tamping, 200
Erecting wood pole H-frames, 203–208
  falling gin method, 203–208
  pole derrick method, 208
Estimating procedure, 5
Explanation of performance diagrams, 235–236
Exponential and hyperbolic functions, 273–274

F

Frequency, 10–11
Functions of a transmission system, 1

G

GMR (geometrical mean radius), 34

H

Hauling and handling poles, 196
Heating factor, 49, 51
H-frame lines, 104–107
  crossarm loading, 107–108
  trend in design, 108–109
Hysteresis, or iron loss, 22

I

Impedance diagram, 7
  linear, 67, 230, 244–245
  load, 265
  surge, 245
Increasing line capacity, 8
Inductive reactance, 7, 19, 34
Installing tower footings, 220–225
Instrument survey, 157
Interconnection of transmission systems, 2
Interpolating table values, 254

K

Kelvin's law equations, 52–53

L

Leading and lagging components of the line, 250–251
Leading and lagging condenser capacity, 231, 236
Line-to-neutral method of comparison, 13–15
Line supplying load at intermediate points, 40–43, 46
Line construction, discussion of, 194–196
Line location and survey, 146
Linear constants, 67, 230, 244
Load carrying capacity of transmission circuit, 35
Loading district map, 77
Long transmission lines, 62
Loss factor, 12
Losses as effected by power factor, 55–56

M

Mathematical solution for intermediate loads (hyperbolic theory), 264–267
Measuring distances, by chaining, 149–151
  by stadia, 151–154
Mechanical design of transmission lines, basis of, 76
Medium length lines, 58
Men required for various operations, 190, 196, 208, 216–217, 225
Meteorological data, 171
Methods, employed in line construction, 190–227
  for determining economical conductor, 12, 48–51
  for facilitating sag calculations, 127
Most efficient receiver power factor, 239
Multiple layer a.c.s.r., 9, 23, 28

N

National Electrical Safety Code, revised, 77–79
New method for calculating sag in long spans, 120–141
Nominal receiver voltages, 4

O

Operation of vectors in complex form, 259–264
  discussion of, 63–64

P

Performance diagrams, 61, 72, 234, 249
Permissible voltage drop, 8
Polar diagram of critical load transmission, 251
Poles, wood, 88–90
  breaking load of, 91
  fiber stress of, 91
Position angles, (complex hyperbolic) θ_{p0}, 254–255
Power transfer, 2
Preliminary survey, 155
Prestretching conductor, 214–215
Properties of unsymmetrical spans, 145
Property information, 157–158

R

Receiving end and line angles, θ_r and θ, 266
Reel mounting and braking, 210–212
Relative strength of single pole and H-frame, 106
Resistance of conductors, basis of, 18–19
  correction factors for single layer a.c.s.r., 22, 27
Running levels, 154–155

S

Sag template, its construction and use, 159–161
Sagging and clipping in, 212–215, 226
  checking with targets, 216
  clipping in ladder, 217, 226
  tension dynamometer, 216
Selection of conductor, 7
Sending-end current locus, 238
Setting poles with dynamite, 201–203
  size of charge, 202
Short-cut methods of calculating voltage drop, 37–45
Short lines, 35
Side-hill elevations, 154
Skin effect, 16, 18
Spans, horizontal, 93, 170
  vertical, 107
Spotting structures, 162–163, 165
Steel as transmission line conductor, 32–34
Steel towers, 177–178
Stress-strain diagram, 215
Stringing conductor, 209–212
  damage to conductor, 211–212
  pin type lines, 209–210
  reel brake, 210–211
  stringing blocks, 212
  suspension type lines, 210–212
Surveying instruments and equipment, 148–150
  station equalities, 156–157

T

Tables, annual cost of conductors, 50
  auxiliary constants, 248
  comparative sag and cable lengths, 137–138
  complex hyperbolic angles and functions, 255
  conductors, ratio strength to weight, 11
  conversion of degrees, minutes and seconds, 290–291
  exponential and hyperbolic functions, 275–288
  pole dimensions, 88–91
  power loss as effected by power factor, 56
  power and power loss, 71
  properties, of a.c.s.r., 23–26, 83–82
    of copper wires and cables, 17, 20–21, 80–81
    of copperweld-copper, 29–31, 84–85
    of steel conductors, 33, 86
  receiver voltages, 5
  relative weights of conductors, single and three-phase, 15
  subdivision of a radian, 292

Tables, tension, temperature and lengths, 116, 117, 129, 137–138
  voltage and current distribution, 71, 232–233, 246–247, 257
  voltage-drop constants, 36
Tension in guys, guying installations, 102–104
Theory, of the catenary curve, 124
  of the parabola curve, 123
Tower footings, 220–225
Tower templates and their use, 187–188
Transposition, of line conductors, 167
Transmission of power at critical load, 240–252
Transmission line devices, etc., 159–188
Transmission line performance with phase control, 228–239
Types of conductors, 8–10
  determining factors, 8–10

U

Unit constants and computed values, 244–245
Unsymmetrical spans, 141–145

V

Vector operations, 259
  addition, 260
  division, 262–263
  multiplication, 261–262
  powers, 263–264
  roots, 263
  subtraction, 261
Vector representation of critical load transmission, 247
Voltage-drop calculations, basis of, 35
Voltage-drop correction, 60
Voltage-drop and performance calculations, 59–61
Voltage rise, 75
Voltage standardization, 3

W

Water holes, 224–225
Wave length of line, 245
Wind velocity, 85–87
Wind and ice loads, 76, 80, 81, 83, 85, 86, 87–88, 111, 115, 125